Multifractal based return interval approach for short-term electricity price volatility risk estimation

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Abstract — Increasing capacities of renewable energy resources have been plugged into the power systems in recent years, while at the same time the ever increasing uncertainties has made the risk management in electricity market a more difficult issue for market participants in the context of optimizing their portfolios. Among numerous risk factors in competitive electricity market environment, the highly volatile electricity price contributes most to the financial risk of the power portfolios, especially in short-term risk management scenarios such as the spot market and real-time balancing market. Research has shown that the fluctuations of electricity prices exhibit multifractal characteristics, yet little work has been done on the price volatility risk evaluation based on the multifractal theory. This paper hence examines the feasibility of applying the multifractal theory to analyze the electricity price fluctuation, and further develops the application of multifractal theory in evaluating financial risk caused by electricity price volatility. A modified return interval approach considering the parameters of multifractal characteristics is employed to estimate the value-at-risk (VaR) of the electricity price. The fluctuant electricity price data series in the Pennsylvania-New Jersey-Maryland (PJM) energy market are adopted as examples to test the effectiveness of the proposed VaR estimation method for short-term electricity price volatility risk evaluation.

Index Terms — electricity market, electricity price, price risk estimation, multifractal theory, return interval approach (RIA)

1. Introduction

Deregulated electricity markets have been established in many countries all over the world in the past few decades [1]. The competitive electricity markets enable market participants to schedule their portfolios in various markets with different time scales, such as the day-ahead (DA) market, real-time (RT) market and forward market [2]. While scheduling their portfolios, market entities also have to consider the uncertainty factors in power systems that would introduce certain risks. These risks need to be analyzed effectively by the participants so as to maximize the economic benefits and at the same time mitigate possible risks.

Typically, the risks in electricity markets are related to the uncertainties of power demands, unplanned outages, system congestion, system reserves, bidding strategies of generation companies and large consumers, and so on [3]. In addition, the electricity producers also face the risks caused by the fluctuation of fuel prices, operational decisions and market conditions [4]. Besides, more uncertainty factors have been introduced into the electricity market as the renewable energy sources are widely being plugged into the power systems. For instance, the uncertainties of wind power and solar radiation not only add up the possible unbalance between power production and consumption, but also enlarge the volatility of electricity prices [5,6]. The difficulties of risk analysis and aversion have hence increased significantly for electricity producers [7].

Although risk analysis methods have been studied for quite a long time in other financial markets such as stock markets, the feasibility of these methods in the electricity market is still questionable because electricity markets need real-time balancing and electricity is difficult and expensive to store. Generally, the financial risk in electricity markets mainly depends on the volatility of electricity prices and the unpredictable unbalance between power production and consumption, known as the price risk and the volume risk, respectively [4]. Compared with the uncertainties of power production and consumption, the fluctuations in electricity prices are more volatile and difficult to predict, especially in the RT balancing market [8]. Considerable research has been done on the risk assessment in electricity markets focusing on the risk caused by the electricity price fluctuation. For example, a historical data based Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) assessment for electricity price risk is proposed in [9]; a stochastic price based unit commitment model is presented in [10] to evaluate the financial risks in electricity markets; the Monte Carlo simulation method is used in electricity price risk management in [11]; the optimal electricity procurement problem for large consumers considering the electricity price fluctuation is discussed in [12]; the price risk in an electricity market with a hybrid structure is addressed in [13]; the price risk of power portfolios in multi-markets is analyzed in [14] based on the well-established mean-variance model.

In addition, different models and methods have been used for simulating the electricity price fluctuation so as to achieve better predictions for risk assessment. In [15], the Gaussian distribution is used to analyze the suppliers' risks in the DA electricity market. The artificial neural network and data of similar days are adopted in [16] to predict electricity prices in the Pennsylvania-New Jersey-Maryland (PJM) electricity market. A fuzzy inference system is employed to forecast the electricity price in the DA electricity market in [17]. Autoregressive models such as the Autoregressive Integrated Moving Average (ARIMA) model and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model are employed for simulating electricity prices in [18] and [19], respectively.

Over the past few years, it has been empirically validated by some researchers that the electricity prices always exhibit multifractal properties [20-23]. However, these works mainly emphasize on the verification of multifractal theory in electricity market environment, little attention has been paid to the application of multifractal analysis in evaluating electricity price risk [8]. For accurate electricity price risk assessment, this paper examines the multifractal characteristics of the electricity price series to verify if the multifractal theory is suitable for price risk estimation in electricity markets. Popular risk assessment methods mostly rely on stochastic simulations, and their applications in assessing the electricity price risk with multifractal characteristics might be limited since the multifractal does not have an explicit expression or a distribution function. Thus, the return interval approach (RIA) is utilized to estimate the electricity price volatility risk with the modification of multifractal characteristics, and the widely used VaR is selected as the risk measure.

The remainder of this paper is organized as follows. Section 2 analyzes the characteristics of electricity price fluctuation with both the Gaussian distribution and the multifractal theory. Section 3 provides a modified RIA method considering the multifractal features of electricity prices and proposes a price risk VaR estimation model. In Section 4, the hourly electricity price data in the PJM RT electricity market is served as an example to verify the effectiveness of the proposed method and further analyses and comparisons are also carried out. Discussions and conclusions are presented in Section 5.

2. Analysis of the electricity price fluctuation

2.1 Limitations of the Gaussian distribution

Typically, the characteristics of electricity price volatility are described and analyzed based on the Gaussian distribution. The parameters of the Gaussian distribution can be easily obtained and the follow-up analyses such as risk assessment and trend prediction have been deeply studied. However, in short-term electricity markets such as the DA and RT markets, the electricity prices may fluctuate violently due to many uncertainties in the electricity market as well as the associated power systems, e.g., changes in demand, fuel price, bidding strategies of generation companies and large consumers. Thus, it is widely believed that the electricity price in a competitive electricity market cannot be very well modeled by a simple Gaussian random process.

The well-established Jarque-Bera (J-B) test [24] and Kolmogorov-Smirnov (K-S) test [25] are used to validate the feasibility of the Gaussian distribution in describing the fluctuation of electricity price series. Hourly RT and DA locational marginal pricing (LMP) data from January 1st, 2008 to December 31st, 2012 in the PJM electricity market are served as test samples [26]. The statistical results of these tests are shown in Table 1. Moreover, the cumulative distribution functions (CDFs) of electricity prices in both the PJM RT and DA electricity markets are shown in Fig. 1.

Table 1 The Gaussian distribution validation tests of electricity prices in PJM electricity markets

Fig. 1 *The CDFs of actual electricity prices compared with the theoretical CDFs of the Gaussian distribution: (i) the RT electricity price data; (ii) the DA electricity price data*

The results in Figs. 1 show that the CDFs of both the RT and DA electricity prices are quite different from that of the standard Gaussian distribution. Moreover, following fluctuation features of electricity prices in the PJM electricity markets can be concluded from the statistical results in Table 1:

1) The kurtosis factor of a set of data that follows the standard Gaussian distribution should be 3. However, the kurtosis factors of both RT and DA data obtained from the J-B test are much larger than 3. In addition, the kurtosis factor of the RT data is also larger than that of the DA data, which indicates the electricity prices in the RT balancing market tend to suffer more peaks and fluctuate more intensely;

2) The skewness factor of a set of data that follows the standard Gaussian distribution should equal to 0. Whereas the skewness factors of both the RT and DA data obtained from the J-B test are larger than 0, which means the electricity prices are more likely to fluctuate above their average value;

3) If the data follows the Gaussian distribution, the threshold of the J-B statistics can be estimated by the distribution of $\chi^2(2)$. The threshold value at the significance level of 5% and 0.1% are 5.9915 and 13.8155, respectively. Apparently the J-B statistics of both the RT and DA data are much larger than the acceptable threshold value;

4) If the data follows the Gaussian distribution, the threshold of the K-S statistics at the significance levels of 5% and 1% are 0.0065 and 0.0078, respectively. The K-S statistics of both the RT and DA data also exceed the acceptable threshold value.

Results from J-B test and K-S test have both rejected the original assumption that electricity prices in the PJM RT and DA electricity markets can be analyzed by the Gaussian distribution. Thus, the electricity market risk assessment based on the Gaussian distribution cannot fully illustrate the fluctuation characteristics of electricity prices, especially in the RT balancing market.

2.2 The multifractal theory

The multifractal theory is an extension of the concept of fractal. In the past few decades, fractal and multifractal have been validated to be effective methods for dealing with complex problems [27,28]. For example, the multifractal theory has been widely studied and applied in fields such as economics [29], geography [30] and medicine [31].

The definition of fractal and multifractal can be inferred from the self-affine process [28]. Considering a time series $X(t)$, like the electricity price, if $\forall c>0$, $\exists H>0$, and accommodates Eq. (1), then $X(t)$ can be seen as self-affine.

$$X(ct) = c^H X(t) \tag{1}$$

By replacing c in Eq. (1), the self-affinity of $X(t)$ can be expressed as:

$$X(t) = t^H X(1) \tag{2}$$

Eq. (2) can be extended to a more general equation as follows:

$$E(|X(t)^{q}|)=t^{Hq}E(|X(1)|^{q}) \tag{3}$$

where $E(\cdot)$ stands for the operation of mathematical expectation.

Define $c(q)$ and $\tau(q)$ as deterministic functions with domain T and Q, respectively. T and Q are defined as real intervals, and $0\in T$, $[0,1]\subseteq Q$. $c(q)$ and $\tau(q)$ can be presented as:

$$c(q)=E(|X(1)|^{q}) \tag{4}$$

$$\tau(q)=Hq-1 \tag{5}$$

By combining Eq. (3)-(5), a multifractal process is defined as:

$$E(|x(t)|^{q})=c(q)t^{\tau(q)+1} \tag{6}$$

The fractal and multifractal theory mainly focus on the self-similarity of a time series. In (6), the scaling function $\tau(q)$ contributes more to the scaling properties of $X(t)$. The scaling function determines the fractal characteristics of the time series. If $\tau(q)$ is linear, then $X(t)$ is called fractal or uni-fractal, while the $\tau(q)$ of a multifractal process should be a non-linear function.

Typically, the multifractal spectrum $f(\alpha)$ and the local Hölder exponent α are served as the characteristic parameters of a multifractal process. The relationships among $\tau(q)$, α and $f(\alpha)$ are described through the Legendre transform [32], as shown in (7) and (8).

$$\alpha=\frac{d\tau(q)}{dq} \tag{7}$$

$$f(\alpha)=\underset{q}{Inf}[q\alpha-\tau(q)] \tag{8}$$

2.3 Estimation of the multifractal spectrum

The box-counting method is adopted to calculate the multifractal spectrum of electricity price series in this paper [33]. For an electricity price series $\{X(t)\}$, t=1,2,…,N, the box-counting steps are as follows.

1) Divide the electricity price data into a number of normalized boxes of size δ, $\delta=\Delta/N$, where $\Delta\in[1,N]$ and N is divisible by all the Δ.

2) Let $P_i(\delta)$ denote the average probability in the divided box i, i=1,2,…,M, and $M=\delta^{-1}$. Then $P_i(\delta)$ can be calculated as follows.

$$P_i(\delta) = \sum_{k=1}^{\Delta} X(i_k) / \sum_{t=1}^{N} X(t) \tag{9}$$

where $X(i_k)$ represents the k-th electricity price in box i. $P_i(\delta)$ obtained by (9) accommodates the power-law distribution displayed in (10).

$$P_i(\delta) \sim \delta^{\alpha} \tag{10}$$

3) Let $S_q(\delta)$ denote the partition function defined in (11). $S_q(\delta)$ also accommodates the power-law distribution shown in (12).

$$S_q(\delta) = \sum_{i=1}^{M} P_i^q(\delta) \tag{11}$$

$$S_q(\delta) \sim \delta^{\tau(q)} \tag{12}$$

where q represents the moment order coefficient and $q \in (-\infty, +\infty)$.

4) Plot the bi-logarithmic curve of $\ln S_q(\delta) - \ln \delta$, and the scaling function $\tau(q)$ can be obtained by calculating the slope value in the linear part of the curve. Afterwards, the Legendre transforms in (7) and (8) are used to estimate the local Hölder exponent α and the multifractal spectrum $f(\alpha)$.

2.4 Characteristics of the multifractal spectrums in electricity markets

The electricity prices normally display self-similarities as in most cases the prices tend to rise at peak demand hours and decline at valley hours, so the volatility of electricity prices can be suitably described and analyzed by multifractal theory and multifractal spectrums. However, the multifractal spectrum $f(\alpha)$ is a nonlinear convex function of α. In order to verify the multifractal characteristics of electricity price data sequences, the hourly electricity prices in the PJM RT and DA electricity markets with different time scales are adopted as test examples. The original price data of PJM RT and DA electricity markets from January 1st, 2008 to December 31st, 2012 are shown in Fig. 2. Fig. 3

demonstrates the multifractal spectrums of RT and DA price series with four different sets of data:

1) Five-year electricity price from January 1st, 2008 to December 31st, 2012

2) One-year electricity price from January 1st, 2012 to December 31st, 2012

3) Summer electricity price from June 1st, 2012 to August 31st, 2012

4) Winter electricity price from October 1st, 2012 to December 31st, 2012

Fig. 2 *Electricity price series: (i) the DA electricity price data; (ii) the RT electricity price data*

Fig. 3 *Multifractal spectrums with different sets of data: (i) the multifractal spectrum of the DA electricity price data sets; (ii) the multifractal spectrum of the RT electricity price data sets*

Intuitive observations of Figs. 2 show that the overall trends of the DA electricity price data and the RT electricity price data are very similar, except that fluctuation in the RT electricity price data can be more volatile. But in Figs. 3, the multifractal spectrums of the DA electricity price are quite different from that of the RT electricity prices with data sets at the same time period. As can be seen in Figs. 3, the multifractal spectrums of the DA (RT) price with the data length of five years and one year have similar shapes. On the other hand, the multifractal spectrums of shorter length of data, in this case the different seasons, can be quite different since the characteristics of electricity prices at different seasons are different as shown in Figs. 2. As a result, the electricity price with different time intervals are validated to exhibit multifractal properties, and the multifractal spectrums will change in respond to the electricity price series with different characteristics.

However, the multifractal spectrums cannot be expressed with a common distribution function, which makes multifractal harder to be employed, special indexes are needed to explain the multifractal spectrum and provide useful information about the fluctuation characteristics of the spectrum for further analyses.

Let α_{max} and α_{min} denote the maximum and minimum value of the local Hölder exponent α, and α_0 denote the value of α when q=0 in (7). The following indexes are employed to describe the characteristics of the multifractal spectrum.

1) The overall magnitude of fluctuation in electricity price series is denoted by $\Delta\alpha=\alpha_{max}-\alpha_{min}$. The larger the value of $\Delta\alpha$ is, the more severe are the fluctuation in the electricity price data.

2) Let S_α denote the overall tendency of the electricity price over the estimated period, which can be obtained by the following equation. If the electricity price tends to rise, then $S_\alpha>1$; otherwise $S_\alpha<1$.

$$S_\alpha = \frac{(1-f(\alpha_{\max}))(\alpha_0-\alpha_{\min})}{(1-f(\alpha_{\min}))(\alpha_{\max}-\alpha_0)} \tag{13}$$

3. The price risk assessment model

3.1 RIA in price risk assessment

Since the fluctuations in the electricity price have been validated to exhibit multifractal characteristics in [20,22] and the test results in Section 2, the fluctuation extent index $\Delta\alpha$ and the tendency index S_α can be adopted to analyze the potential risk in the electricity market caused by the electricity price fluctuation. Though the proposed indexes are able to illustrate the fluctuation characteristics of the electricity price series, they still cannot be regarded as those commonly used risk assessment indexes with explicit economic meanings such as VaR and CVaR. As a consequence, the multifractal indexes as well as the original electricity price data should be subjected to further analysis. The RIA methodology is employed as an indirect approach to estimate the VaR of the electricity price volatility with the multifractal characteristics.

The idea of return interval focuses on the analysis and estimation of the occurrence of extreme events, which is similar to the definition of risk. The probability distribution of the return interval statistics can be quite different due to the different characteristics of the original data. Generally, return interval based analyses have been proved to be effective when dealing against time series with temporal scaling properties and long-term correlations, as well as multifractal characteristics [34,35].

Consider an electricity price series $\{X(t)\}$, t=1,2,…N. The return interval r_i is defined as the time interval between two adjacent extreme events $X(t_{j1})$ and $X(t_{j2})$ which exceed a certain threshold V. In other words, $r_i=t_{j2}-t_{j1}$, $t_{j1}<t_{j2}$, $X(t_{j1})\geq V$, $X(t_{j2})\geq V$, and $\forall t_s\in(t_{j1},t_{j2})$, $X(t_s)<V$. An illustration of the return interval is shown in Fig. 4. If $\{r_i\}$ denotes the return interval series for a specific threshold V, i=1,2,…,N_V, then the average return period R_V is defined as follows.

$$R_V = 1/\int_V^{\infty} P(X)dX = N/N_V \tag{14}$$

where $P(X)$ denotes the probability density function of the electricity price series $X(t)$.

Fig. 4 *Demonstration of the return intervals under the effect of V*

The statistics of the return interval series $\{r_i\}$ contain the information of non-linear correlations in the original electricity price data $\{X(t)\}$. Let $P_V(r)$ denote the probability density function of $\{r_i\}$ under the effect of threshold V. The existing research has validated that when the original data series have multifractal characteristics, a power-law behavior of $P_V(r)$ as shown in (10) exists [35].

$$P_V(r) \sim (r/R_V)^{-\theta(V)} \tag{15}$$

where the power exponent $\theta(V)$ is a monotonically decreasing function of V and accommodates $\theta(V)$>1.

Assume that the current moment is t_c and the last extreme event when V is exceeded occurs at Δt_l before. Let

$B_V(\Delta t_l,\Delta t)$ denote the probability that another extreme event will happen in the following time period Δt starting from t_c, as shown in (16).

$$B_V(\Delta t_l, \Delta t) = \int_{\Delta t_l}^{\Delta t_l + \Delta t} P_V(r)dr \Big/ \int_{\Delta t_l}^{\infty} P_V(r)dr \tag{16}$$

According to the power-law behavior in (15), a simplified result can be obtained in (17) when Δt is much less than Δt_l.

$$B_V(\Delta t_l, \Delta t) = 1 - (1 + \frac{\Delta t}{\Delta t_l})^{1-\theta(V)} \approx (\theta(V) - 1)\frac{\Delta t}{\Delta t_l} \tag{17}$$

However, if Δt is not much less than Δt_l, the result in (17) will be inaccurate. In this case, (17) can be modified as follows.

$$B_V(\Delta t_l, \Delta t) = \frac{(\theta(V) - 1)\Delta t}{\Delta t_l + (\theta(V) - 1)\Delta t} \tag{18}$$

When analyzing the wildly fluctuant electricity price at the confidence level of c, an appropriate method must estimate the VaR value such that the probability of electricity prices exceeding the VaR is, at most, $1-c$. Combined with the concept of the return interval, the estimated VaR value is equivalent to the threshold value V if (19) holds

$$R_V = 1/(1 - c) \tag{19}$$

3.2 Modification of RIA

The key step of estimating VaR by RIA is the calculation of the power exponent $\theta(V)$. In practice, $\theta(V)$ can be obtained through linear fitting in the bi-logarithmic graph of $P_V(r)$. However, the size of electricity price data is relatively limited compared with other mature financial markets such as the stock market and, therefore, the linear fitting of the electricity price by RIA might introduce a certain amount of error. Taking the electricity price data of the PJM RT electricity market as an example, Fig. 5 shows the fitting results when R_V is set to 5 and 20, respectively.

Fig. 5 *The linear fitting results through bi-logarithmic graph of $P_V(r)$*

In order to enhance the estimation accuracy of $\theta(V)$, the indexes of multifractal characteristics are used to modify the RIA analysis results through compensation. Considering that $\Delta\alpha$ reflects the extent of the price fluctuation and S_α indicates the price tendency, the original probability of extreme event occurrence in (18) shall be compensated as

$$B_V(\Delta t_l, \Delta t) = S_\alpha \log_{10}(1+\Delta\alpha)\frac{(\theta(V)-1)\Delta t}{\Delta t_l + (\theta(V)-1)\Delta t} \tag{20}$$

In (20), the multifractal characteristics of the electricity price time series have been added in the RIA analysis. If the time series suffers more violent fluctuation or tends to rise, the probability of the extreme event occurrence increases and vice versa.

3.3 RIA procedures of VaR estimation in electricity markets

With consideration of the multifractal characteristics of electricity price data, RIA can be used to estimate the VaR of the electricity price fluctuation in the following time period Δt starting from t_c. The procedures are as follows.

1) Set up the initial threshold value V according to the confidence level c of VaR and the available hourly electricity price data. V is adopted as the initially estimated VaR of the next trading hour.

2) Analyze the multifractal spectrum of the available historical price data and calculate indexes $\Delta\alpha$ and S_α.

3) Estimate the power exponent $\theta(V)$ under the effect of the threshold V through RIA.

4) Δt_l and Eq. (20) are adopted to calculate the value of $B_V(\Delta t_l,\Delta t)$.

5) If $|B_V(\Delta t_l, \Delta t) - (1-c)| \le \xi$, then V will be considered as the appropriate VaR at confidence level c, where ξ stands for the permissible error of the VaR estimation. Otherwise, proceed to Step 6.

6) If proceeding to this step for the first time, record the result of the sign function shown in (21), denoted as S_{g0}, and proceed to Step 7; otherwise, compare the sign result with S_{g0}. If the signs are the same, then proceed to Step 7; otherwise, the current threshold value V should be regarded as the VaR.

$$S_{g0} = \mathrm{sgn}(B_V(\Delta t_l, \Delta t) - (1-c)) \quad (21)$$

7) If $B_V(\Delta t_l,\Delta t)>(1-c)$, add a fixed value ΔV to threshold V, then repeat the RIA from Step 3. Correspondingly, if $B_V(\Delta t_l,\Delta t)<(1-c)$, deduct V by ΔV each time this step is followed.

The flowchart of the VaR assessment through the modified RIA is shown in Fig. 6.

Fig. 6 *The flowchart of the VaR estimation through the modified RIA*

4. Case study and simulation results

The hourly electricity price in the PJM RT balancing market is employed to illustrate the validity of the proposed VaR estimation method for assessing the short-term price risk caused by the electricity price volatility. For demonstration, the VaR of the electricity price volatility in 2012 is estimated by analyzing the available historical data from 2008 to 2011.

4.1 VaR estimation

Let ΔT_m and ΔT_r denote the size of available data used in the multifractal spectrum calculation and RIA, which are set to 720 and 8760, respectively. More data are needed in RIA to enhance the accuracy of the $\theta(V)$ estimation, while less data are used in multifractal spectrum calculation because the fluctuation characteristic of the electricity price data in the RT balancing market displays a substantial correlation with the recent past electricity price data in the same market. In this test, Δt, ξ and ΔV are set to 1 hour, 0.1% and 0.05\$/(MW·h), respectively. The VaRs of the RT electricity prices estimated by the proposed method at confidence levels of c=99% and c=95% are shown in Fig. 7.

Fig. 7 *The estimated VaR value of the RT electricity price in 2012: (i) c=99%; (ii) c=95%*

4.2 Result analysis

As can be seen from Figs. 7, the VaR estimation results by the proposed modified RIA method coincide essentially with the peak price in practice. Detailed analysis is carried out to examine the efficiency of the proposed method. The statistics of the VaR estimation results are listed in Table 2.

Table 2 The statistics of the VaR estimation results with the modified RIA method

If the estimated VaR of the RT electricity price is lower than the actual value, it is considered an estimation failure since the predicted VaR underestimates the peak fluctuation in the incoming trading hour. As shown in Table 2, the failure rate of the VaR estimation of the PJM RT balancing market in 2012 is slightly lower than the theoretical failure rate at both confidence levels of c=99% and c=95%. Thus, the effectiveness of the proposed method for the short-term VaR estimation stands verified.

The effectiveness of VaR estimation in price risk analysis and aversion is deeply related to the estimation accuracy. For example, a fixed VaR of 300$/(MW·h) has a much lower failure rate but hardly contributes to electricity price risk management. Define the estimation error series as the estimated VaR series minus the actual electricity price data, and the failure series as the absolute estimation errors when the VaR is lower than the actual electricity price. The following indexes are used to analyze the accuracy of the VaR estimation, and the statistics of the deviations are listed in Table 3.

1) The average value of the estimated VaR series of electricity price fluctuation, denoted as V_{aver};

2) The average estimation error of the error series, denoted as A_{error};

3) The standard deviation of the error series, denoted as D_{error}.

4) The estimation error summation of the failure series, denoted as F_{sum}.

Table 3 Statistics of VaR estimation deviations

Several conclusions can be drawn from the statistics in Table 3 and Figs. 7. Although the estimation results contain certain amount of error, the VaR estimated by the proposed RIA method can still be regarded to have trends very similar to the actual RT electricity price considering that the VaR estimation is different from the electricity price prediction, which means smaller A_{error} and D_{error} do not necessarily indicate better estimation results. The estimation errors A_{error} and D_{error} decrease significantly with the confidence level and the same conclusion can also be reached by observing the curves in Figs. 7. The value of F_{sum} corresponding to the failure series indicates the total possible financial loss caused by the electricity price fluctuation, and the value of F_{sum} at c=99% is much smaller than that at c=95%. Thus, different confidence levels can be specified according to the risk preferences of participants in electricity markets.

4.3 Comparisons

Two traditional VaR estimation methods are used for comparison. One is the Gaussian distribution based VaR estimation, by employing the average price and the standard deviation. The other is based on the statistics of historical data regardless of its distribution. Fig. 8 demonstrates the VaR estimation results at c=99% and c=95% by these two reference methods considering the most recent 100 available data. Further analysis of the estimation deviations and validity are listed in Table 4.

Fig. 8 *Estimated VaR value of RT electricity price in 2012 based on Gaussian distribution and historical data: (i) c=99%; (ii) c=95%*

Table 4 Statistics of VaR estimation validity and deviations of the two reference methods

As shown in Table 4, both the Gaussian distribution based and historical data based methods have higher estimation failure rates than the proposed RIA method. Furthermore, the estimation deviations indicate that the results obtained by the Gaussian distribution based VaR are apparently more accurate than those by the historical data based VaR at c=95%, while the results obtained by the historical data based VaR show significant advantages at c=99%. However, neither of these two methods is better than the proposed modified RIA method in terms of the validity and estimation accuracy, and these can be intuitively observed from Figs. 7 and 8 as well.

One possible explanation for the higher estimation failure rates of both Gaussian distribution based and historical data based methods may lie in their statistical features. Both these methods rely on the statistics parameters of historical electricity price data, which means the short-term electricity price volatilities and fluctuation characteristics will be covered up by the overall statistics. In fact, the failure rates and accuracies of VaR estimation by the Gaussian distribution based and historical data based methods can become even worse with more historical electricity price data taken into account. So the Gaussian distribution based and historical data based methods may be suitable in estimating the long-term trend of electricity prices, but they are not able to efficiently illustrate the volatilities of electricity price in short-term cases, e.g. in the DA and RT market.

5. Discussions and conclusions

As the fluctuation characteristics of electricity prices cannot be fully revealed by the most commonly used

Gaussian distribution, this paper has discussed and verified the multifractal nature of electricity prices in the PJM electricity market. Then, a modified RIA based VaR estimation method is proposed to analyze the financial risk caused by price fluctuation in electricity markets. Through the example of the PJM RT market where the electricity price data has severe volatility, the proposed modified RIA method has been validated to be an effective and accurate method for estimating the VaR of the electricity price. Compared with the Gaussian distribution based VaR estimation method and another widely used historical data based method, the proposed method shows significant advantages. Thus, more accurate analysis can be carried out to help electricity market participants manage the financial risk caused by the electricity price fluctuations by applying the proposed modified RIA method with characteristics of multifractal taken into account.

On the other hand, the proposed method requires sufficient historical data and the estimation process needs more computational time than the traditional methods such as the Gaussian distribution based method because of its computational complexity. Furthermore, risk management in electricity markets goes beyond the risk of electricity price volatility. Other risks such as volume risk and operational risk also need further studies to accommodate the future power systems with more energy sources as well as uncertainty factors integrated.

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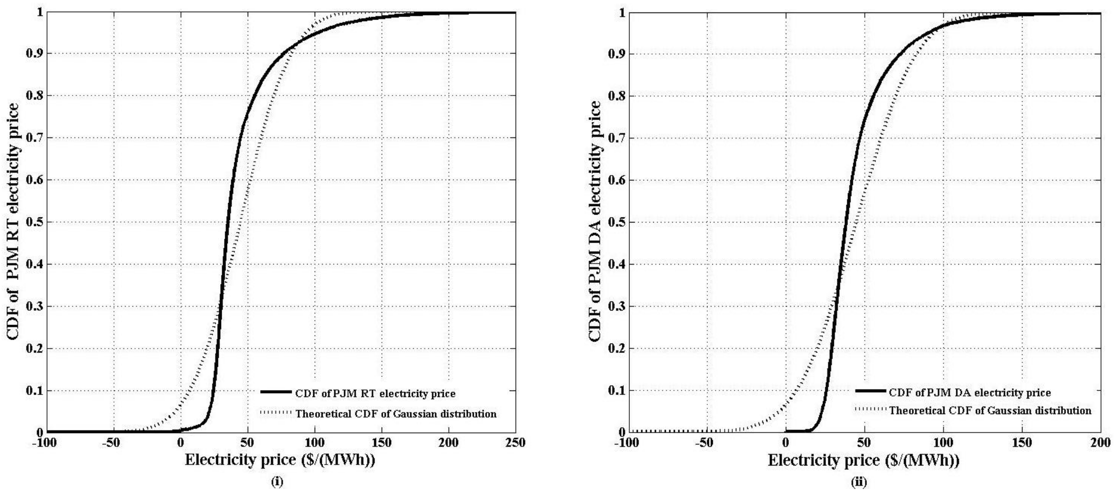


Fig. 1 *The CDFs of actual electricity prices compared with the theoretical CDFs of the Gaussian distribution: (i) the RT electricity price data; (ii) the DA electricity price data*

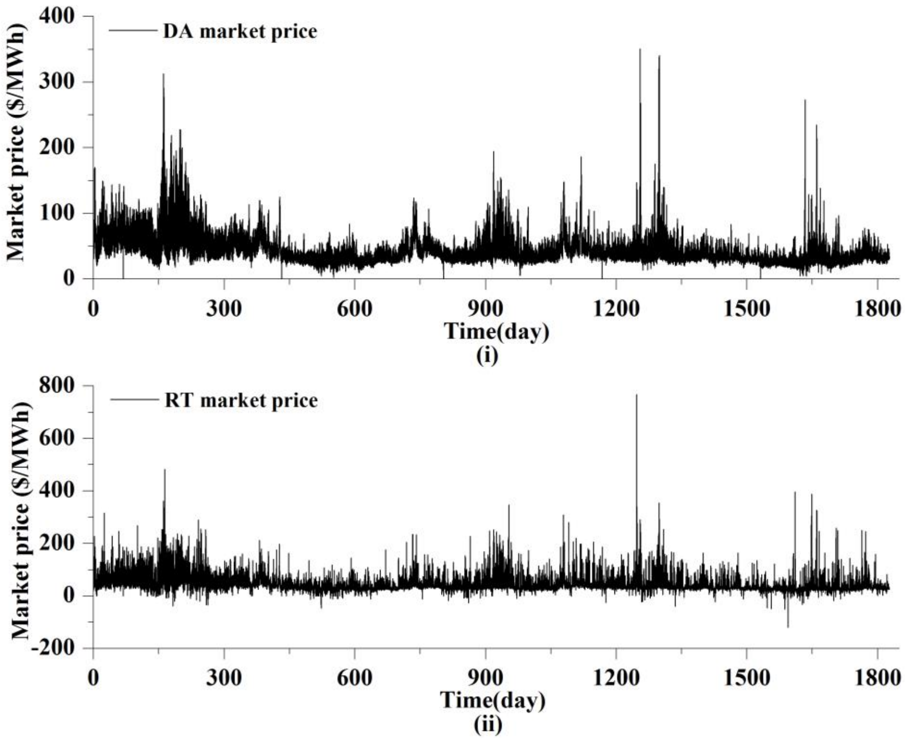
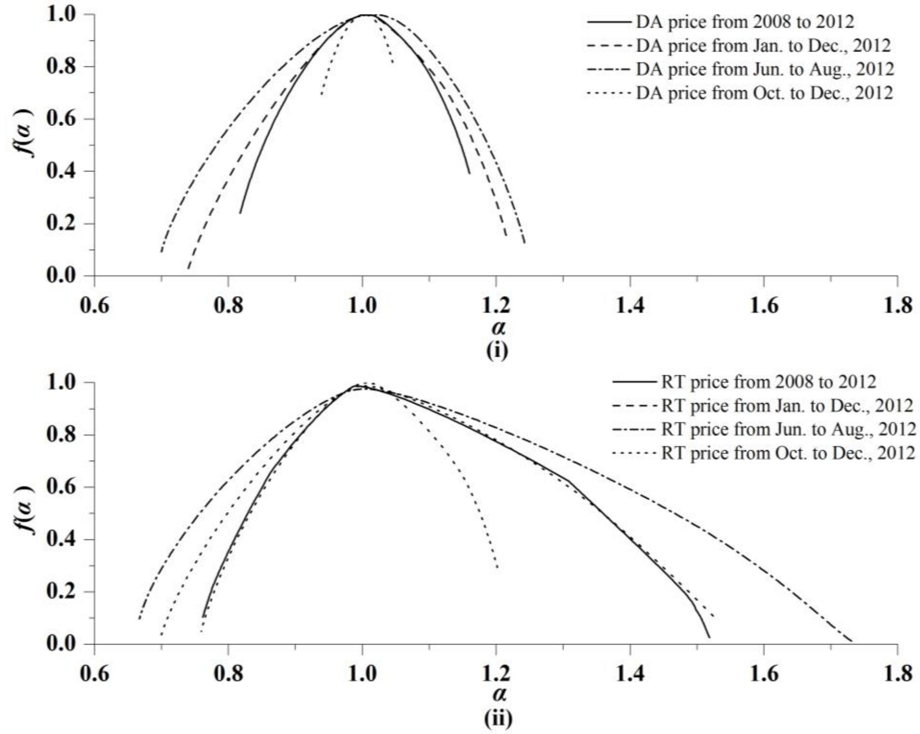


Fig. 2 *Electricity price series: (i) the DA electricity price data; (ii) the RT electricity price data*



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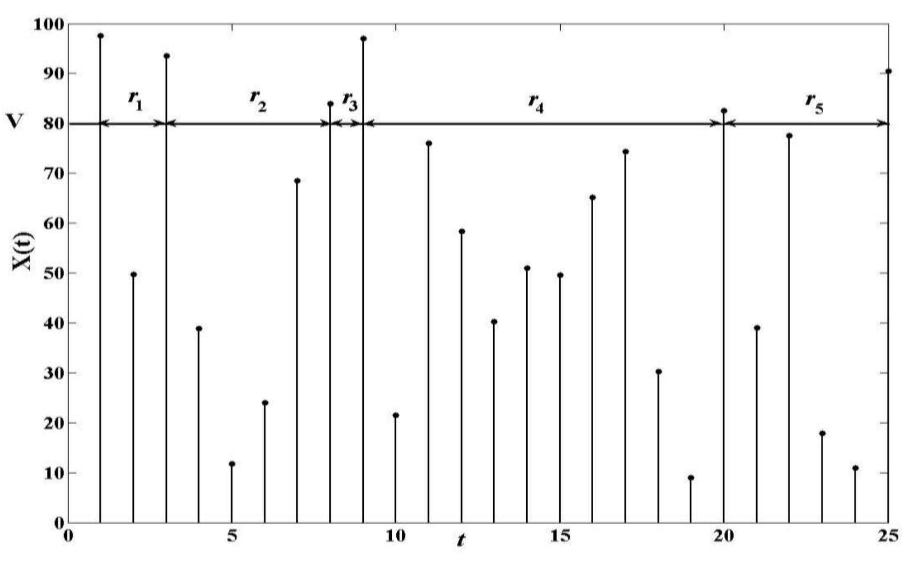

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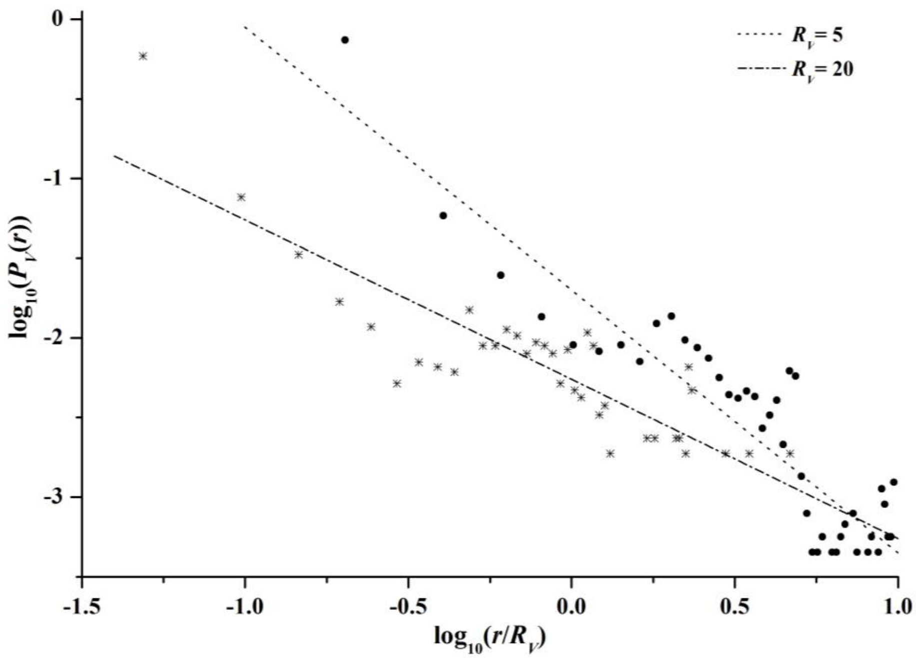

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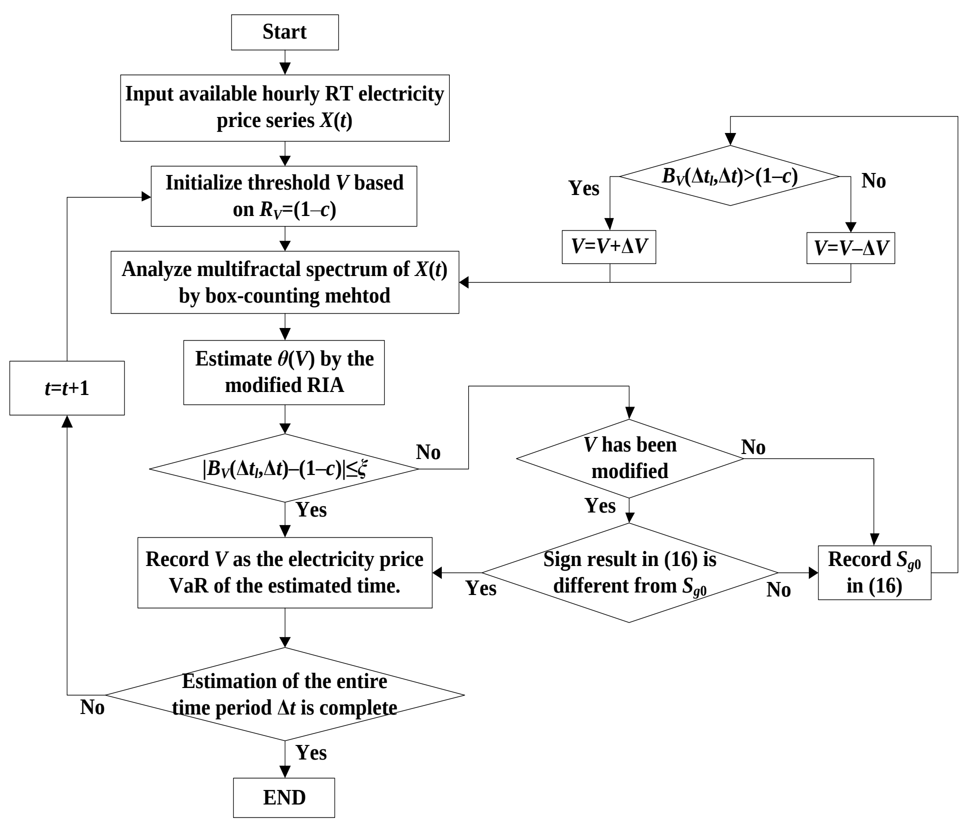

Fig. 6 *The flowchart of the VaR estimation through the modified RIA*

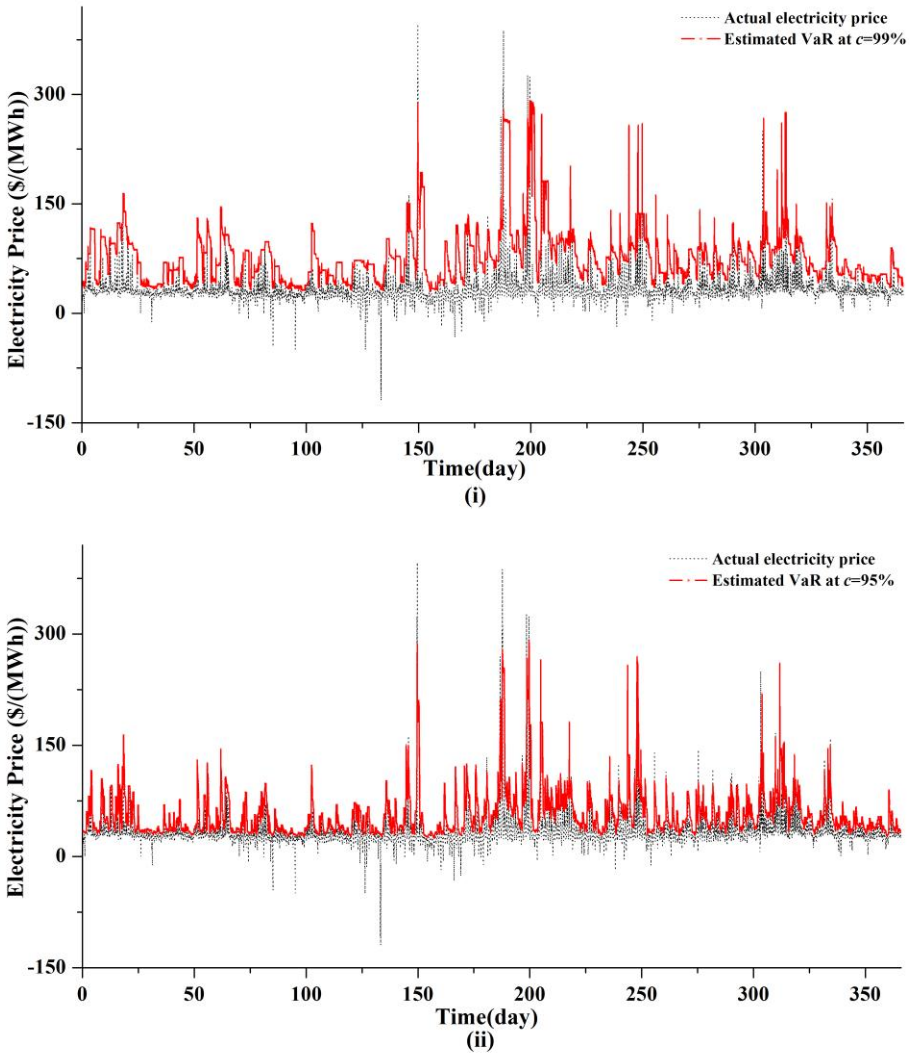

Fig. 7 *The estimated VaR value of the RT electricity price in 2012: (i) c=99%; (ii) c=95%*

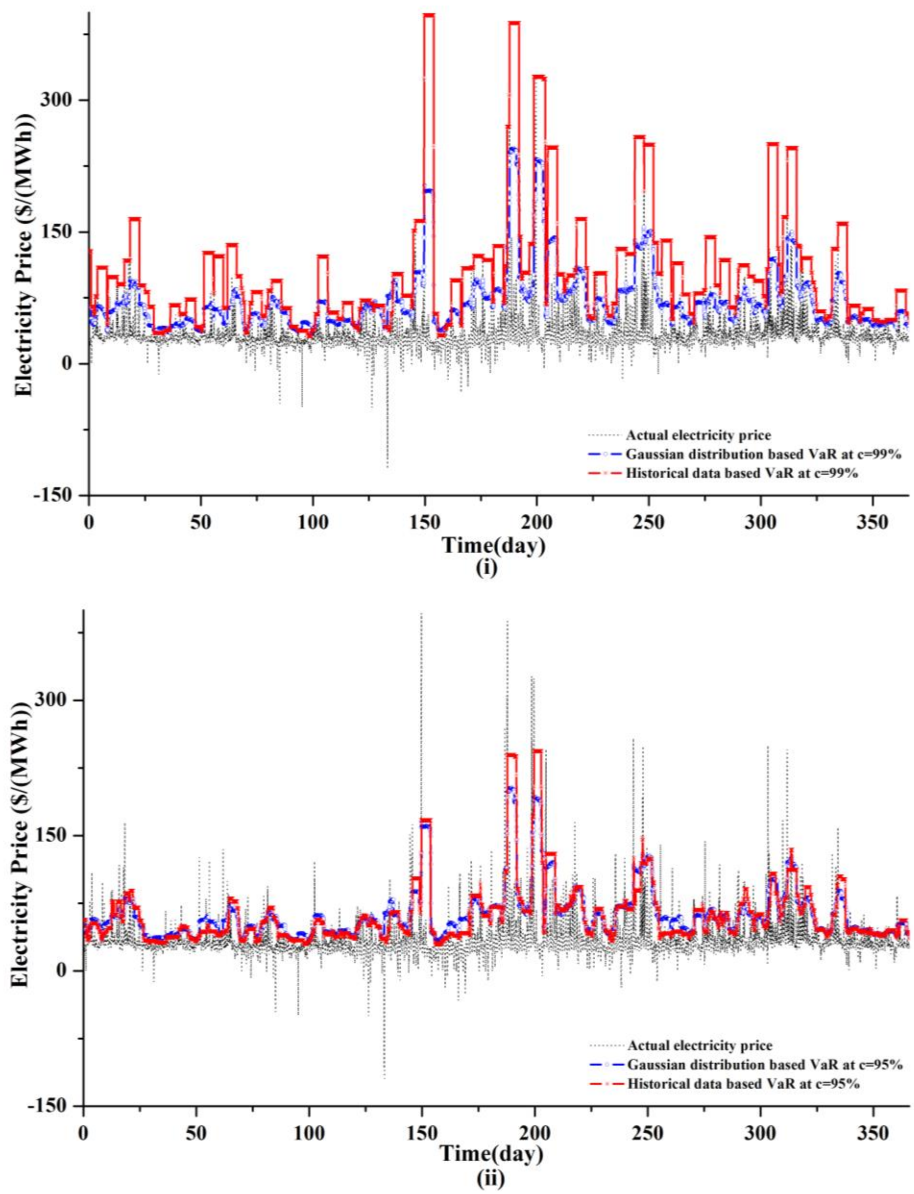

Fig. 8 *Estimated VaR value of RT electricity price in 2012 based on Gaussian distribution and historical data: (i) c=99%; (ii) c=95%*

List of Tables

Table 1 The Gaussian distribution validation tests of electricity prices in PJM electricity markets

Data source	Sample size	Kurtosis factor	Skewness factor	J-B statistics	K-S statistics
DA	43848	18.6877	2.9105	511540.3	0.9998
RT	43848	36.8942	3.7103	2199488.6	0.9930

Table 2 The statistics of the VaR estimation results with the modified RIA method

Confidence level	Sample size	Statistics of estimation failure	Failure rate	Theoretical failure rate
c=99%	8784	70	0.80%	1.0%
c=95%	8784	412	4.69%	5.0%

Table 3 Statistics of VaR estimation deviations

Confidence level	c=99%	c=95%
Average actual electricity price ($/(MW·h))	33.06	33.06
V_{aver} ($/(MW·h))	76.44	56.36
A_{error} ($/(MW·h))	43.38	23.30
D_{error} ($/(MW·h))	35.70	27.54
F_{sum} ($/(MW·h))	1797.3	6445.8

Table 4 Statistics of VaR estimation validity and deviations of the two reference methods

	Gaussian Distribution Based Method		Historical Data Based Method	
Confidence level	c=99%	c=95%	c=99%	c=95%
Statistics of estimation failure	301	420	134	531
Failure rate	3.43%	4.78%	1.53%	6.05%
V_{aver} ($/(MW·h))	72.81	63.26	109.20	62.44
A_{error} ($/(MW·h))	39.75	30.20	76.14	29.38
D_{error} ($/(MW·h))	37.95	32.01	70.43	37.62
F_{sum} ($/(MW·h))	8749.9	11906.0	3223.0	12847.2