CAP High School Prize Exam

8 April 2004
9:00 – 12:00

Competitor's Information Sheet

The following information will be used to inform competitors and schools of the exam results, to determine eligibility for some subsequent competitions, and for statistical purposes. Only the marking code, to be assigned by the local examination committee, will be used to identify papers for marking.

Marking Code:

This box must be left empty.

PLEASE PRINT CLEARLY IN BLOCK LETTERS.

Family Name: ______________________ Given Name: ______________

Home Address: ____

____

______________________ Postal Code: ______________

Telephone: (   ) ________________ E-mail: ______________________

School: ______________________________ Grade: ______________

Physics Teacher: ______________________________

Date of Birth: ______________________________ Sex: ______________

Citizenship: ______________________

For how many years have you studied in a Canadian school? ____

Would you prefer further correspondence in French or English? ____

Sponsored by:

Canadian Association of Physicists
Canadian Chemistry and Physics Olympiads

Canadian Association of Physicists
2004 Prize Exam

This is a three-hour exam. National ranking and prizes will be based on a student's performance on both sections A and B of the exam. Performance on the multiple-choice questions in part A will be used to determine whose written work in part B will be marked for prize consideration by the CAP Exam National Committee. The questions in part B have a range of difficulty. Do be careful to gather as many of the easier marks as possible before venturing into more difficult territory. If an answer to part (a) of a question is needed for part (b), and you are not able to solve part (a), assume a likely solution and attempt the rest of the question anyway. No student is expected to complete this exam and parts of each problem may be very challenging.

Non-programmable calculators may be used. Please be careful to answer the multiple-choice questions **on the answer card/sheet** provided; most importantly, write your solutions to the three long problems on **three separate** sheets as they will be marked by people in different parts of Canada. Good luck.

Data

Speed of light	$c = 3.00 \times 10^8$ m/s
Gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Acceleration due to gravity	$g = 9.80\,\text{m/s}^2$
Fundamental charge	$e = 1.60 \times 10^{-19}$ C
Mass of electron	$m_e = 9.11 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.673 \times 10^{-27}$ kg
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s
Coulomb's constant	$1/4\pi\epsilon_o = 8.99 \times 10^9$ J·m/C²
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K

Part A: Multiple Choice

Question 1

You are standing upright in a room in front of a vertical mirror. In this mirror, you can see from your position only the upper two-thirds of your body. You wish to see the entire length of your body reflected in the mirror. Which combination of the following three courses of action will achieve this?

(I) Move away from the mirror;
(II) move toward the the mirror;
(III) use a mirror whose height will allow you to see your whole image when you are at your initial position.

(a) (I) only; (b) (II) only;
(c) (III) only; (d) either (I) or (III).

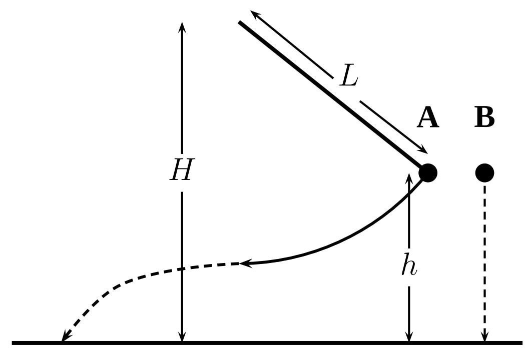
Question 2

A 5 kg cart collides on a horizontal surface with a 20 kg cart. Which cart experiences the smaller force because of the collision?

(a) The 5 kg cart; (b) the forces are equal;
(c) the 20 kg cart; (d) it depends if the collision is elastic.

Question 3

Two monkeys with the same mass stand on a branch at height h above the horizontal jungle floor. Monkey A steps off the branch holding the end of an inextensible rope of length L whose other end is tied to another branch at height H, lets go at the bottom of the swing, and falls freely to the floor, as shown below. Monkey B steps off and falls straight downward. Then, neglecting air resistance but not the tension in the rope, the total work W done on each monkey and the speed v with which each hits the floor are as follows:



(a) $W_A < W_B$, $v_A < v_B$; (b) $W_A = W_B$, $v_A < v_B$;
(c) $W_A = W_B$, $v_A = v_B$; (d) $W_A < W_B$, $v_A = v_B$.

Question 4

A person, standing on a train that is accelerating forward at $3.3\,\text{m/s}^2$, throws a ball vertically upward. Neglecting air resistance, the magnitude of the ball's acceleration relative to the train is

(a) $9.8\,\text{m/s}^2$; (b) $10.3\,\text{m/s}^2$;
(c) $7.0\,\text{m/s}^2$; (d) $13.1\,\text{m/s}^2$.

Question 5

I must cross a river in the shortest possible time. Water flows downstream at a constant 5.0 m/s between the two parallel shores. Taking the direction of the flow as reference, if my boat has a maximum speed of 10 m/s, it should head at

(a) 90°; (b) 120°;
(c) 150°; (d) 27°.

Question 6

A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal,

(a) the hoop has a greater translational speed than the cylinder;
(b) the cylinder has a greater translational speed than the hoop;
(c) the hoop and the cylinder have the same translational speed;
(d) the hoop has a greater rotational speed than the cylinder.

Question 7

A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be

(a) zero everywhere.

(b) dependent on the location along the string.

(c) not zero anywhere.

(d) non-zero and in the same direction everywhere.

Question 8

In vacuum, a potential difference V is maintained between points a distance d apart. The corresponding electric field $\boldsymbol{E}$ accelerates an electron from rest to a speed v over that distance. Which one of the following statements is true?

(a) E does not depend on d; (b) E depends on V, not on d;

(c) E depends only on d: (d) v depends on V, not on d.

Question 9

Light waves propagate along the length of a pipe containing a perfect vacuum and whose end caps are perfect reflectors. A standing-wave pattern is formed with frequency f and wavelength λ. When a gas, with index of refraction n, is introduced into the pipe, which of the following changes occurs?

(a) λ increases; (b) f increases;

(c) λ decreases; (d) f decreases.

Question 10

Students claim that in a lab experiment they witnessed a head-on *elastic* collision between two balls on a horizontal surface which resulted in the balls being both at rest. No external horizontal force was acting at any time on the masses. Which of these comments is the most appropriate about this process?

(a) Initial speeds and masses must have been different;

(b) initial speeds and masses must have been identical;

(c) initial speeds, but not necessarily masses, were identical;

(d) the process cannot have occurred as claimed.

Question 11

The potential across a $3\,\mu$F capacitor is $12\,$V when it is not connected to anything. It is then connected in parallel with an uncharged $6\,\mu$F capacitor. At equilibrium, the charge q on the $3\,\mu$F capacitor and the potential difference V across it are

(a) $q = 12\,\mu$C, $V = 4\,$V; (b) $q = 24\,\mu$C, $V = 8\,$V;

(c) $q = 36\,\mu$C, $V = 12\,$V; (d) $q = 12\,\mu$C, $V = 6\,$V;

Question 12

In subatomic physics, one often associates a characteristic wavelength λ with a particle of mass m. If $\hbar = h/2\pi$ (h being Planck's constant) and c the speed of light, which of the following expressions is most likely to be the correct one?

(a) $\lambda = \hbar c/m$; (b) $\lambda = \hbar/mc^2$;

(c) $\lambda = \hbar m/c$; (d) $\lambda = \hbar/mc$;

Question 13

At equilibrium, the electric field at a point on a closed conducting surface, whether the surface is charged or neutral, can never be

(a) tangent to the surface;

(b) perpendicular to the surface;

(c) zero;

(d) directed inward since it must vanish inside.

Question 14

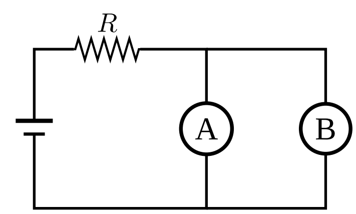
A uniform cubical box of mass M rests on a horizontal floor with one edge against a small obstruction fixed to the floor. Can a horizontal force of magnitude F applied on the box at the centre of the side opposite the obstruction tip the box?

(a) No, never; (b) yes, only if $F > mg$;

(c) yes, $F > mg/2$ is sufficient; (d) yes, only if $F > 2mg$.

Question 15

A circuit consists of a battery, a resistor R , and two light bulbs A and B as shown below:



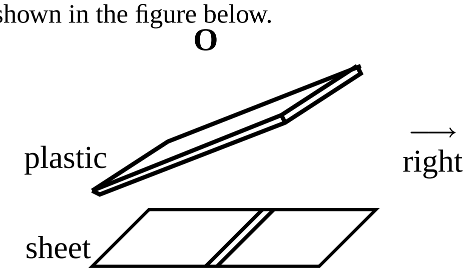
If the filament in lightbulb A burns out, then the following is true for light bulb B:

(a) It is turned off; (b) its brightness does not change;

(c) it gets dimmer; (d) it gets brighter.

Question 16

An observer at O views two closely spaced lines on the bottom sheet through an angled slab of plastic with parallel faces, exactly as shown in the figure below.



Compared to when there is no plastic, the lines appear to the observer

(a) the same but shifted to the right;

(b) shifted to the left and spaced further apart;

(c) shifted to the right and spaced closer together;

(d) exactly as they do without the plastic slab.

Question 17

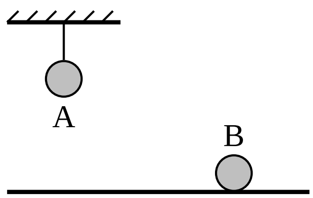
If all of Earth's polar ice were suddenly to melt into the oceans, in the short term the length of the day would

(a) increase; (b) remain the same;

(c) decrease; (d) first decrease, then increase.

Question 18

Two identical balls, A and B, of uniform composition and initially at the same temperature, each absorb exactly the same amount of heat. A is hanging down from the ceiling while B rests on the horizontal floor in the same room. Assuming no subsequent heat loss by the balls, which of the following statements is correct about their final temperatures, T_{A} and T_{B}, once the balls have reached their final dimension?



(a) $T_{\mathrm{A}} < T_{\mathrm{B}}$; (b) $T_{\mathrm{A}} > T_{\mathrm{B}}$;

(c) $T_{\mathrm{A}} = T_{\mathrm{B}}$; (d) $T_{\mathrm{A}} \leq T_{\mathrm{B}}$.

Question 19

Two uncharged balls A and B, each very light and coated with a conducting material, hang vertically side by side just touching each other. A positively-charged glass rod is brought near ball A without touching it. Now A and B are separated and then the glass rod is removed. If Q_{A} and Q_{B} represent the electric charges on A and B, respectively, you conclude that

(a) $Q_{\mathrm{A}} < 0$ and $Q_{\mathrm{B}} < 0$; (b) $Q_{\mathrm{A}} < 0$ and $Q_{\mathrm{B}} > 0$;

(c) $Q_{\mathrm{A}} > 0$ and $Q_{\mathrm{B}} < 0$; (d) $Q_{\mathrm{A}} > 0$ and $Q_{\mathrm{B}} > 0$;

Question 20

You are travelling at constant speed in an airtight car with a balloon floating motionless next to you. Suddenly, you slam on the brakes so as to stop the car quickly. During decceleration, with respect to the car the balloon

(a) moves forward; (b) remains motionless;

(c) moves backward; (d) can move forward or backward.

Question 21

Due to tidal friction on Earth, the radius R of the Moon's orbit is increasing at the rate of a few centimetres per year. During this process, the Moon's angular momentum

(a) remains constant since its speed decreases;

(b) remains constant but its total energy increases;

(c) increases as $\sqrt{R}$ while its total energy increases;

(d) decreases as $\sqrt{R}$ while its kinetic energy decreases.

Question 22

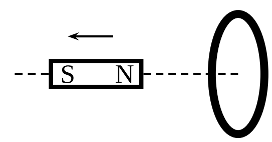
In order to measure the speed v of blood flowing through an artery, a uniform magnetic field $\boldsymbol{B}$ is applied in a direction perpendicular to the flow and a voltmeter measures the voltage across the diameter D of the artery, at right angles to $\boldsymbol{B}$. If positive and negative ions in the blood are longitudinally at rest with respect to the flow, the speed of the flow is closest to

(a) $v = V/BD$; (b) $v = BD/V$;

(c) $v = VD/B$; (d) $v = B/VD$.

Question 23

A bar magnet with its north (N) and south (S) poles as shown below is initially moving to the left, along the axis of, and away from a circular conducting loop. A current I is induced in the loop, with $\boldsymbol{a}$ the acceleration of the magnet due to this current. As seen from the magnet looking in the direction of the loop,



(a) I runs clockwise and $\boldsymbol{a}$ points to the left;

(b) I runs counterclockwise and $\boldsymbol{a}$ points to the right;

(c) I runs clockwise and $\boldsymbol{a}$ points to the right;

(d) I runs counterclockwise and $\boldsymbol{a}$ points to the left.

Question 24

You are moving a negative charge $q < 0$ at a small constant speed away from a conducting spherical shell on which resides a negative charge $Q < 0$. The electrostatic field of Q is $\boldsymbol{E}$. Let U be the total energy of q, W_{a} the work done by the force $\boldsymbol{F}_{\mathrm{a}}$ you exert on q, and W_{E} the work done by the electrostatic force $\boldsymbol{F}_{\mathrm{E}}$ on q. Then, as q is being moved,

(a) $W_{\mathrm{a}} = -W_{\mathrm{E}}$, and therefore U remains constant;

(b) $\boldsymbol{F}_{\mathrm{a}} = -\boldsymbol{F}_{\mathrm{E}}$, and therefore U remains constant;

(c) U increases;

(d) U decreases;

Question 25

An ice cube of pure fresh water floats on pure fresh water in a glass. A huge ice shelf, also of pure fresh water, floats on the ocean away from Antarctica. Neglecting the contribution due to the density of air, as the ice cube and the iceberg melt,

(a) the water level rises both in the glass and in the ocean;

(b) the water level does not change in either case;

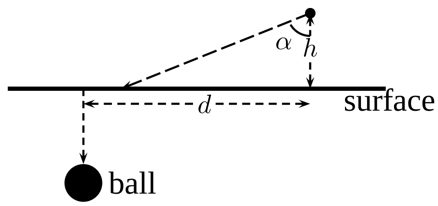
(c) the water level stays the same in the glass but rises in the ocean;

(d) the water level decreases in the glass but stays the same in the ocean.

Part B

Problem 1

A uniform ball of mass $M = 1.00\,\text{kg}$ is released from rest at the surface of a pond and sinks vertically into the water. An observer, wishing to determine the density of this ball, measures the time-dependence of the angle α between the vertical at his position, a distance h above the surface, and the direction of the ball's apparent instantaneous position (see figure). The horizontal distance between the observer and the initial position of the ball is d.



The drag force acting on the ball as it sinks is well approximated at low speed v by $F_{\text{drag}} = -bv$, where b is a constant. We also take the density of water to be $\rho_{\text{w}} = 1000\,\text{kg/m}^3$ and its refraction index as $n_{\text{w}} = 1.33$.

(a) Derive an expression for the instantaneous depth y of the ball as a function of α, h, and d.

(b) Having calculated the depth of the ball as a function of time, the observer can then construct the following table for the speed of the ball as a function of time:

Time [s]	Speed [m/s]	Time [s]	Speed [m/s]
0.000	0.000	0.225	0.8425
0.010	0.0910	0.275	0.8767
0.025	0.2086	0.325	0.8966
0.075	0.5081	0.375	0.9082
0.125	0.6823	0.425	0.9145
0.175	0.7836	0.475	0.9155

From these data, find the initial acceleration of the ball and estimate at which value its speed will stabilise. Then calculate the density of the ball and obtain a value for the drag coefficient b.

Problem 2

Bicycle headlights are often powered by a generator, with the rim of the generator's shaft rolling against the tire's rim due to the rotation of the wheel. The shaft is rigidly connected to a coil within the generator which rotates in a magnetic field $\boldsymbol{B}$.

In one such generator, the coil has 125 turns of wire and a cross-sectional area of $0.0010\,\text{m}^2$, immersed in a field of magnitude $0.080\,\text{T}$. At the area of contact with the tire, the rim of the generator's shaft has a radius of $1.25\,\text{cm}$. The tire's diameter is 66 cm.

(a) If the lightbulb in the headlight needs $5.0\,\text{W}$ of average electrical power, corresponding to a voltage amplitude of $4.0\,\text{V}$, to produce a decent amount of light, calculate the linear speed of the bicycle needed, assuming no slipping anywhere in the problem.

(b) Calculate the torque that must be supplied by the bicycle wheel to produce the required average electrical power for the bulb. State briefly two other simplifying assumptions that must be made.

(c) What is the amplitude of the current induced in the coil under these conditions?

(d) To the extent that the lightbulb obeys Ohm's Law, and that its resistance is constant, by what factor would the power delivered to the bulb increase if the cyclist tripled the bicycle's speed (presumably downhill or at the Tour de France)?

Problem 3

The long jump as an Olympic sport dates back at least as far as 700 BC. Unlike in the modern Olympics, it was practised not only from a running start, but also from a standing start. The length of a jump was measured (as it still is) from the point where the back of the feet touched the ground on landing. Paintings on ancient Greek vases depict athletes jumping from a standing start while holding compact weights in both hands. Examples of these have been dug up by archaeologists, and typical ones, made of stone or lead, each have a mass of around 3 kg.

(a) Write a short paragraph explaining qualitatively how athletes could boost their performance, ie. jump further from a standing start, by carrying such weights. More precisely, describe what they would have done with the weights during the jump.You can accept that, as biomechanics research has shown, a loaded body could take off at the same speed and angle as an unloaded one.

(b) Estimate by how much the length of a 3 m jump could be increased for a 65 kg athlete jumping at an angle of 50°. Remember that this takes place from a standing start, not a running start like today's long jump. You can take the centre of mass of the body to be at a height of 1 m above ground and the length of the arms to be 65 cm. Also, at the moment of take-off, the athlete is leaning forward so that his shoulders are about 15 cm in front of the centre of mass of his body.

* * * *