UNIVERSITY OF SASKATCHEWAN

Department of Physics and Engineering Physics

Phys 223.3 Mechanics I

Final Examination

Instructor: Yansun Yao

April 8th, 2019 Time: 2:00 PM ~ 5:00 PM

ANSWER ALL FIVE QUESTIONS.

FULL MARK IS 100.

MARKS PER EACH QUESTION ARE INDICATED.

WRITE YOUR ANSWERS IN THE EXAM BOOKLETS.

Q1. CONSERVATIVE FORCE. Two forces are given in Cartesian coordinates:

$$\mathbf{F}_1 = \hat{\mathbf{i}}y - \hat{\mathbf{j}}x + \hat{\mathbf{k}}z^2$$
$$\mathbf{F}_2 = \hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z^3$$

(a) (**6 marks**) Determine whether $\mathbf{F}_1$ is a conservative force.

(b) (**6 marks**) Determine whether $\mathbf{F}_2$ is a conservative force.

(c) (**8 marks**) Find the potential energy $V(x, y, z)$ for the force(s) that are conservative, if any. Set the zero point at the origin (0, 0, 0), e.g., $V(0, 0, 0) = 0$.

Q2. PROJECTILE. A cannon is capable of firing shell at constant muzzle speed v_0. One shell is fired at an elevation angle of 30° above horizon and it hits a target on the ground that is a distant R ahead of the canon. Ignore air resistance and assume constant g.

(a) (**8 marks**) Find the expression for R.

(b) (**8 marks**) The second shell is fired at a different elevation angle but it hits the same target. Find the elevation angle at which the second shell is fired.

Q3. 2D HARMONIC OSCILLATOR. A particle of mass m executes two dimensional harmonic oscillations under a restoring force **F**. The position of the particle is described by,

$$x = A\cos(\omega t)$$
$$y = A\sin(2\omega t)$$

Here ω and A are positive constants.

(a) (**6 marks**) Determine the restoring force **F** in Cartesian coordinates.

(b) (**12 marks**) Determine which of the following quantity, or quantities, will not change with time.

(1) Total energy E (2) Angular momentum L (3) Speed v

Show necessary work to establish your conclusion.

(c) (**6 marks**) Determine the time interval t at which the particle passes the same point in the trajectory with the same velocity.

Q4. GRAVITIONAL FORCE FIELD. A satellite of mass m and angular momentum L moves around the Earth under the gravitational force,

$$F(r) = \frac{K}{r^2}.$$

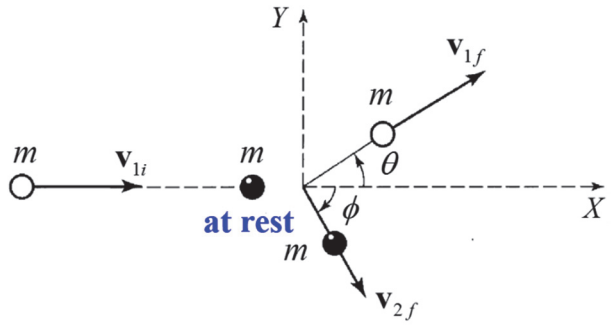
Here r is the distance between the satellite and Earth's center, and K is a negative constant. Assume the zero point of $V(r)$ is at infinity.

(a) (**8 marks**) Find all extrema in the effective potential $V_{\text{eff}}(r)$ and sketch the effective potential.

(b) (**8 marks**) The satellite is initially in **a circular orbit**. Find the radius of the orbit (r_0) and total energy of the satellite (E_0).

(c) (**8 marks**) The satellite is then sent to **an elliptical orbit** by firing an on-board thruster along the radial direction. In the new orbit, the total energy of the satellite is $E_0/2$. Find the distances of the closest and farthest approaches (r_{min} and r_{max}) of the satellite to Earth's center.

Q5. ELASTIC COLLISION. Particle 1 of mass m and velocity $\mathbf{v}_{1i}$ makes an elastic collision with particle 2 of equal mass that is stationary. After the collision, particle 1 is deflected through an angle θ above the incident direction with the velocity $\mathbf{v}_{1f}$. Particle 2 recoils at an angle ϕ below the incident direction with the velocity $\mathbf{v}_{1f}$. See figure.



(a) (8 marks) Show that the recoil angle ϕ is given by,

$$\tan\phi = \frac{v_{1f}\sin\theta}{v_{1i} - v_{1f}\cos\theta}.$$

(b) (8 marks) Show that the speed of particle 1 after the collision is given by,

$$v_{1f} = v_{1i}\cos\theta.$$

********** END OF EXAM **********