UNIVERSITY OF SASKATCHEWAN

Department of Physics and Engineering Physics

Phys 223.3 Mechanics I

Final Examination

Instructor: Yansun Yao

April 16th, 2018 Time: 9:00 AM ~ 12:00 PM

ANSWER ALL FIVE QUESTIONS.

FULL MARK IS 100.

MARKS PER EACH QUESTION ARE INDICATED.

WRITE YOUR ANSWERS IN THE EXAM BOOKLETS.

Q1. PROJECTILE

A rigid ball of mass m is thrown at an initial speed v_0 and a 45° angle above the ground toward a wall located at a distance d away. The ball hits the wall at the height h and bounces off, before it hits the ground at a distance R from the wall. The magnitude of the free-fall acceleration is g. Assume that the collision of the ball with the wall is **perfectly elastic and ignore air resistance.**

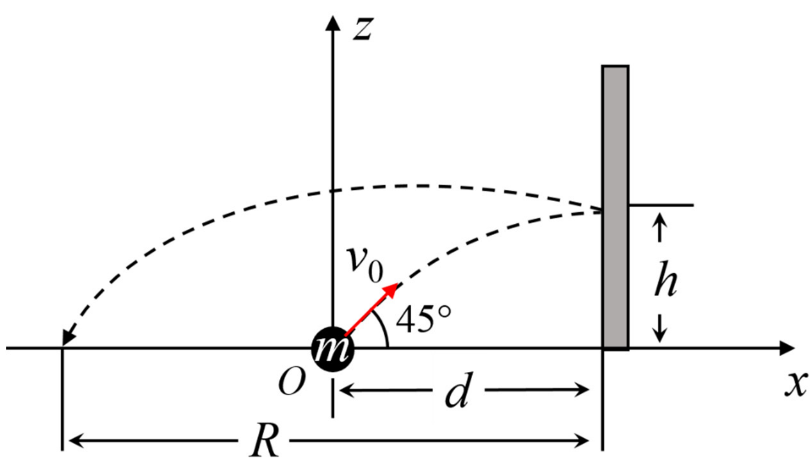
(a) **(4 marks)** Find the height h.

(b) **(6 marks)** Find the velocity of the ball at the instant it hits the wall. Write the result in component form (x- and z-values).

(c) **(10 marks)** Show that the distance R is,

$$R = \frac{v_0^2}{g} - d$$

Hint for (c): you may move the origin of the coordinates to the point of collision and reverse the x-direction.



Q2. CENTRAL FORCE

A particle of mass m is subject to a restoring force and executes two-dimensional isotropic harmonic oscillations. The time-dependent position of the particle is described by (in Cartesian coordinates),

$$x = A\cos(\omega t)$$
$$y = A\sin(\omega t)$$

where ω is the angular frequency and A is the amplitude of the oscillation.

(a) **(6 marks)** Find the restoring force $F(r)$ in plane polar coordinates.

(b) **(6 marks)** Prove that the oscillator's angular momentum **with respect to the force center** remains constant at all time,

$$\frac{d\mathbf{L}}{dt} = 0$$

(c) (**8 marks**) Find the magnitude of the angular momentum L of the oscillator **with respect to the force center**.

Q3. ORBITS IN CENTRAL FORCE FIELD

The motion of an object around the Sun is dictated by the gravitational pull from the Sun,

$$F(r) = -G\frac{Mm}{r^2}.$$

Here M and m are the masses of the Sun and the object, respectively; r is the distance between the Sun and the object, and G is the gravitational constant.

(a) (8 marks) Assume that the Earth has **a circle orbit** of the radius R (due to a very small eccentricity). Show that the speed of the Earth in the orbit is,

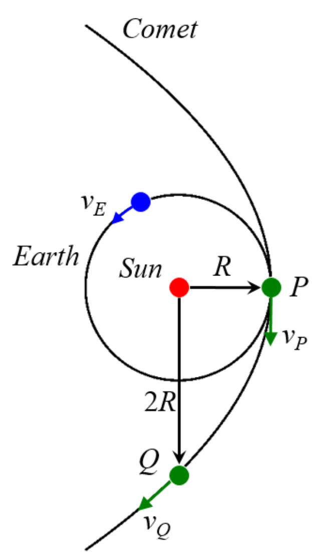
$$v_E = \sqrt{\frac{GM}{R}}$$

(b) (8 marks) A comet moves in **a parabolic orbit** in the same plane as the Earth's orbit. The comet's orbit is tangential to the Earth's orbit at the point P. Show that the speed of the comet when it passes the point P is,

$$v_P = \sqrt{\frac{2GM}{R}}$$

(c) (8 marks) Show that the speed of this comet when it passes the point Q which is at a distance $2R$ from the Sun is,

$$v_Q = \sqrt{\frac{GM}{R}}$$



Q4. ROCKET

Consider a one-stage rocket shooting straight up from ground from rest. The mass of the rocket is m_0 at launching and $m_0/4$ after the fuel is burned out. Assume a constant free-fall acceleration **g**, a constant rate of change in the mass of the rocket $dm/dt = -k$, and a constant fuel exhaust velocity **u** **with respect to the rocket**. Ignore air resistance.

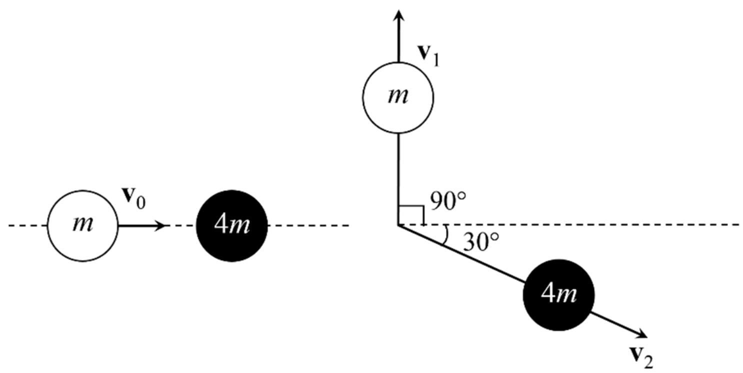
(a) (10 marks) Find the speed of the rocket at the end of the burn.
(b) (10 marks) Find the altitude of the rocket at the end of the burn.

You may use these integrals: $\int \frac{1}{x}dx = \ln|x| + C$, $\int \ln x dx = x \ln x - x + C$

Q5. COLLISION

A particle of mass m with the speed v_0 strikes a particle of mass $4m$ at rest. After collision the particle of mass m is scattered at an angle of 90° above the incident direction while the particle of mass $4m$ proceeds at an angle 30° below the incident direction, see figure.

(a) (8 marks) Find the speeds v_1 and v_2 of the two particles after collision.
(b) (8 marks) Find the disintegration energy Q for this collision.



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