On Using a Time Variable Infiltration with the Israelson Border Irrigation Equation

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It is shown why a time variable infiltration equation cannot be used with the Israelson differential equation for flow down an irrigation border.

1. Introduction

Israelson's differential equation for flow down an irrigation border is

\[ q \frac{dt}{dt} = I \times \frac{dx}{dt} + k \times dx \]  

where \( q \) = flow rate per unit border width

\( t \) = time

\( x \) = distance down the border

\( h \) = average depth of water on the surface

\( I \) = infiltration rate.

Originally this equation was proposed for a constant infiltration rate, \( I \); however, the solution did not generally fit experimental data. Singh\(^2\) attempted to overcome this difficulty by using a time variable infiltration rate of the form

\[ I = ce^{-t} \]  

where \( c \) is a constant. This equation is known to be valid for horizontal infiltration and for vertical infiltration into fine textured soils.

The solution of Eqn (1), subject to Eqn (2), is

\[ x = \frac{q}{c} e^t - \frac{qh}{2c^2} \]  

Eqn (3) is a straight line when \( x \) is plotted versus \( t^{1/2} \). Example data used by Singh\(^2\) fitted a plot of this type when data near \( t \) equal to zero was ignored. Consequently, it appeared that \( c \) and \( h \) could be determined from the slope and intercept of the best fit line. However, Singh\(^2\) found that the \( c \) value obtained from infiltrometer tests had to be "corrected" to match the \( c \) value obtained from his solution. The following discussion will clarify why this would be expected.

2. Discussion

Eqn (1), as mentioned, was originally proposed for a constant infiltration rate. This equation is quite valid for this assumption and the assumptions of constant \( q \) and \( h \). However, this equation cannot be used for a constant infiltration rate, as Singh\(^2\) has done, because this implies that the infiltration rate over the entire length of the border varies with time when time is measured from the commencement of irrigation at the head end of the border. Or in other words, at any given time the infiltration rate over the entire border is a constant, which means that the infiltration rate decreases at a point before water even reaches the point. Consequently, this solution greatly over estimates the distance \( x \) for a given time \( t \) when specific values of \( c \) and \( h \) are used. Conversely, when the field data is used to determine \( c \) and \( h \), \( c \) is over-estimated.

Philip and Farrell\(^3\) obtained an exact solution to the infiltration-advance problem that Singh\(^2\) set out to solve, namely the problem of advance down a border assuming a constant inflow at

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the head end, a constant depth of flow, and an infiltration equation of the form given in Eqn (2). Their solution was:

\[
x = \frac{2t^{1/2}}{q} = \frac{h}{\pi c} \left[ \text{erfc} \left( \frac{\pi c t^{1/2}}{h} \right) - 1 \right]
\]

Philip and Farrell\(^4\) also obtained solutions for the same problem using other commonly used infiltration equations. One of these solutions was for the infiltration equation

\[
I = ct^{-1/2} + A
\]

where \(A\) is a constant.

Norum\(^4\) has developed a curve matching technique for the determination of the constants \(c\) and \(A\) using the Philip and Farrell exact solutions and field data. This method also includes the possibility of \(A\) equal to zero which gives the solution for values of \(c\) and \(h\) as Singh\(^2\) set out to do.

Figs 1, 2 and 3 are examples showing the data used by Singh\(^2\), Singh's solution, the exact solution using Singh's parameters, and the exact solution using the parameters determined by Norum's curve matching technique. The data in Figs 1 and 3 fit the exact solution when infiltration equations of the form given in Eqn (2) are used. However, the data in Fig. 2 does not fit this type of equation; consequently, an infiltration equation of the form given in Eqn (5) is used.

As expected, in each case Singh's solution over-estimates the value of \(c\). Data for Fig. 1 was originally taken from Criddle \etal\(^5\) who also gave ring infiltrometer data for the same field. The \(c\) value obtained from the infiltrometer data was 0.0322 ft min\(^{-1/2}\), while Singh's solution gave 0.0463 ft min\(^{-1/2}\) and the solution by Norum's method gave 0.0263 ft min\(^{-1/2}\).

REFERENCES

   Handbook 82, SCS, USDA, 1956

Fig. 1. Time-distance relationship for bare soil. Data from Criddle et al.\(^5\)
Fig. 2. Time-distance relationship for clover crop. Data from Israelson\(^1\)
Fig. 3. Time-distance relationship for alfalfa crop. Data from Israelson\(^1\)