

# CAP High School Prize Exam

April 5th, 2012

9:00 - 12:00

## Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

### PLEASE PRINT CLEARLY IN BLOCK LETTERS

Family Name: \_\_\_\_\_ Given Name: \_\_\_\_\_

Home Address: \_\_\_\_\_

\_\_\_\_\_ Postal Code: \_\_\_\_\_

Telephone: (     ) \_\_\_\_\_ Email: \_\_\_\_\_

School: \_\_\_\_\_ Grade: \_\_\_\_\_

Physics Teacher: \_\_\_\_\_

Date of Birth: \_\_\_\_\_ Sex: Male  Female

Citizenship: \_\_\_\_\_

or Immigration Status: \_\_\_\_\_

For how many years have you studied in a Canadian school? \_\_\_\_\_

Would you prefer further correspondence in French or English? \_\_\_\_\_

Sponsored by:

Canadian Association of Physicists,  
Canadian Physics Olympiad,  
The University of British Columbia,  
Department of Physics and Astronomy.

## Canadian Association of Physicists 2012 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on students' performance on sections A and B of the exam. Performance on the questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. Part A consists of twenty-five multiple-choice questions. The questions in part B span a range of difficulties, and may require graphing. Be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

**Notice:** Full marks will be given to a student who provides any full correct solution to the long problems. Partial marks will be given for partial solutions. There are no penalties for incorrect answers. The questions are not of equal difficulty. Remember we are challenging the best physics students in Canada; it is possible that even the best papers may not achieve an overall score of 80%. This is meant to be tough!

### Data

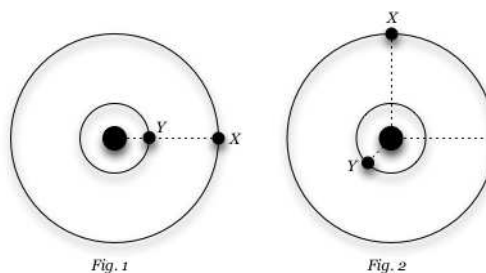
Speed of light  $c = 3.00 \times 10^8$  m/s  
 Gravitational constant  $G = 6.67 \times 10^{-11}$  Nm<sup>2</sup>/kg<sup>2</sup>  
 Acceleration due to gravity  $g = 9.80$  m/s<sup>2</sup>  
 Normal atmospheric pressure  $P_0 = 1.01 \times 10^5$  Pa  
 Density of fresh water  $\rho = 1.00 \times 10^3$  kg/m<sup>3</sup>  
 Specific heat of water  $C_w = 4186$  kJ/kgK  
 Specific heat of ice  $C_i = 2050$  kJ/kgK  
 Latent heat of water  $L_w = 2260$  kJ/kg  
 Latent heat of ice  $L_i = 334$  kJ/kg  
 Density of ice  $\rho_i = 916$  kg/m<sup>3</sup>  
 Fundamental charge  $e = 1.60 \times 10^{-19}$  C  
 Mass of electron  $m_e = 9.11 \times 10^{-31}$  kg  
 Mass of proton  $m_p = 1.67 \times 10^{-27}$  kg  
 Planck's constant  $h = 6.63 \times 10^{-34}$  Js  
 Coulomb's constant  $1/(4\pi\epsilon_0) = 8.99 \times 10^9$  Nm<sup>2</sup>/C<sup>2</sup>  
 Boltzmann's constant  $k = 1.38 \times 10^{-23}$  J/K  
 A.U. Astronomical Unit  $1.49598 \times 10^{11}$  m: The approximate distance from the Sun to the Earth.  
 Radius of the Earth  $R_e = 6.371 \times 10^6$  m  
 Radius of the Sun  $R_s = 6.96 \times 10^8$  m

### Part A: Multiple Choice

Each multiple choice question is worth 1 point.

#### Question 1

Two planets X and Y travel counterclockwise in circular orbits around a star as shown in the figure below:



The radii of their orbits are in the ratio 3 : 1. At some time, they are aligned as in Fig. 1, making a straight line with the star. After five Earth years, the angular displacement of planet X is 90.0°, as in Fig. 2. What is the angular displacement of planet Y at this time?

- 35°
- 90°
- 180°
- 360°
- 468°

#### Question 2

You are holding a bottle of sparkling water inside a car moving forward. When the driver applies the brakes:

- Bubbles in the middle of the liquid will start to move forward with respect to the bottle.
- Bubbles will start to move backward with respect to the bottle.
- Bubbles will stay at the same horizontal location in the water.
- Depending on the speed of the car, bubbles might move forward or backward.

#### Question 3

An Earth satellite revolves in a circular orbit at a height  $h$  from the surface of the Earth. If  $R$  is the Earth's radius and  $g$  is the acceleration due to gravity at the surface of the Earth, then the speed of the satellite is given by:

- $\sqrt{gR}$
- $\sqrt{g(R+h)}$
- $\sqrt{g\frac{R^2}{R+h}}$
- $\sqrt{g\frac{(R+h)^2}{R}}$

#### Question 4

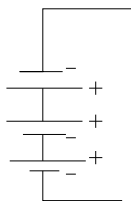
Assume that sodium produces monochromatic light with a wavelength of  $5.89 \times 10^{-7}$  m. At what approximate rate would a 10 watt sodium-vapour light be emitting photons? Assume that the efficiency of the light bulb is about 30%.

- $8.9 \times 10^{18}$  photons/sec.
- $3.0 \times 10^{19}$  photons/sec.
- $9.9 \times 10^{19}$  photons/sec.
- $2.0 \times 10^{20}$  photons/sec.

**Question 5**

Three batteries are placed in series. Each battery has an internal resistance  $r$ . If one of the batteries is placed the wrong way around as shown in the picture, what will be the total resistance of the three cells now?

- a)  $r/3$
- b)  $r/2$
- c)  $r$
- d)  $2r$
- e)  $3r$



**Question 6**

At the designed intensity, the two beams circulating in the Large Hadron Collider (LHC) at CERN consist of 5616 bunches (2808 in each direction) of approximately  $1.15 \times 10^{11}$  protons per bunch. A small commercial hydrogen cylinder contains 40 L of gas at a pressure of 10 MPa and a temperature of  $25^\circ\text{C}$ . Assuming an injection efficiency of 70%, how many times could the LHC be filled at the designed intensity using a single, perfectly hermetic cylinder?

- a)  $1.1 \times 10^{11}$
- b)  $1.5 \times 10^{11}$
- c)  $2.1 \times 10^{11}$
- d)  $1.1 \times 10^{14}$
- e)  $1.5 \times 10^{14}$

**Question 7**

The Earth is constantly receiving energy from the Sun. To stay at approximately the same temperature, the Earth loses energy by:

- a) Conduction
- b) Radiation
- c) Convection
- d) Evaporation
- e) The Earth does not lose energy, this is why we have global warming.

**Question 8**

A car starts to move at time  $t = 0$ . If the engine of the car is able to provide constant power, which of the following statements is correct about the speed of car at the beginning of the motion?

- a) The speed is constant.
- b) The speed increases proportionally to time passed ( $v \propto t$ ).
- c) The speed increases as the square root of time ( $v \propto \sqrt{t}$ ).
- d) The speed increases as time squared ( $v \propto t^2$ ).

**Question 9**

A detector far away from the source of a wave is detecting pulses of that wave every 0.2 second. If the detector starts to move towards the source at a speed of

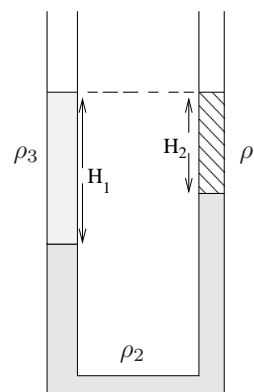
6.0 km/h, then it would detect a total of 18200 pulses per hour. What is the speed of the wave?

- a) 100 m/s
- b) 150 m/s
- c) 200 m/s
- d) 300 m/s

**Question 10**

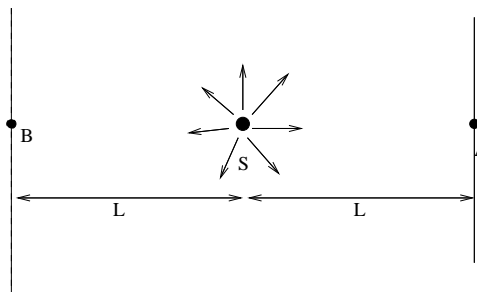
There are three different liquids, with densities  $\rho_1$ ,  $\rho_2$  and  $\rho_3$ , in a U-shaped container as shown in the picture. The lengths shown are  $H_1 = 15$  cm and  $H_2 = 10$  cm. Which of the following equations gives the correct relation between the densities of the fluids in the container?

- a)  $3\rho_3 = 2\rho_1 + \rho_2$
- b)  $\rho_3 = 2\rho_1 + 3\rho_2$
- c)  $2\rho_3 = 3\rho_1 + \rho_2$
- d)  $\rho_3 = 3\rho_1 + 2\rho_2$



**Question 11**

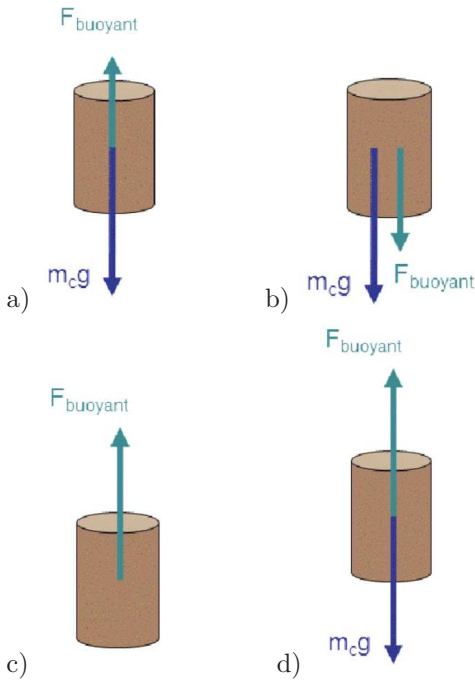
The intensity of light from a source decreases with the distance  $x$  from the source, as  $\frac{1}{x^2}$ . In the following picture, the intensity of light from the source  $S$  at point  $A$  on a curtain is  $8.1U$  where  $U$  is some unit. We then add a big mirror parallel to the curtain at point  $B$ . The distances  $AS$  and  $SB$  are equal, and the points  $A$ ,  $S$  and  $B$  are in line perpendicular to the curtain and the mirror. What is the intensity of light reaching point  $A$  now?



- a) 9.0 U
- b) 10.15 U
- c) 10.8 U
- d) 16.2 U

**Question 12**

Which of the following is the correct free-body diagram for a cylinder floating in water?



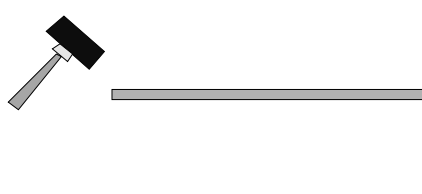
**Question 13**

Vancouver's latitude is about  $49^\circ$ , and Earth's axial tilt is about  $23^\circ$ . The power delivered by the Sun reaching a horizontal surface in Vancouver at noon differs in winter and summer. What is the approximate ratio of this quantity measured in winter, compared to that measured in summer?

- a) 1 : 2
- b) 1 : 3
- c) 1 : 4
- d) 1 : 5

**Question 14**

Consider a metal rod firmly attached to a wall. When you strike the rod with a hammer, which kind of wave will you excite?



- a) A longitudinal wave.
- b) A transverse wave.
- c) Either kind, or both, depending on where and how you hit the rod.
- d) No wave will be excited, the metal is too strong.

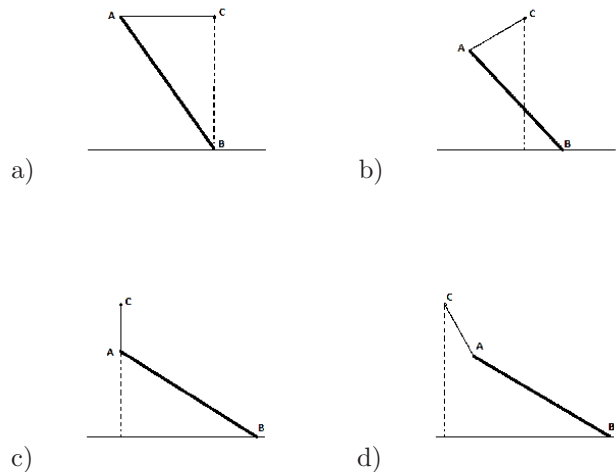
**Question 15**

Assume that you videotaped the fall of a ball due to gravity in vacuum, and are now playing the video in reverse at the same speed. Which of the following statements is correct about the acceleration of the ball seen in these conditions, compared to that of the actual falling ball?

- a) The accelerations are the same in both cases.
- b) They have the same value but opposite directions.
- c) They have different values but the same direction.
- d) Both the values and directions differ.

**Question 16**

A rod (AB) is attached to a fixed point (C) using a light rope (AC). The other end of the rod (B) is sitting on ice with negligible friction and the system is in stationary position. Which of the following can be the equilibrium configuration of this system?



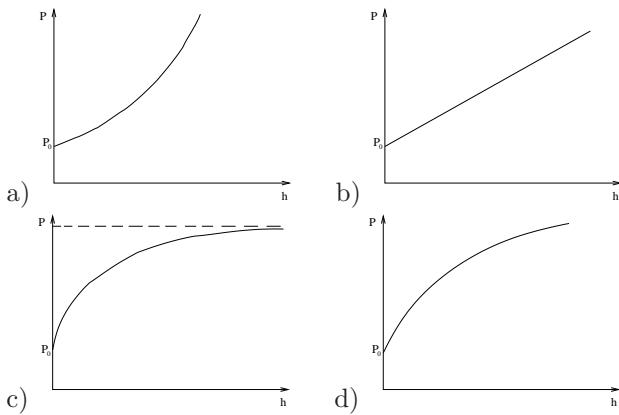
**Question 17**

Which of the following is the closest to the total mass of the atmosphere around the Earth?

- a)  $10^{13}$  kg
- b)  $10^{16}$  kg
- c)  $10^{19}$  kg
- d)  $10^{22}$  kg

**Question 18**

At shallow depth,  $h$ , the pressure in the ocean is simply given by  $P = P_0 + \rho gh$ , in which  $\rho$  is the density of water and  $P_0$  is the air pressure. As we go deeper, the high pressure causes the water to compress and become denser. Which of the following sketches illustrates the correct dependence of the pressure on the depth  $h$ ?



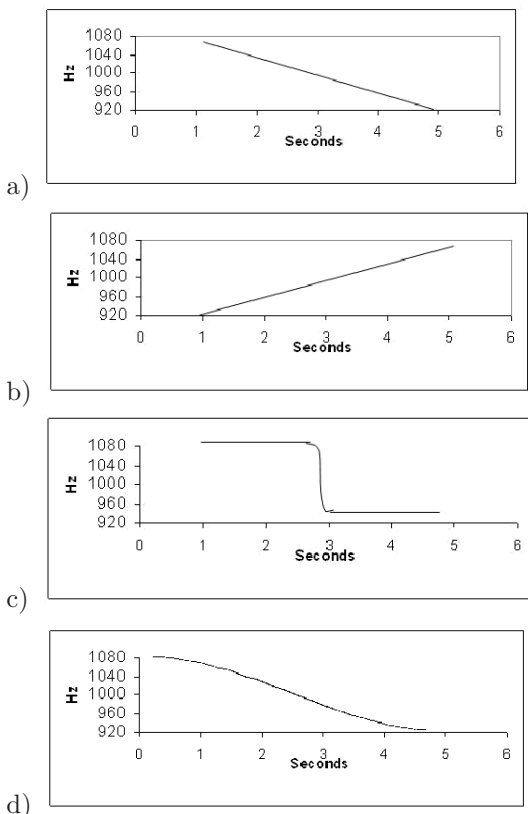
**Question 19**

An aluminium plate and a glass plate are left in a room for a long time. Putting one ice cube on each plate, you notice that the ice melts faster on the aluminium plate. Why?

- a) The ice is in thermal equilibrium with the glass plate, but not with the aluminium plate.
- b) Aluminium conducts heat to the ice more rapidly than glass.
- c) The aluminium plate holds more heat.
- d) The aluminium plate is warmer.

**Question 20**

You are on the side of the highway, listening to the siren of a fast-approaching ambulance. When the ambulance is not moving the frequency of its siren is 1000 Hz. Which of the following graphs best describes the frequency that you hear as the ambulance approaches and then passes you?



**Question 21**

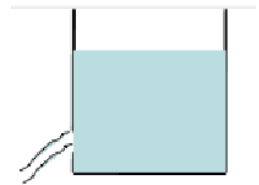
Which of the following radiation types has the longest wavelength?

- a) Radio waves
- b) Visible light
- c) X rays
- d) Infrared light
- e) They all have at the same wavelength

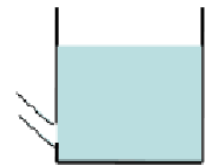
**Question 22**

A cup filled with water has a hole in the side, through which the liquid is flowing out. If the cup is dropped from a height, what will happen to the water flowing from the cup?

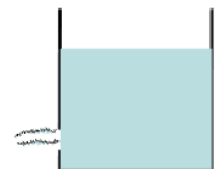
- a) It will keep coming out, flowing at the same rate as before:



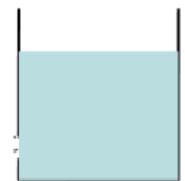
- b) It will keep coming out, but it will flow a bit slower than before.
- c) It will keep coming out, but start to flow upwards relative to the cup:



- d) It will keep coming out, flowing horizontally with respect to the falling cup:



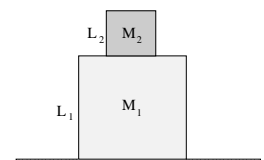
- e) It will stop flowing.



**Question 23**

Two cubes, with respective masses  $M_1$  and  $M_2$  and side lengths  $L_1$  and  $L_2$ , are lying on a smooth table as shown. What is the pressure exerted by the cubes on the table?

- a)  $\frac{M_1 g}{L_1^2} + \frac{M_2 g}{L_2^2}$
- b)  $\frac{M_1 g}{L_1^2} + \frac{M_2 g}{L_1^2}$
- c)  $\frac{M_1 g}{L_2^2} + \frac{M_2 g}{L_2^2}$
- d) None of the above.



### Question 24

Consider an object that floats in water but sinks in oil. When the object floats in water, half of it is submerged. If we slowly pour oil on top of the water so it completely covers the object, the object:

- a) moves up
- b) stays in the same place
- c) moves down

### Question 25

The pressure in a stream of water flowing out of a faucet is:

- a) largest at the bottom
- b) largest near the faucet
- c) the same everywhere in the stream

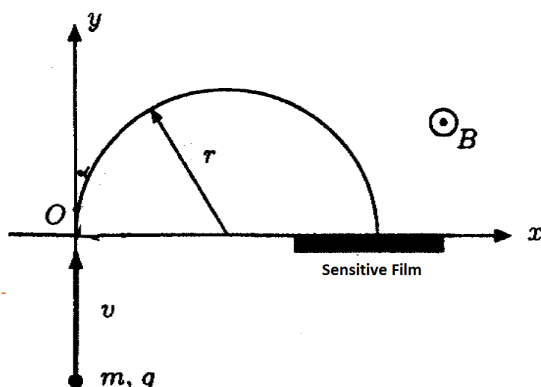


## Part B: Problems

### Problem 1 (10 points)

Mass spectrometry is a way of measuring the mass-to-charge ratio of charged particles as well as distinguishing different components of a beam of particles. As an example of such an experiment, consider ions of two isotopes of Potassium,  $K_{19}^{39}$  and  $K_{19}^{41}$ , with positive charge  $+e$  entering as a beam of particles into a region with magnetic field  $B$  in the  $z$  direction, as shown in the picture. The beam is injected at point  $O$ . Due to the magnetic field, all the ions travel in a circle with radius given by  $r = \frac{mv}{qB}$ , in which  $v$  is the velocity of ions,  $m$  is the mass of ions and  $q$  is the charge. After these two types of ions travel through the magnetic field, they reach a sensitive film and record bright spots on it as show in the picture.

Consider the distance from point  $O$  to the bright spot produced by  $K_{19}^{39}$  ion to be  $L_1$ , and the distance to the spot recorded by  $K_{19}^{41}$  ion to be  $L_2$ . We use  $L$  for the average of  $L_1$  and  $L_2$ .



(a) Assuming that the energy of both isotopes is the same, compute the ratio  $\frac{L_2 - L_1}{L}$

(b) Since these ions have different masses, one can distinguish them by observing the two bright spots on the film. However, some experimental uncertainties can complicate the situation. For example, tuning the energy of the beam to a specific value is impossible. If the energy of particles in the beam varies from  $7.9 \times 10^{-17} \text{ J}$  to  $8.1 \times 10^{-17} \text{ J}$ , is it possible to distinguish these two types of ions?

(c) Furthermore, adjusting the ions to enter exactly in the  $y$  direction, as shown in the picture, is also impossible. Assuming that the beam enters at a small angle  $\alpha$  from the  $y$  direction, compute  $L$  as a function of  $\alpha$ .

(d) Now assume that the particles in the incident beam can have any angle  $\alpha$  between  $0$  and  $3^\circ$  from the  $y$  direction, due to the uncertainty in the angle. Is it possible to distinguish the two types of ions in this case?

### Problem 2 (10 points)

Andrzej's electric bike is equipped with a motor built into the front wheel hub. The stator of the electric motor (stationary part with the coils) is fixed solidly to the axle and the magnets are attached to the wheel. He made the following measurements on the bike:

1. If he runs it on a horizontal road at 20 km/h and then switches the motor off, it takes the bike 50 m to stop.
2. At full power, it can accelerate up a  $5^\circ$  slope from 5 to 20 km/h in 6 seconds.
3. The total mass of the bike plus the rider, is 106 kg.
4. The diameter of the wheels is 66 cm.
5. The diameter of the front wheel axle is 11 mm.
6. The bike uses a 48 V battery with a capacity of 8 Ah (ampere-hour).
7. The motor is 85% efficient.

Find out:

- (a) What is the current flowing through the motor when the bike is driven at 20 km/h on a horizontal road without pedaling?
- (b) How far can it go on one battery charge at 20 km/h?
- (c) What is the net torque acting on the front axle?
- (d) What is the current flowing through the motor when the bike is accelerating up a  $5^\circ$  slope from 0 to 20 km/h in 20 seconds?
- (e) What is the full mechanical power of this motor?
- (f) What is the maximum slope one can bike up at constant speed using the motor only (not pedaling)?
- (g) What is the current flowing through the motor at maximum power?
- (h) What was the most important simplifying assumption that you had to make in order to solve this problem?

**Problem 3**

This problem has two independent parts.

**Problem 3A (5 points)**

A Russian nuclear icebreaker named *50 Let Pobedy* has a length of 160 m, a width of 30 m and two nuclear reactors, each delivering about 30 MW of power. It can clear a path 35 m wide in 2 m thick ice (at  $-10^{\circ}\text{C}$ ) at a speed of 5 knots (2.5 m/s). Somebody suggested that instead of breaking the ice, one should melt it with a gigantic heater mounted at the front of the ship.

(a) For this to work, what power should this heater have, in order to achieve the same performance as the icebreaker?

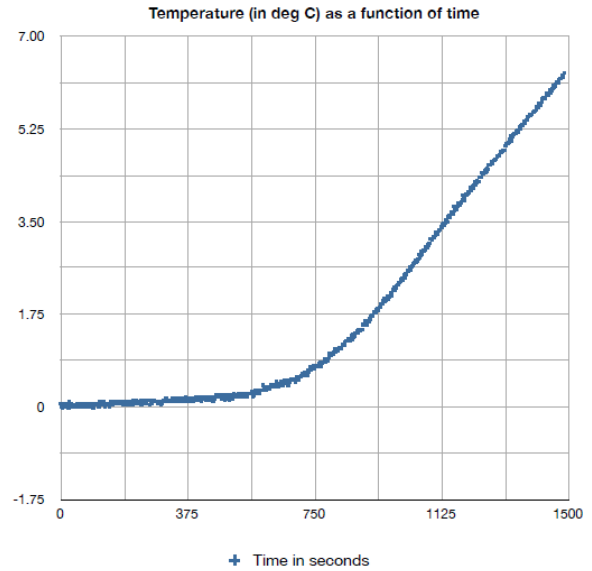
(b) Explain in one sentence whether such a heater would work or not on this icebreaker.

**Problem 3B (5 points)**

You are given a graph of temperature as a function of time of a container with a mixture of ice and water slowly heated at constant rate. When all the ice melts, there is 850 mL of water in the container. In the following questions, neglect the evaporation of water.

(a) At which rate is the container heated?

(b) How much ice was initially present?



Question 1	a	b	c	d	e
Question 2	a	b	c	d	
Question 3	a	b	c	d	
Question 4	a	b	c	d	
Question 5	a	b	c	d	e
Question 6	a	b	c	d	e
Question 7	a	b	c	d	e
Question 8	a	b	c	d	
Question 9	a	b	c	d	
Question 10	a	b	c	d	
Question 11	a	b	c	d	
Question 12	a	b	c	d	
Question 13	a	b	c	d	
Question 14	a	b	c	d	
Question 15	a	b	c	d	
Question 16	a	b	c	d	
Question 17	a	b	c	d	
Question 18	a	b	c	d	
Question 19	a	b	c	d	
Question 20	a	b	c	d	
Question 21	a	b	c	d	e
Question 22	a	b	c	d	
Question 23	a	b	c	d	
Question 24	a	b	c		
Question 25	a	b	c		