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SMALL-SCALE SPATIAL STRUCTURE OF SHALLOW SNOWCOVERS

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ABSTRACT

The results of a field study of the small-scale spatial structure of the depth of shallow seasonal snowcovers in prairie and arctic environments are presented. It is shown that the spatial distribution of snow depth is fractal at small scales, becoming random at scales beyond some limiting length. This is due to the autocorrelation of depth at small sampling distances. The transition of fractal to random behaviour is indexed by a 'cutoff length', which is defined by the intersection of the 'fractal' slope and horizontal tangent of a logarithmic plot of the standard deviation of depth versus sampling distance.

The magnitude of the cutoff length is related to the degree of macroscopic variability of the underlying topography. An increase in length due to the effects of macroscopic topographic variability on snowcover accumulation is confirmed by de-trending field measurements. The de-trended data shown a cutoff length for wheat stubble and fallow surfaces of approximately 30 m, which is consistent with the distance determined from measurements on 'flat' fields.

The implications of the transition of snow depth from fractal to random structure on the scales of snow sampling and modelling are presented. The cutoff length may provide a statistic for stratifying shallow snowcovers, by linking snowcover properties to the underlying topography.

KEY WORDS shallow snowcover; spatial structure; scale; fractal; random

INTRODUCTION

The frequency characteristics of the depth and water equivalent of a snowcover are used in the measurement of these properties, the development of synthetic snowcovers, the stratification of landscapes for snow sampling, and the determination of the area depletion in snow-covered areas during ablation. At large scales, the spatial distributions of the depth and water equivalent of snowcovers within relatively homogeneous climatological regions of the Canadian Prairies are influenced by vegetation and topography. McKay (1970) and Steppuhn and Dyck (1974) found that stratifying a watershed according to terrain and vegetative variables, then sampling snowcovers of the same landscape class, reduced the coefficient of variation of the water equivalent. Stratification in snow sampling attempts to divide a snow field into smaller units that have common statistical properties. Also, the procedure ensures that all portions of the landscape are sampled. Small subregions within a landscape may contain large quantities of water (McKay, 1970, Wood and Marsh, 1978) and stratification prevents these areas from being omitted.

Stratification procedures for snow measurement require a method for dividing an area into individual landscape classes that have similar accumulation and retention properties. In the Canadian Prairies, classification of vegetation is fairly straightforward as there are relatively few vegetation types. Classification of topography is more difficult because: (a) objective measurements of topography are difficult to obtain and (b) it is necessary to link topographic features to snow accumulation. Steppuhn (1976) developed a landscape classification system for snowcover that consisted of nine landform and six land use classes. A primary weakness of the system is that the criteria used to designate the various classes are subjective. Shook (1995) demonstrated that the classes suggested by Steppuhn (1976) did

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t group seasonal snowcovers in a consistent manner. Snowcover stratification cannot be completely ccessful without a method of quantifying topographic features by their snow accumulation and reten-n characteristics.

Accurate predictions of snowmelt runoff from a watershed require information on the temporal variation snow-covered area during ablation. The extent of the gross area of a watershed that is snow covered ects runoff primarily in two ways: (a) it influences the melt rate and (b) it governs the contributing a of runoff. The highest rate of meltwater production on a watershed is most likely to occur when product of the average melt rate and snow-covered area is a maximum.

Many hydrologists (e.g. Dunne and Leopold, 1978; Ferugson, 1984; Martinec, 1985; Buttle and McDon-ll, 1987; Shook *et al.*, 1993) have demonstrated that the frequency distribution of the water equivalent in nowcover strongly influences the areal depletion of snowcover during ablation. In addition, Shook and ay (1994) showed that it is the semi-fractal distribution of the water equivalent within a shallow, contin-us snowcover that is responsible for the formation of the fractal patches of soil and snow that develop ring ablation. The geometries of patches influence the contributing area of snowmelt and runoff and, ing to the small-scale advection of energy from bare ground to the remaining snowcover, the rate of lting. Because patchy snowcover has a two-dimensional structure, an ablation model of these snow-vers must be at least two-dimensional. Therefore, it is necessary to measure and to model the fractal spa-l distribution of water equivalent properly in order to predict ablation successfully.

This paper investigates the spatial distributions of depth of shallow snowcovers in open, exposed, low-ief terrain in prairie and arctic environments. Special attention is given to the change of snow depth m fractal to random behaviour. A procedure is presented for defining a 'cutoff length', an index of distance in which the transition occurs, from field data. It is shown that the cutoff length, although inde-ndent of vegetation, varies with topography. The importance of these results to snow measurement and delling is stressed.

FRACTAL NATURE OF SHALLOW SEASONAL SNOWCOVERS

operties of fractals

Fractal geometry has been used to categorize and measure irregular shapes, such as those found in nature andelbrot, 1983). Fractal shapes are distinguished by two related properties: self-similarity and lack of a aracteristic scale. A self-similar object looks the same at any magnification. Any degree of magnification l reveal more detail—the shape never becomes a conventional line, curve or surface. Lacking a charac-istic scale, a fractal object has no obvious size and its size can only be determined by imposing an exter-l scale.

Fractal objects are of two types and, in the absence of standardized names, they are designated herein as act and statistical. Exact fractals are generated mathematically by recursively applying simple geometric es and include the Koch curve and the Sierpinski gasket (Mandelbrot, 1983). Exact fractals show exact f-similarity, the magnification of any part exactly resembling the whole. They also lack any characteristic le, their self-similarity existing over an infinite range of scales.

Whereas exact fractals are a mathematical ideal, statistical fractals are objects that are generated by ran-m or chaotic processes (either natural or artificial) and whose mathematical properties resemble those of act fractals. Statistical fractals show statistical self-similarity, magnifications showing the same statistical gree of roughness as shown by the whole object. Because natural processes are generally subject to limit-scales, the objects created may only display fractal properties over a restricted range of scales.

The fundamental parameter characterizing a fractal object is D, the fractal dimension. Conventional clidean objects have dimensions of zero (points), one (lines) two (planes) or three (space). Fractal objects ve non-integer dimensions. The magnitude of D is an index of the relative roughness and irregularity of object.

The value of D, for statistical fractals, is established from scaling behaviour. There are many ways of asuring scaling and, consequently, many ways of estimating D. One useful fractal dimension is D_p,

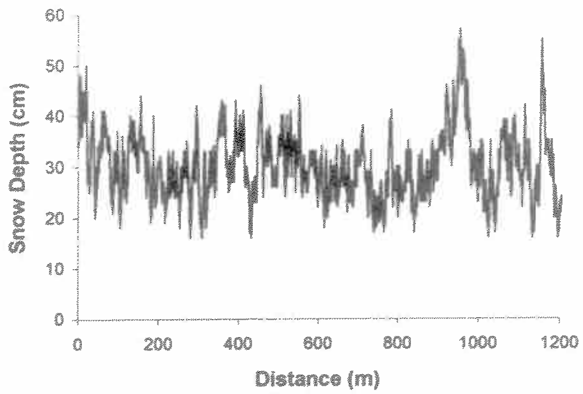


Figure 1. Snow depth transect. Stubble field, Kernen Farm, SK, 28 February 1994

the fractal dimension of the perimeter–area relationship of a group of fractal objects. D_p is calculated from the slope of the best-fit line on a plot of log(perimeter) against log(area).

FREQUENCY CHARACTERISTICS OF SNOW DEPTH

The following analyses concern field measurements of snow depth, although it is recognized that hydrologists are often most concerned with the water equivalent of a snowcover. There are two reasons for this approach. Firstly, measurements of depth are easier, and, being less time-consuming to monitor, a larger set of data can be obtained. Secondly, shallow prairie snowcovers do not display a statistically significant relationship between depth and density. Therefore, snow depth and water equivalent are strongly correlated (Shook and Gray, 1994). The spatial variability in snow water equivalent is due primarily to the variability of down depth, i.e. variations in snow density are of minor importance.

Shook (1995) demonstrated that the distributions of depth and water equivalent of shallow seasonal snowcovers in the Canadian Prairies can be described by the two-parameter log-normal probability density function. It is possible, therefore, to approximate the frequency distribution of water equivalent in prairie snowcovers from the mean and standard deviation of snow depth and an estimate of the mean snow density.

Over characteristic distances less than 100 m (micro-scale), differences in accumulation patterns of snowcovers result from variation in air flow patterns and snow transport. In addition to atmospheric variables, the primary factors affecting snow accumulation are surface roughness and snow supply and erodability. On the prairies, snow depth and water equivalent monitored along a snow course exhibit high autocorrelation.

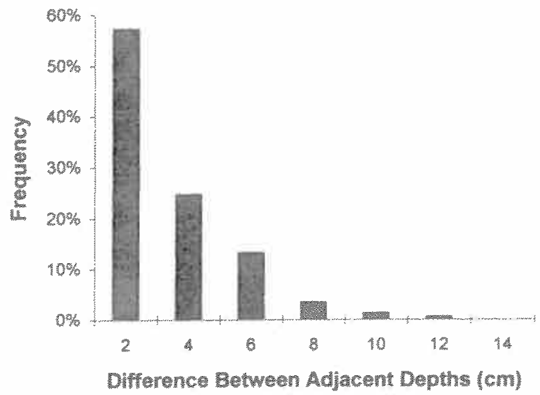


Figure 2. Histogram of differences between adjacent snow depths. Stubble field, Kernen Farm, SK, 28 February 1994

Table I. Snow depth measurements. Stubble field, Kernen Farm, 28 February 1994

Sampling direction	Measurement interval (m)	Samples
SE–NW	0·1	411
NE–SW	1	581
NW–SE	1	1204

;ure 1 shows a cross-section of snow depths measured at 1-m intervals on a relatively flat field of wheat stub-. Figure 2 plots the frequency distribution of the differences between snow depths at adjacent sampling ints in the transect. The large number of small differences in depth reflects high autocorrelation between acent measurements.

A fractal data series displays positive autocorrrelation; that is, the value at a point is similar to values at arby points. For an evenly sampled fractal series, positive autocorrelation causes the standard deviation to be a power function of sample size, (T) (Turcotte, 1992),

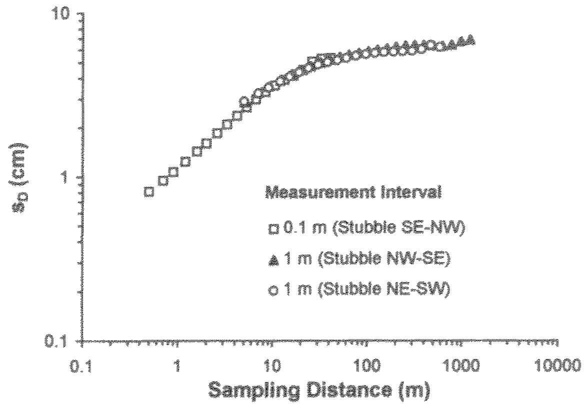
$$s \propto T^{\mathrm{H}}, \tag{1}$$

ere, H = Hausdorff measure (constant) = $2 - D$, in which D is the fractal dimension. H may be deter-ned from the slope of the best-fit line of a logarithmic plot of standard deviation versus sample size. :he data are randomly distributed, the standard deviation is independent of sample size, resulting in a rizontal plot of s vs T, and $H = 0$.

alysis of standard deviation

The application of Equation (1) to the spatial distribution of snow depth was tested using observations nitored along transects on a macroscopically flat stubble field at the Kernen Farm (Lat. 52° 8′ N, Long. 5° 30′ W), near Saskatoon, Saskatchewan, Canada. The depths were measured with a standard snow oth rod at intervals of approximately 10 cm and 1 m, along three transects in the field (see Table I). e precision of depth measurement was ±1 cm.

The measurements were sampled sequentially to determine an average value for the standard deviation of oth for a specific sampling distance. For example, for $T = 10$, which corresponds to a sampling distance 10 m with measurements taken at an interval of 1 m, the standard deviation of the first ten samples was culated. Then, the sampling point was moved along one position and the standard deviation of the next values was calculated. The process was repeated until all data were used. Then, the average standard



ıre 3. Variation in the standard deviation of snow depth with sampling distance. Stubble field, Kernen Farm, SK, 28 February, 1994

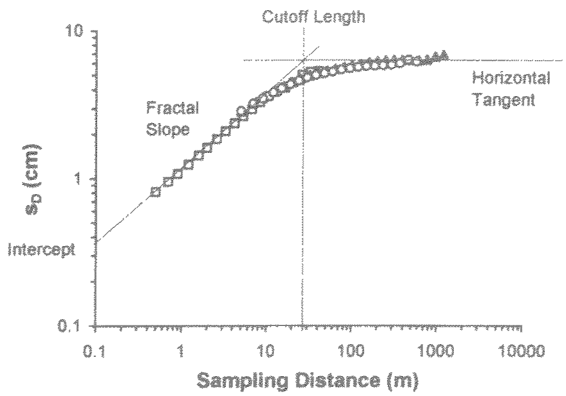


Figure 4. The transition between fractal and random structure as defined by the intersection of the fractal slope and the horizontal tangent to the points

deviation for the group was computed. For each data set, average standard deviations were calculated for T values between five and the total number of samples. Because the data were collected at different spacings (0·1 m and 1 m), the sampling distance, which is the product of T and measurement interval, is used in place of T in the following analyses.

The effect of sampling distance on the standard deviation of snow depth for the three data sets listed in Table I is shown in Figure 3. The plots are so similar as to suggest that the measurement interval and direction (the prevailing wind direction is approximately NW–SE) have no appreciable effect on the relationship. Therefore, the standard deviations for the three data sets were 'pooled' and plotted against sampling distance. The results of this procedure showed the standard deviation increasing as a power law with increasing sampling distance for distances less than 20 m ($H = 0{\cdot}47$, $r^2 = 0{\cdot}99$). At distances between 20 and 100 m the slope of the curve decreased progressively and for sampling distances greater than 100 m $H = 0{\cdot}062$, which is comparable to the value for a set of randomly sampled snow depths ($H = 0{\cdot}043$) measured by Shook (1995). The flattening of the curve suggests an upper limit (characteristic length) of the fractal distribution of depth between 20 and 100 m. When the sampling distance is greater than the characteristic length, the distribution becomes random.

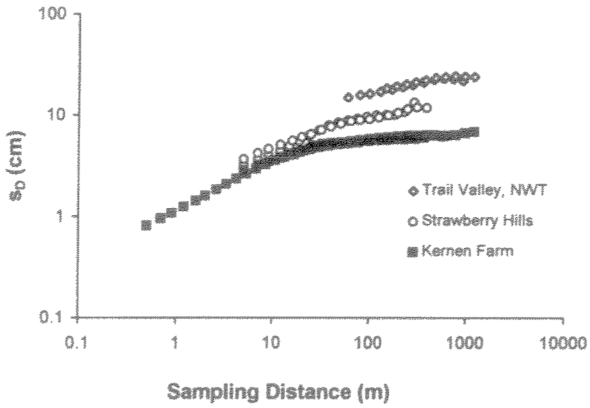
The stubble in the field was of uniform height and was completely filled with snow. Since there was no other structure capable of causing autocorrelation of snow depth over many metres of this flat field, it is hypothesized that the autocorrelation is caused primarily by the formation of dunes. The supply of blowing snow and the degree of surface roughness limit the maximum size of snow dunes on a flat field. Autocorrelation caused by a dune must terminate at scales greater than the length of the dune. The fact that snow dunes have a variety of lengths may account for the gradual transition of the spatial distribution from fractal to random structure.

Because the change from fractal to random behaviour is gradual, the point of intersection of the lines describing the respective distributions, is taken as the scaling or 'cutoff length' (see Figure 4). Although this point is arbitrary, it is an objective index of the scale of transition because it is derived from physically defined components. The 'fractal' slope is a function of the fractal dimension of snow depth. The horizontal tangent represents a physical property (random distribution) that the data *eventually* approach.

Cutoff length

The cutoff length is an index of the upper limit in the sampling distance for fractal structure. In the absence of major changes in vegetation, it is believed that this limit is established primarily by topography.

As expressed above, the variability in snow depth at small scales (shorter than the cutoff length) is believed to be due to the interactions between snow and small-scale surface properties. The largest snow features that can accumulate at these scales are snow dunes, whose maximum size is limited by the roughness of the surface. In cultivated areas of the prairies, the surface covers of fields in fallow and stubble



;ure 5. Variation in the standard deviation of snow depth with sampling distance. Data from Kernen Farm and Strawberry Hills, near Saskatoon, SK and Trail Valley Watershed, near Inuvik, NWT

: relatively uniform at scales ranging from the plant spacing (generally <20 cm) to the field length (*ca.* :m). On these land units, field-scale variation in surface roughness can only be due to similar-scale varia- ity in topography. Large-scale topographic features (hills, valleys) create drifts of snow, which are essen- lly large dunes, that extend the autocorrelation of depth to larger scales.

The influence of large-scale topography on the cutoff length is demonstrated in Figure 5. These plots were nstructed from measurements of snow depth made on various landforms at the Kernen Farm and the ·awberry Hills, near Saskatoon, SK and the Trail Valley Creek Watershed, near Inuvik, NWT. The :asurements at Trail Valley were taken at approximately 10-m intervals, in transects across a large valley. e vegetation was predominantly tundra, with scattered brush on the valley sides and heavier brush ır the creek.

The cut-off lengths for snow depth at the various stations, estimated from the data in Figure 5, are listed Table II. These values suggest that the magnitude of the cut-off length may be related to the degree of ge-scale topographic relief. Although the quality of data for Trail Valley is relatively poor, the measure- nts being few and collected at large measurement intervals over terrain with varying vegetation, the cut- length for the snowcover at this location appears to be much greater than the cut-off lengths for the)wcovers at Kernen Farm and Strawberry Hills.

'ects of distance trends

Since the data from Kernen Farm, the Strawberry Hills and Trail Valley were collected at different times 1 on different vegetation, the connection between cut-off length and topography suggested by the data in ble II is not conclusive. The hypothesis of a relation between topography and cut-off length was tested by ninating vegetation as a causal factor.

Snow depths were measured on adjacent stubble and fallow fields in the Strawberry Hills on 3 March)5. Figure 6 shows that the plots of standard deviation of depth versus sampling distance for the fields

)le II. Topographic relief (change in elevation in horizontal distance) and snow depth cut-off length for Kernen m, the Strawberry Hills and Trail Valley Watershed

›w survey site	Geomorphology	Relief (approximate)	Approximate cut-off length (m)
·nen Research Farm	Lacustrine Plain	30 m in 7 km	30 m
ıwberry Hills	Moraine	70 m in 2 km	80 m
il Valley Watershed	Valley	60 m in 0·5 km	500 m

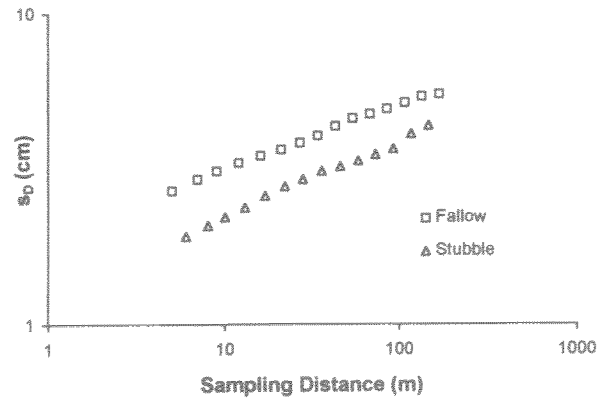


Figure 6. Variation in the standard deviation of snow depth with sampling distance. Stubble and fallow fields. Strawberry Hills, 3 March 1995

demonstrate similar slopes. The absence of a cutoff length in the graphs indicates that the transects were shorter than any cut-off length imposed by topography.

The snow depths measured on the Strawberry Hills display long-distance trends, which appear to be due to topographic variability (see Figures 7 and 8). In open, exposed prairie regions, snow accumulation is inversely related to topographic position (elevation) because snow is eroded from the tops of hills and accumulates in depressions. The measurements on stubble (Figure 7) show snow depths decreasing along a traverse from the bottom to the top of a hill. This trend is characteristic in observations that have been made along a slope that faces the prevailing wind. The data on fallow (Figure 8) were collected over a U-shaped depression. Snow depths are shallower at the tops of the hills than in the depression. If the transect had been longer, the large-scale variation in snow depth would most likely have become cyclic, as the underlying topography is hummocky.

To test the influence of large-scale trends in snow depth on the cutoff length, the data were de-trended. The steps were:

1. A straight line was fitted to the stubble data (see Figure 7).
2. A second-order polynomial was fitted to the fallow data (see Figure 8).
3. The snow depths calculated by the fitted curves were subtracted from the corresponding observed values.

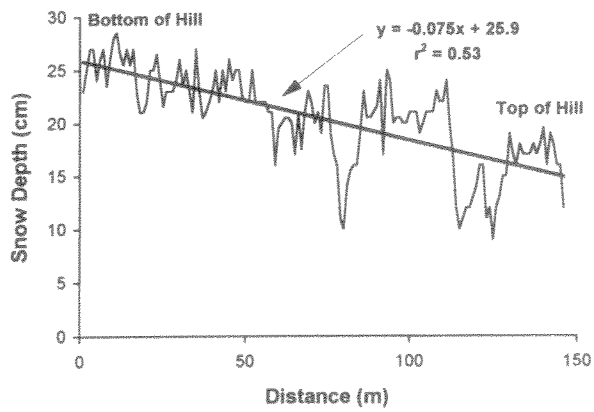
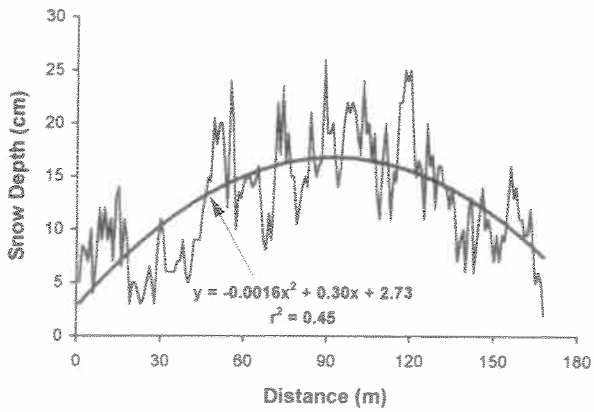


Figure 7. Snow depth transect on stubble field. Strawberry Hills, 3 March 1995. The fitted line was used to de-trend the data for topographic effects



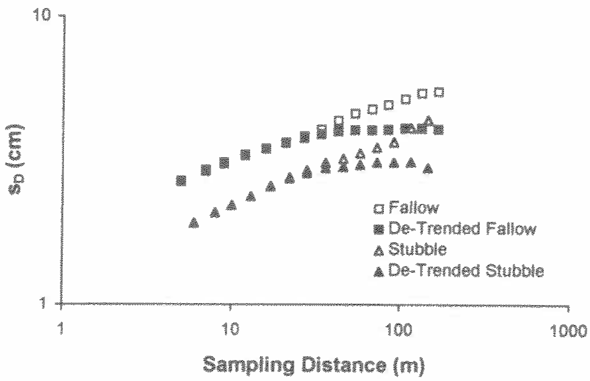
ure 8. Snow depth transect on fallow field. Strawberry Hills, 3 March 1995. The fitted line was used to de-trend the data for topographic effects

e subtraction of depths (item 3) removed most large-scale trends. Curves (linear and polynomials of ;ree two to five) fitted to the residuals were essentially flat with r^2 values less than 0·005.

Standard deviations were calculated from the de-trended data for various sampling distances. Figure 9)ws that the fractal slopes of the curves for the de-trended data are the same as those for the original :a. However, the de-trended data display a cut-off length of approximately 30 m, which agrees with cutoff length for relatively flat fields of stubble and fallow at the Kernen Farm (see Table II).

[he decrease in the cutoff length produced by de-trending the data confirms that: (a) the autocorrelation snow depth is due to both small-scale and large-scale surface features, and (b) the effect of large-scale tures is to increase the cutoff length. Autocorrelation at small scales is believed to be a result of snow 1es; whereas large-scale autocorrelation is believed to be owing to drifts caused by large-scale variations topography. The cutoff length for the de-trended data is believed to represent the limiting scale of the :tal autocorrelation, which appears to be almost the same for both stubble and fallow fields.

[he close agreement between the values for the cutoff length determined from the de-trended data for ow and stubble suggests that the magnitude of the distance is relatively insensitive to land use. There- e, for transects of snow depth measured on rolling topography, the cutoff length is an index of the 'izontal extent of snow drifts, i.e. a measure of the effects of topographic variability on snow accumu- on. Further research is required to establish quantitative relationships between cutoff length and face condition and topographic relief.



re 9. Variation in the standard deviation of snow depth with sampling distance. Natural and de-trended data. Stubble and fallow fields. Strawberry Hills, 3 March 1995

IMPLICATIONS OF SEMI-FRACTAL DISTRIBUTION OF SNOW DEPTH

The semi-fractal small-scale structure of snow depth has several important implications with respect to the measurement and modelling of snow cover.

1. The cutoff length defines the minimum scale of a snow course; i.e. the transect must be longer than the cutoff length to produce an unbiased estimate of the standard deviation. To avoid bias, the scale of the sample set must be greater than the cut-off length. This can be accomplished by ensuring that the minimum circle capable of enclosing the sample points has a diameter greater than the cut-off length.
2. Ablation models for shallow snowcovers must reproduce the geometries of patches of soil and snow to calculate small-scale advection correctly. This requires: (a) the scale of a model snowpack to be sufficiently small to incorporate the semi-fractal spatial distribution of water equivalent, and (b) observations of snow depth and water equivalent at scales less than the cutoff length.
3. The dependency of cutoff length on topographic relief will influence the scales of model snowcovers on complex landscapes. Since the length represented by one array element must be smaller than the cutoff length, a complex landscape comprising two or more topographic types will have to be represented either by separate arrays or by separate regions within one array. The flattest landscape unit within the snow field to be modelled, which will have the smallest cutoff length, will define the scale of the model.
4. An association between cutoff length and topography may provide a method of independently subdividing landscapes for snowcover accumulation.

CONCLUSIONS

Knowledge of the standard deviation of snow depth is important for measurement and for modelling snowmelt of shallow, seasonal snowcovers. At small scales, the spatial distribution of snow depth is fractal, at larger scales it is random. The fractal character, which is due to autocorrelation, causes the standard deviation of snow depth to be a power function of the sampling distance. Beyond some limiting cutoff scale the spatial distribution of snow depth becomes random and the standard deviation becomes independent of sampling distance.

Measurements of snow depth suggest that the cutoff length is related to the macroscopic variability of the underlying topography. On flat land, the cutoff length is approximately 30 m, independent of surface condition (wheat stubble or fallow). The distance increases on hummocky terrain and across a wide valley floor. Removing the large-scale trends in topography from the measurements, produces a common cutoff for hummocky and flat terrain. It is concluded that the variability in large-scale topography is the primary cause of the increase in the distance of the transition of snow depth from fractal to random structure.

The cutoff length defines the minimum scales of snow surveys and snowmelt models. The parameter may provide an objective criterion for stratifying shallow snowcovers for modelling and for measurement.

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