UNIVERSITY OF SASKATCHEWAN

Department of Physics and Engineering Physics

PHYS 223.3 Midterm Examination

Instructor: Yansun Yao

Feb 15th, 2019 Time: 7:00 PM ~ 9:00 PM

ANSWER ALL FIVE QUESTIONS.

FULL MARK IS 100.

MARKS PER EACH QUESITON ARE INDICATED.

1. 1D Time Dependent Force. A particle of mass m initially resting on a frictionless surface is subject, beginning at $t = 0$, to a force,

$$F(t) = F_0 \cos^2(\omega t).$$

(a) Prove the following identity for any angle α,

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 \qquad \textbf{(10 marks)}$$

(b) Show that the velocity of the particle for all time is,

$$v(t) = \frac{F_0}{2m}t + \frac{F_0}{4m\omega}\sin(2\omega t) \qquad \textbf{(10 marks)}$$

2. 1D Velocity Dependent Force. A metal block of mass m slides on a horizontal surface along positive direction ($v > 0$). The surface has been lubricated with a heavy oil so that the block suffers a resistance that varies as the 3/2 power of the velocity:

$$F(v) = -cv^{3/2},$$

where c is a positive constant. If the initial velocity of the block is v_0 at $x = 0$, show that the block cannot travel farther than

$$\frac{2m\sqrt{v_0}}{c}.$$ **(15 marks)**

3. 1D Position Dependent Force. A particle of mass m moves in a motion with the velocity v varying with position x according to the equation,

$$v(x) = \frac{k}{x^2},$$

where k is a positive constant.

(a) Find the potential energy $V(x)$. Set the zero point at $x = \infty$. (**10 marks**)

(b) Show that the total energy E of the particle is always zero. (**10 marks**)

(c) Find the position of the particle as a function of time, $x(t)$. Assume initial condition $x = 0$ when $t = 0$. (**10 marks**)

4. Simple Harmonic Motion. A particle of mass m undergoes a simple harmonic motion with natural angular frequency ω_0 and amplitude A. The maximum acceleration of the particle is $|a_{max}|$ and the maximum speed is $|v_{max}|$.

(a) Show that the amplitude A is described by,

$$A = \frac{|v_{max}|^2}{|a_{max}|}.$$ **(10 marks)**

(b) Show that the period T of oscillation is described by,

$$T = 2\pi \frac{|v_{max}|}{|a_{max}|}.$$ **(10 marks)**

5. Driven Harmonic Oscillator. An undamped harmonic oscillator ($b = 0$) of mass m and natural angular frequency ω_0 is subject to an driving force,

$$F_d(t) = F_0 \cos(2\omega_0 t) .$$

At $t = 0$, the oscillator is at rest at the equilibrium point ($x = 0$). Show that the position of the oscillator is given by,

$$x(t) = \frac{F_0}{3m\omega_0^2}\left[\cos(\omega_0 t) - \cos(2\omega_0 t)\right]. \qquad \textbf{(15 marks)}$$

********** **END OF EXAM** **********