# CAP High School/CEGEP Prize Exam

April 11, 2018

9:00 - 12:00

## Student Information Sheet

The following information will be used to inform students and schools of the exam results, to determine eligibility for some subsequent competitions and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

| Marking Code: |
|---|

This box must be left empty.

**PLEASE PRINT CLEARLY IN BLOCK LETTERS**

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| Student First Name | | | | | | | | | | | | | | | | | | | | | | | | |
| Student Last Name | | | | | | | | | | | | | | | | | | | | | | | | |
| Teacher Name | | | | | | | | | | | | | | | | | | | | | | | | |

Home Address: ______________________

Postal Code: ______________________

Telephone: (      ) ______________ Email: ______________

School: ______________ Grade: ______________

Date of Birth: ______________

Gender: ______________ Citizenship: ______________

If you are not a Canadian citizen, what is your Immigration Status? ______________

For how many years have you studied in a Canadian school? ______________

Would you prefer further correspondence in French or English? ______________

*By signing this page I agree that, if my score on this Exam is in the top 20 nationally or in the top 6 in my province, my name, my school's name, and my ranking can be published.*

Student's signature: ______________________

Sponsored by:

Canadian Association of Physicists;
Canadian Physics Olympiad;
Department of Physics and Astronomy,
University of British Columbia.


## 2018 Canadian Association of Physicists Highschool/Cegep Prize Exam

This is a three-hour exam. National ranking and prizes will be based on students' performance on sections A and B of the exam. Performance on questions in section A will be used to determine whose written work in section B will be marked for prize consideration by the CAP Exam National Committee. Section A consists of 25 multiple-choice questions. The questions in section B span a range of difficulty, and may require graphing. Be careful to gather as many of the easier marks as possible before venturing into more difficult territory. When you are unable to solve any part of a question, you may assume a likely answer to that part and attempt the rest of the question anyway.

Non-programmable calculators may be used. Answer the multiple-choice questions ***on the answer sheet*** provided. Most importantly: write your solutions to the three long problems on ***three separate sheets*** as they will be marked by people in different parts of Canada. Good luck.

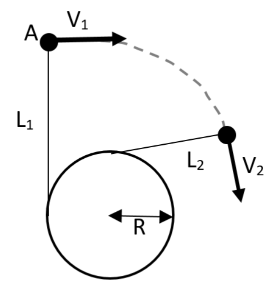
**Notice**: Full marks will be given to students who provide full correct solutions to problems in Section B. Partial marks will be given for partial solutions. There are no penalties for incorrect answers. The questions are not of equal difficulty. Remember that we are challenging the best physics students in Canada; it is possible that even the best papers may not achieve an overall score of 80%. This Exam is meant to be tough!

**Data**

Speed of light $c = 3.00 \times 10^8$ m/s
Gravitational constant $G = 6.67 \times 10^{-11}$ N·m$^2$/kg$^2$
Acceleration due to gravity $g = 9.81$ m/s$^2$
Standard atmospheric pressure $P_0 = 1.01 \times 10^5$Pa
Density of fresh water $\rho = 1.00 \times 10^3$ kg/m$^3$
Specific heat of water $C_w = 4186$ J/(kg·K)
Specific heat of ice $C_i = 2050$ J/(kg·K)
Latent heat of water $L_w = 2260$ kJ/kg
Latent heat of ice $L_i = 334$ kJ/kg
Density of ice $\rho_i = 916$ kg/m$^3$
Fundamental charge $e = 1.60 \times 10^{-19}$ C
Mass of an electron $m_e = 9.11 \times 10^{-31}$ kg
Mass of a proton $m_p = 1.67 \times 10^{-27}$ kg
Planck's constant $h = 6.63 \times 10^{-34}$ J·s
Electrostatic constant $k = 8.99 \times 10^9$ N·m$^2$/C$^2$
Boltzmann's constant $k_B = 1.38 \times 10^{-23}$ J/K
Astronomical Unit (approximate distance from the Sun to the Earth) 1 au $= 1.49598 \times 10^{11}$ m
Radius of the Earth $R_E = 6.371 \times 10^6$ m
Radius of the Sun $R_S = 6.96 \times 10^8$ m
Stefan's constant $\sigma = 5.6704 \times 10^{-8}$ W/(m$^2$· K$^4$)
$H_2$ Molar mass 2.016 g/mol
$O_2$ Molar mass 31.998 g/mol
$N_2$ Molar mass 28.013 g/mol
Mass of the Earth $5.97 \times 10^{24}$ kg
Mass of the Sun $1.99 \times 10^{30}$ kg

# Section A

1) A ball is attached to a cylinder by a string and is given an initial speed $V_1$ at time $t_1$. The ball moves along a frictionless surface and the cylinder is fixed and cannot rotate. What is the speed of the ball at a later time $t_2$?




a) $V_2 = \frac{L_1}{L_2} V_1$

b) $V_2 = \frac{L_1+R}{L_2+R} V_1$

c) $V_2 = \frac{L_1^2+R^2}{L_2^2+R^2} V_1$

d) $V_2 = \frac{\sqrt{L_1^2+R^2}}{\sqrt{L_2^2+R^2}} V_1$

e) $V_2 = V_1$

2) In the previous question, at $t = t_1$, at what rate does the string wind around the cylinder (i.e. at what speed does L1 get shorted)?

a) $\frac{L_1}{R} V_1$

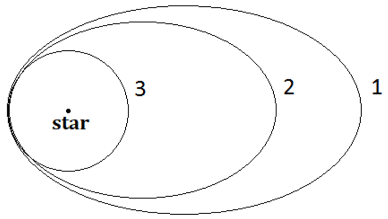
b) $\frac{L_1+R}{R} V_1$

c) $\frac{R}{L_1} V_1$

d) $\frac{R^2}{L_1^2} V_1$

e) $\frac{L_1^2}{R^2} V_1$

3) Three planets, each with same mass, orbit in different orbits around the same star. Which statement is true about the total energy of the three orbits? Assume that $m_{\text{planets}} \ll M_{\text{star}}$.




a) $E_1 < E_2 < E_3$

b) $E_2 < E_1 < E_3$

c) $E_3 < E_2 < E_1$

d) $E_3 < E_1 < E_2$

4) Charged particles with mass $m$ and charge $Q$ are injected with speed $v$ into a region with a constant magnetic field. The particles are injected in a direction perpendicular to the magnetic field lines and are found to move in circular orbits with period $T$. If the experiment is repeated for particles with mass $2m$, charge $Q/4$, while the magnetic field strength is tripled, what is the orbital period of the particles in the new experiment?

a) 8T/3

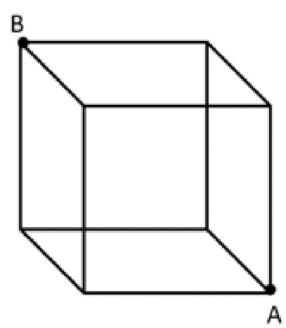
b) 3T/2

c) 4T/3

d) 2T

e) T/3

5) In the cube shown below, each edge is a wire with resistance $R$. What is the equivalent resistance between points A and B?




a) $R$

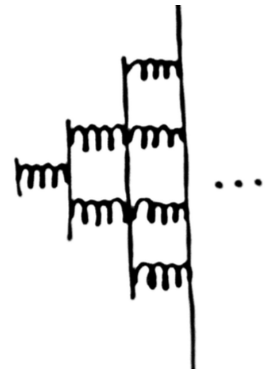
b) $\frac{1}{3}R$

c) $\frac{2}{3}R$

d) $\frac{1}{2}R$

e) $\frac{5}{6}R$

6) Consider the arrangement of springs shown below. All springs shown have a spring constant $k$. The first layer contains 1 spring, the next 2 springs, the next 4, and so on, with the $n$th layer containing $2^{n-1}$ springs. What is the value of the spring constant of the entire assembly in the limit where the number of layers n goes to infinity ($n \to \infty$)



a) $k/2$

b) $k/3$

c) $2k$

d) $3k$

e) $4k$

7) Knowing that Toronto's coordinates are 43.6532°N, 79.3832°W, what is the ratio of the solar power through a horizontal surface at noon on the shortest day of the year to the solar power through the same horizontal surface at noon on the longest day of the year? (The tilt of the Earth is about 23.4°)

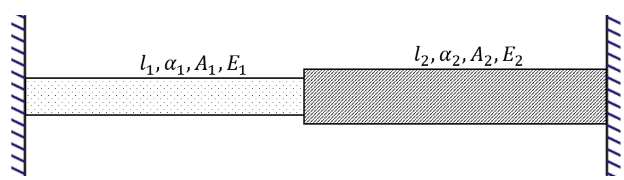
a) 0.21

b) 0.33

c) 0.42

d) 0.62

8) Two rods of the length $l_1$ and $l_2$ are held between two rigid walls $l_1 + l_2$ apart. Let $\alpha_1, E_1, A_1$ and $\alpha_2, E_2, A_2$ be the thermal coefficient of expansion, Young modulus, and the area of each rod respectively. Which of the following conditions guarantees that the joint between the rods will move left if the temperature of the system is increased? (Assume that both rods preserve their cylindrical shapes)




a) $\alpha_1 l_1 > \alpha_2 l_2$

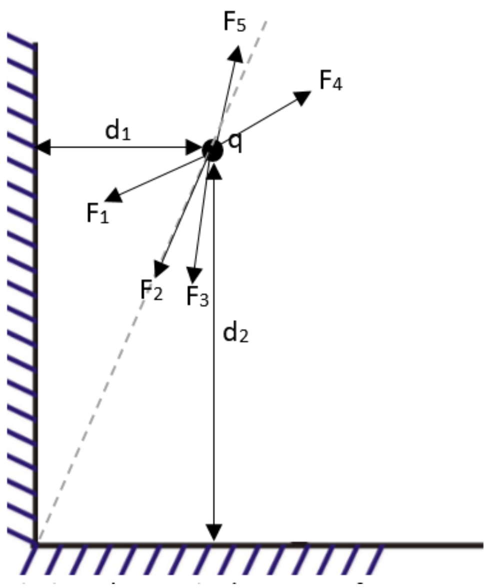
b) $E_1 \alpha_1 l_1 > E_2 \alpha_2 l_2$

c) $\frac{E_1 \alpha_1 l_1}{A_1} > \frac{E_2 \alpha_2 l_2}{A_2}$

d) $\frac{E_1 \alpha_1}{A_1} > \frac{E_2 \alpha_2}{A_2}$

e) $\frac{E_1}{l_1 A_1} > \frac{E_2}{l_2 A_2}$

9) A positive charge $q$ is placed in front two semi-infinite conducting walls as shown below. The walls are grounded. Which arrow best represents the direction force on the charge $q$, if $d_1 < d_2$?




a) $F_1$

b) $F_2$

c) $F_3$

d) $F_4$

e) $F_5$

10) A person is knitting a sweater using a spherical ball of yarn with an initial radius of 15 cm (at t = 0). The radius of the ball of yarn is 10 cm after 5 hours ($t = 5$ h) of knitting. What is the radius of the ball of yarn at time $t = 7$ h, assuming that the person is knitting at a constant rate?

a) 8 cm

b) 7.07 cm

c) 6.16 cm

d) 3.68 cm

e) 0 cm (the yarn would run out)

11) The pump shown below is used to inflate a ball and is made of two compartment with volumes $V_1$ and $V_2$ and two one-way valves shown as $A_1$ and $A_2$. A one-way valve is valve with two modes of operation:

**Mode 1**: the valve is open and there is a flow in the direction shown in the figure. For this mode

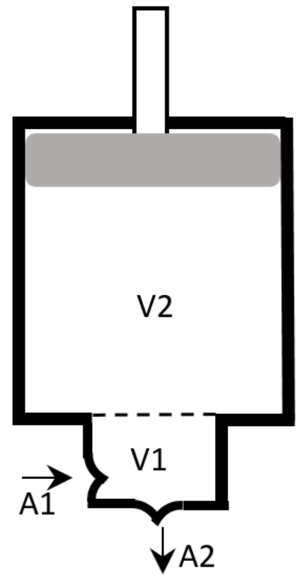
Pressure(inflow) = Pressure(outflow).

**Mode 2**: the valve is closed and flow is stopped since

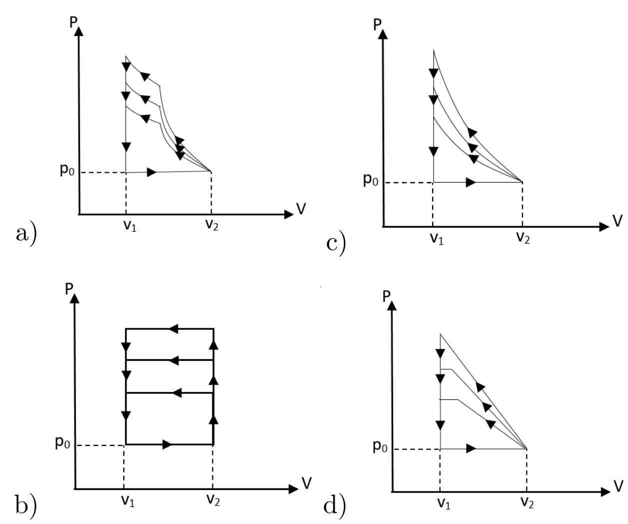
Pressure(inflow) $<$ Pressure(outflow).

**Note**: the one-way valve assures that

Pressure(inflow) $\leq$ Pressure(outflow).




Which of the figures below best represents the $P - V$ diagram of the air in the pump as the ball is inflated?




12) As shown in the figure below, two balls of radii $R_1 = 2$ cm and $R_2 \gg R_1$, and with masses $m_2 \gg m_1$ fall from a height $h = 1$ m. Ignoring friction, assuming that all impacts are perfectly elastic and that the initial distance between the balls is small, which of the choices below is closest to the maximum height of ball 2 after one bounce?

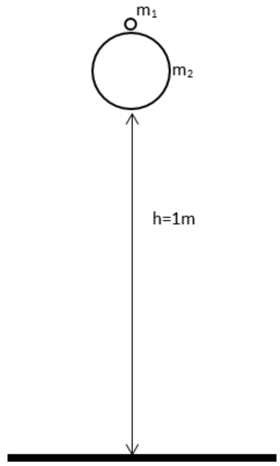
a) 1 m

b) 2 m

c) 3 m

d) 4 m

e) 9 m




13) A ball is dropped from the top of a building with no initial speed. After 1 s, a second ball is thrown downward with an initial vertical speed of 26 m/s. What is the minimum height of the building if the second ball hits the ground earlier than the first ball?

a) 0.45 m

b) 4.91 m

c) 6.93 m

d) 8.33 m

e) 12.65 m

14) A current of 12 mA passes through a red LED diode with a forward voltage of 3 V. Which of the following is a good estimate for the number of photons per second this LED produces?

a) $1 \times 10^{12}$

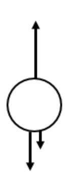
b) $3 \times 10^{14}$

c) $1 \times 10^{17}$

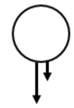
d) $3 \times 10^{19}$

e) $1 \times 10^{22}$

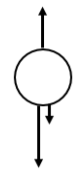
15) A ball is thrown upwards. Which of the following free body diagrams best describes the forces acting on the ball when it is in the air moving upwards. Assume that air drag (friction) is proportional to speed.



a)



b)



c)

d)

16) While the ball in the previous question is still going upwards, the magnitude of the net force acting on the ball in the previous problem is:

a) Increasing

b) Decreasing

c) Remains constant with the height of the ball

17) Relative to an inertial frame at the centre of the earth, approximately how fast is a person in Vancouver (latitude 49.3° N) moving due to the Earth's rotation around its axis?

a) 700 m/s

b) 300 m/s

c) 450 m/s

d) 400 m/s

18) Which is more likely: that a single throw of a die will result in '1', or that three throws of a die will each result in an odd number?

a) Single die throw resulting in '1' is more likely

b) Three throws each resulting in an odd number is more likely

c) These two events have the same probability

19) Collision of cosmic-rays entering the upper atmosphere of the Earth produce large numbers of particles called muons. These muons move vertically through the atmosphere at very high speed (comparable to the speed of light, c), forming a beam of flux I. Because muons decay into other particles, the flux changes with time according to the exponential decay law $I = I_0 e^{-\frac{t}{\tau}}$, where $\tau = 2.2$ $\mu$s is the avarage lifetime in the rest frame of the muon, $I_0$ is the muon flux at time $t = 0$, and $t$ is measured in the rest frame of the muon. If we place a detector at a

height on 10 km above the ground, we detect a muon flux of $I_0 = 1000$ part/(s$\cdot$ m$^2$). If we place the same detector on the ground, we detect a muon flux of $I_g = 49$ part/(s$\cdot$ m$^2$). Assuming that the muons in the beam are all moving at the same relativistic speed, what is that speed?

a) $v = 0.81\ c$

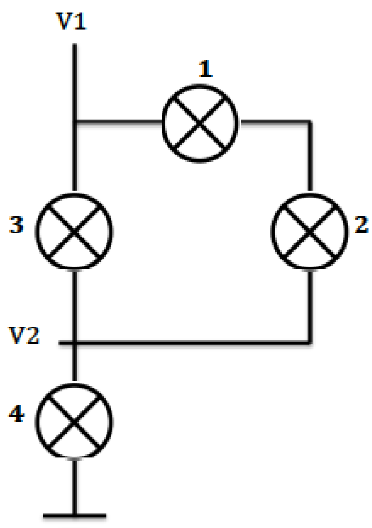
b) $v = 0.85\ c$

c) $v = 0.90\ c$

d) $v = 0.98\ c$

e) $v =\ c$

20) Given the circuit below, which light bulb do you have to unplug so that $V_2 = \frac{1}{3}V_1$? The four light bulbs are identical.




a) Light bulb 1

b) Light bulb 2

c) Light bulb 3

d) Light bulb 4

e) None of the above

21) A pulsar is a highly magnetized, rotating neutron star (or white dwarf), with high density and short, regular rotating period. You have just discovered a new pulsar that spins around its axis 20 times per second. Assuming that the object is spherical, what is its minimum density?

a) $\rho = 5.6 \times 10^{13}$ kg/m$^3$

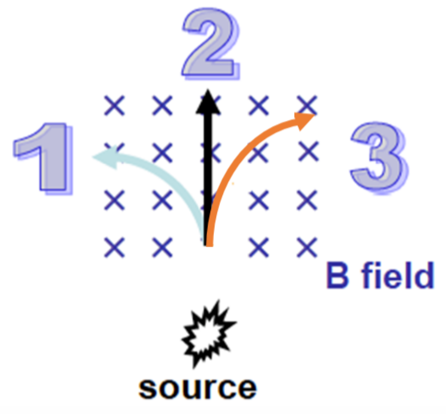
b) $\rho = 7.4 \times 10^{8}$ kg/m$^3$

c) $\rho = 6.1 \times 10^{10}$ kg/m$^3$

d) $\rho = 2.2 \times 10^{8}$ kg/m$^3$

e) $\rho = 1.7 \times 10^{13}$ kg/m$^3$

22) A radioactive substance decays, and an emitted particle passes through a uniform magnetic field pointing into page, as shown. Which labeling is correct?




a) 1=alpha; 2=beta; 3=gamma

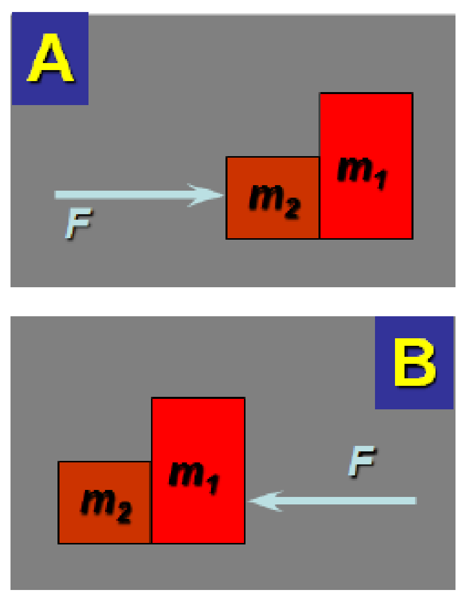
b) 1=alpha; 3=beta; 2=gamma

c) 2=alpha; 1=beta; 3=gamma

d) 2=alpha; 3=beta; 1=gamma

e) 3=alpha; 1=beta; 2=gamma

23) If you push with force $F$ on either the heavy box ($m_1$) or the light box ($m_2$), in which of these two cases is the contact force between the two boxes smaller? Ignore friction.

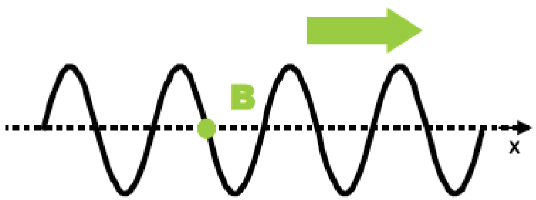



a) case A

b) case B

c) same in both cases

24) Consider an infinite wave moving to the right on a non-stretchy string, as shown below. What is the direction of the velocity of a particle at the point labeled B?




a) 

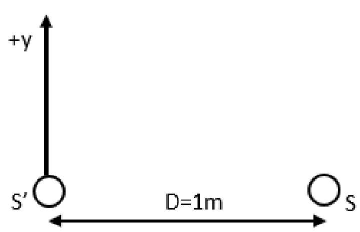
b) 

c) 

d) 

e) velocity is zero

25) A person is moving away from two speakers S and S' along the y direction as shown in the figure below. The speakers are in phase with each other and emit a sound wave with wavelength 6 cm. As the person moves away from speaker S', they mark the points on the y axis where there is no sound. Which of the plots below shows the position of the marked points correctly?




a) 


b) 


c) 


d) 


# End of Section A

# Section B starts on next page

1) A closed 5-litre cylinder containing 0.25 g of a substance in solid and liquid forms was slowly heated using a constant power. A graph of the temperature as a function of time is shown below. The heat capacity of the liquid is 2.43 J/(g · K), the latent heat of melting is 105 J/g, and the molar density is 46 g/mol.

a) What is the melting point of the substance?

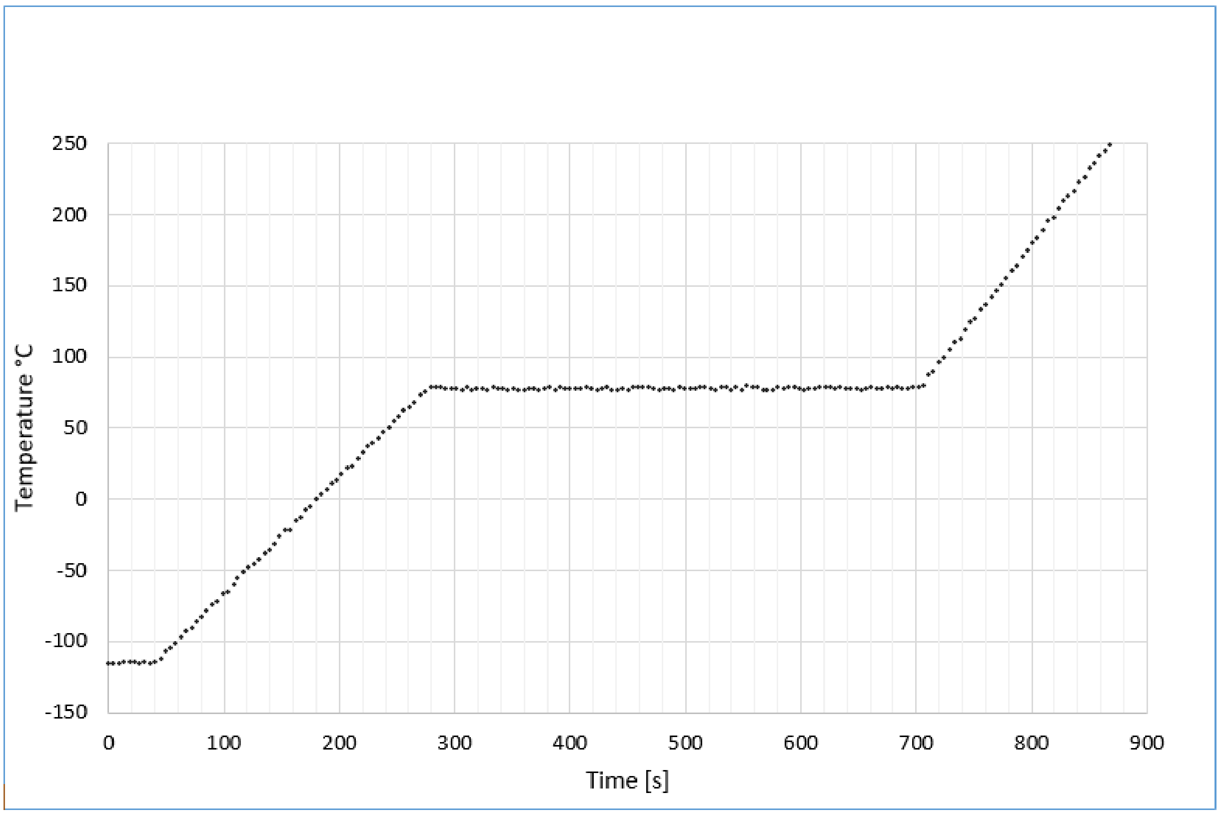
b) What is the boiling point of the substance?

c) How much of the substance was solid at the beginning?

d) What is the latent heat of evaporation of the substance?

e) What is the specific heat of the gaseous state of the substance?

f) What is the pressure in the container at 200°C?



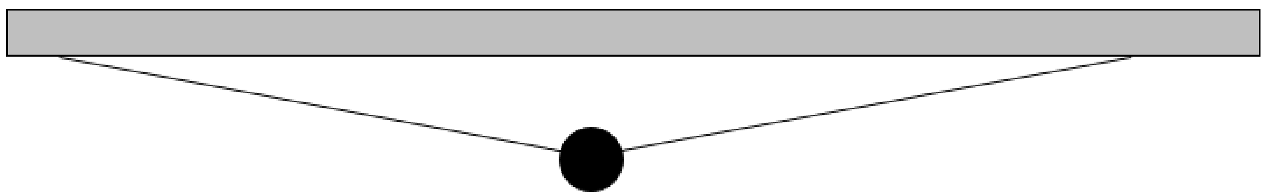
2) A ball of mass $m = 200$ g is hung in the middle of horizontal elastic cord of length $L = 1.5$ m, attached to the ceiling, as shown in figure below.

The elastic constant of the cord is $k = 300$ N/m.

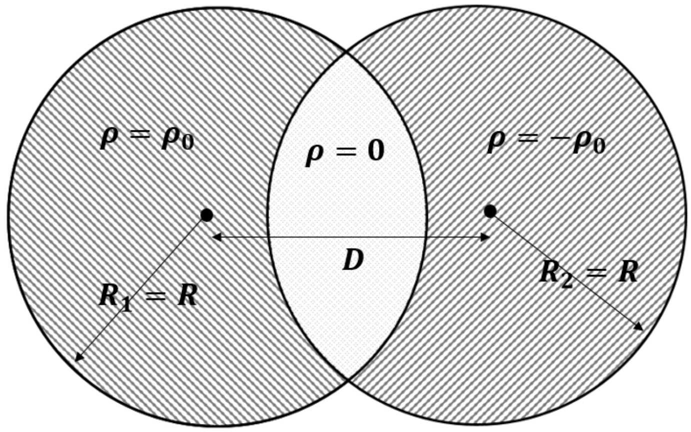
a) What is the distance from the ceiling to the center of the ball, assuming that the cord was not stretched when it was horizontal?

b) Calculate the period of very small oscillations from the equilibrium point.

c) Prove or disprove: when we pull the ball up to the ceiling and let it go, the resulting oscillations will be exactly described by simple harmonic motion.

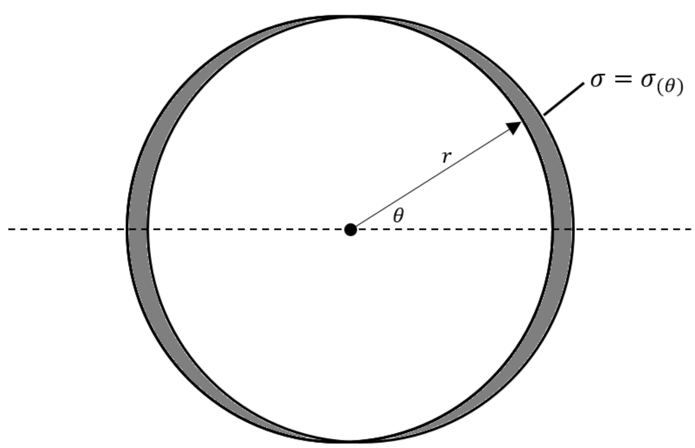


3) The charge distribution shown below has three regions with charge densities of $\rho_0$, 0, and $-\rho_0$. The system can be thought of as the superposition of two spheres of the same radius $R_1 = R_2 = R$ and opposite charge density $\rho_0$ and $-\rho_0$, with their centers placed at a distance $D$ from each other, with $D < 2R$.




a) What is the electric field in the overlapping region?

b) Assume that $D \ll R$. We can approximate the charge distribution by a surface charge density $\sigma(\theta)$ distributed on the surface of the sphere at radius r=R. Find $\sigma(\theta)$ as a function of $\rho$, $D$, $\epsilon_0$, and $\theta$ .




c) Using the charge distribution found in part b and the corresponding electric field from part a, find the surface charge distribution $\sigma_c(\theta)$ of a neutral conducting sphere in a constant electric field $E_0$.

| | | | | | | |
|---|---|---|---|---|---|---|
| Question 1 | a | b | c | d | e | f |
| Question 2 | a | b | c | d | e | f |
| Question 3 | a | b | c | d | e | f |
| Question 4 | a | b | c | d | e | f |
| Question 5 | a | b | c | d | e | f |
| Question 6 | a | b | c | d | e | f |
| Question 7 | a | b | c | d | e | f |
| Question 8 | a | b | c | d | e | f |
| Question 9 | a | b | c | d | e | f |
| Question 10 | a | b | c | d | e | f |
| Question 11 | a | b | c | d | e | f |
| Question 12 | a | b | c | d | e | f |
| Question 13 | a | b | c | d | e | f |
| Question 14 | a | b | c | d | e | f |
| Question 15 | a | b | c | d | e | f |
| Question 16 | a | b | c | d | e | f |
| Question 17 | a | b | c | d | e | f |
| Question 18 | a | b | c | d | e | f |
| Question 19 | a | b | c | d | e | f |
| Question 20 | a | b | c | d | e | f |
| Question 21 | a | b | c | d | e | f |
| Question 22 | a | b | c | d | e | f |
| Question 23 | a | b | c | d | e | f |
| Question 24 | a | b | c | d | e | f |
| Question 25 | a | b | c | d | e | f |