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ACCURACY IN ESTIMATING SNOW COVER WATER EQUIVALENTS

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ABSTRACT

Hydrologic models often require accurate estimates of mean water equivalent and associated error, $\overline{W}$ ± error, for basin snow cover. Determining the mean water equivalent as the product of mean depth and mean density, that is, $\overline{W} = \overline{\rho}\,\overline{d}$ + covariance, reduces sampling, increases confidence, and yields additional information from the survey. Depth measurements of maximum winter snows covering Canadian Prairies often reveal non-normal, positively skewed areal frequency distributions, which demand care in estimating errors. A natural system of areal stratification, applicable to surveys of snows on agricultural prairie, appears capable of estimating $\overline{W}$, $\overline{d}$, and $\overline{\rho}$ with their associated errors, and features (1) limited sampling for density, (2) confidence in areal extrapolation, (3) flexibility in application, and (4) service in calibrating index snow-courses or Nipher-shielded precipitation gauges.

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SNOW WATER EQUIVALENTS

Solutions to many hydro-meteorological problems, especially those derived from models, often require accurate estimates of the average liquid water equivalent for the snow covering extensive land areas. Areal estimates of water equivalent are very difficult to obtain. Direct extrapolation of snowfall caught in precipitation gauges ignores the redistribution effects of wind, non-uniform melt, etc. Traditional snow cover measurement by weighing vertical cores at various locations encounters an enormous sampling problem. If one may assume that each measurement accurately describes the absolute snow water equivalent covering the immediate one square meter of land, a sample of ten observations over 1000 km^2 results in a sample density of

$$\frac{10\ \text{m}^2}{1000\ \text{km}^2} = \frac{1}{100{,}000{,}000}\ .$$

Aerial measurement of snow cover by sensing the attenuation of terrestrial gamma radiation may ease sampling, but will not eliminate it. If for the same cost, aerial surveys increased sample coverage from 10 m^2 to 10 km^2, sampling density will still equal only 1%.

The snow water equivalent, W, at a point is commonly computed as the product of the snow depth, d, and the vertically integrated snow density, ρ, obtained from a snow core. If snow water is pure, with a specific gravity of one, working units for W, d and ρ are cm(liquid), cm(snow), and cm(liquid)/cm (snow), respectively per unit land area. For an areal mean water equivalent, this product becomes

$$\bar{W} = \bar{\rho}\,\bar{d} + \text{covariance} = \bar{\rho}\,\bar{d} + r_{\rho d}\, s_{\rho}\, s_{d}$$

where $r_{\rho d}$ is the correlation coefficient, if significant, between depth and density, s_{ρ} and s_{d} are standard deviations for sample observations of ρ and d, or their linear transforms, and symbols with bars, ‾, denote areal means. With this equation and each component may be determined separately by independent sampling schemes.

ERROR

Mean-value estimates of any absolute quantity should include a measure of the most likely confidence associated with the estimate. Consequently, an estimate of mean areal water equivalent should be expressed by $\bar{W} \pm s_{W} D_{W}$, where D_{W} is the areal frequency distribution of W (frequency density function), and

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s_w is the standard error of W. Likewise, valid estimates of depth and density should also include bands of confidence, that is, $\bar{d} \pm s_d D_d$ and $\bar{\rho} \pm s_\rho D_\rho$. The standard error, s, encompasses primarily natural areal variability and sampling error. Because samples are small and cover limited areas, these errors can range quite large. In comparison, errors from instrumentation (Work, _et al_, 1965) and measurement (Turcan and Loijens, 1975) are minor and can often be ameliorated by various corrections.

While standard errors can be evaluated by statistical analyses of the samples, areal frequency distributions must be assessed prior to the snow survey. Figure 1 presents a set of distributions for point observations of d, ρ, and W from a March 1972 snow cover over prairie pastures in Saskatchewan. The similarity between water equivalent and depth distributions suggests a dominance of depth over density in influencing resultant water equivalent. The Figure also indicates that which repeated symmetry tests have confirmed, namely, that over similar landscapes density distributions tend to be normal, while those for depth are commonly non-normal. An intensive snow survey in March 1974 involving 20 different landscapes over a Prairie agricultural watershed showed normal distributions for density in 85% and for depth in 15% of the survey. Coefficients-of-variation from a 1972 Prairie snow survey, compared in Table 1, averaged twice as great for depth as for density.

Table 1. Summary of Snow Cover Statistics by Landscape Class, 1972, Bad Lake, Saskatchewan.

Landscape Class	No. of Snow Courses	Depth (cm)		Density (cm/cm)	
		Mean	Coeff. of Variation	Mean	Coeff. of Variation
Upland Stubble	2	34	.291	.14	.163
Lowland Stubble	1	53	.282	.16	.215
Plain Stubble	3	21	.320	.22	.154
Upland Pasture	9	14	.703	.16	.370
Lowland Pasture	9	62	.634	.27	.339
Plain Pasture	7	27	.608	.19	.363
Classes Combined		34	.935	.20	.440

Lower variances combined with normal frequency distributions justify a reduction in sample numbers for density, a welcome practice, considering the comparative difficulty associated with density measurement.

Actual areal frequency distributions for snow depth need not be defined if sampling is planned wisely. The central limit theorem assures the existence of a normal distribution for means derived from a group of samples taken randomly from a population whose distribution is unknown. Thus, a set of mean depths resulting from a series of snow-course samples will be normally distributed irrespective of within-course distribution.

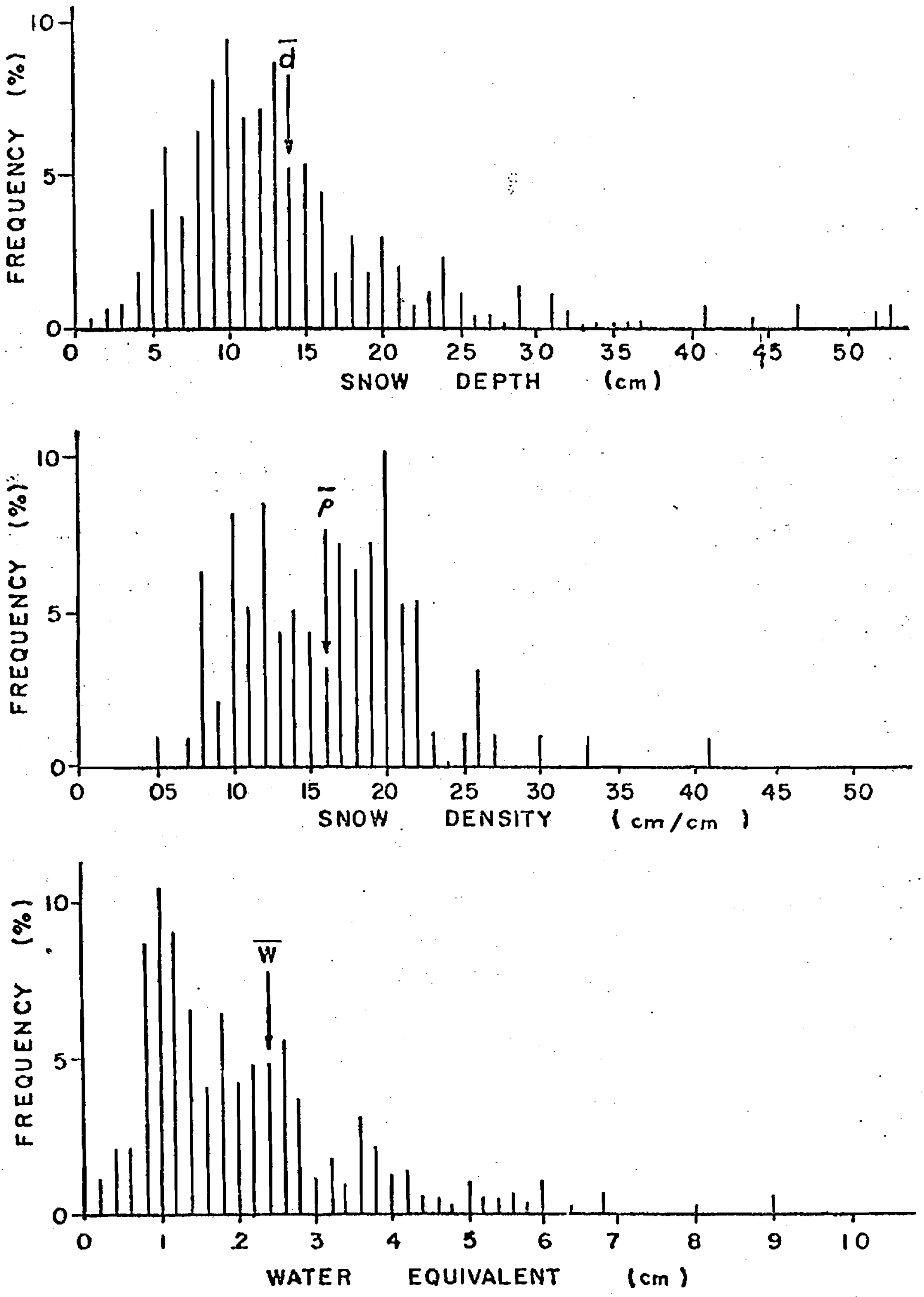

Figure 1. Frequency Distributions of 312 Snowcover Measurements over Upland-Pasture, Bad Lake Basin, Saskatchewan

UNITIZED SNOW SURVEYING

Unitized snow surveying combines the inherent advantages of the central limit theorem with those associated with stratified sampling. Each survey under this method is limited to a climate zone where areal variation in snowfall is either small or results in uniformity after a period of accumulation. Procedures basic to the method are outlined as follows:

1. Aerial photographs, field inspections, satellite imagery, etc. are used to unitize a base map of the area under survey. Units are defined according to landscape classes for vegetation and terrain which will most likely influence snow cover. Classes listed in Table 2 have proven applicable to prairie snow. Comparisons of variation data in Tables 1 and 2 also show, as expected, that snow depth varies significantly more between landscape classes than within them.

Table 2. Snow Cover Depth Statistics by Landscape Class, 1974, Bad Lake, Saskatchewan

Landscape Class	n	Mean Depth (cm)	Coeff. of Variation
Plain			
Fallow	10	41.5	.155
Stubble	6	46.4	.133
Rolling Plain			
Fallow	19	49.4	.151
Stubble	5	58.8	.083
Pasture	16	56.2	.174
Gradual Slope			
Fallow	9	50.4	.202
Stubble	5	47.4	.147
Scrub	7	65.6	.240
Slough			
Fallow	5	46.1	.110
Stubble	3	46.6	.139
Sharp Slope			
Pasture	12	111.5	.199
Scrub	21	126.5	.239
Broad Lowland			
Fallow	11	101.1	.188
Stubble	12	95.2	.084
Pasture	6	97.0	.114
Scrub	15	112.3	.183
Topland			
Fallow	14	22.9	.277
Stubble	6	37.6	.136
Pasture	5	21.2	.434
Farm Yard	2	129.1	.182
Classes Combined		68.1	.502

2. The number, n, of land parcels within each class required for the depth survey can be determined for a specified accuracy; more often, however, n will be dictated by the time and money available. Random selection of the parcels to be sampled is desirable, but not essential. Extrapolation within a landscape class is valid.

3. During the survey, depth is observed along a randomly-placed snow

course within each parcel selected for sampling and classed by landscape, while density observations are taken at random throughout each class. The number of density observations per class should approach 30 to assure good statistical spread. The number of depth observations per snow course should be sufficient to insure adequate areal coverage over the parcel and define the natural areal variability. Repeated samples of snow depth taken in Saskatchewan suggest about 32 - 36 observations per land parcel.

4. Once obtained, the snow survey data can be tested for homogenity of variance, equivalence of means, and possible class combination.

5. Data calculations yield: (1) mean depth, $\bar{d}_i$, for each snow course parcel, i; (2) mean depth, $\bar{d}_h$, density, $\bar{\rho}_h$, water equivalent, $\bar{W}_h$, and associated confidences for each class, h. For the 1972 Bad Lake, Saskatchewan survey these were:

Landscape Class	$\bar{W}_h$ (cm)	$\bar{d}_h$ (cm)	$\bar{\rho}_h$ (cm/cm)
Upland Stubble	5.1 ± 0.9	34 ± 2.7	.14 ± .02
Lowland Stubble	9.3 ± 2.5	53 ± 5.9	.16 ± .06
Plain Stubble	4.5 ± 0.6	21 ± 2.0	.22 ± .05
Upland Pasture	2.4 ± 0.2	14 ± 0.9	.16 ± .01
Lowland Pasture	18.6 ± 1.6	62 ± 4.0	.27 ± .02
Plain Pasture	6.1 ± 0.5	27 ± 2.2	.19 ± .02

(confidences at the 0.1 alpha error probability)

6. Estimates of mean depth, density, and water equivalent which represent the entire area under survey are calculated by summing values for landscape classes weighed by their respective surface areas.

SUMMARY

Unitized surveying features flexibility in that only preferred landscape classes need be surveyed. Extrapolation of the results are possible (1) areally by the natural relationship of snow to landscape, and (2) temporally by calibrating index snow-courses of Nipher-shielded precipitation gauges. The method also provides a procedure for determining "ground truth" in evaluating new techniques and instruments. If for specific storms, or specific times during winter accumulation, snow cover were areally uniform, landscape stratification would be superfluous, unitized sampling unnecessary. The method remains to be tested in climates where significant winter precipitation includes rainfall or in areas with wide ranges in elevation.

LITERATURE CITED

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