# A Synchrophasor Measurement-Based Fault Location Technique for Multiterminal Multi-section Nonhomogeneous Transmission Lines 

Ting Wu ${ }^{{ }^{*}}$, C.Y. Chung ${ }^{2}$, Innocent Kamwa ${ }^{3}$, Jiayong Li $^{1}$, and Mingwen Qin ${ }^{1}$<br>${ }^{l}$ Department of Electrical Engineering, The Hong Kong Polytechnic University, Hong Kong<br>${ }^{2}$ Department of Electrical and Computer Engineering, University of Saskatchewan, Saskatoon, Canada<br>${ }^{3}$ Hydro-Québec/IREQ, Power System and Mathematics, Varennes, Canada<br>*ting.wu@connect.polyu.hk


#### Abstract

This paper presents a fault-location technique for multi-terminal multi-section nonhomogeneous transmission lines which combine overhead lines with underground power cables, by using voltage and current synchrophasors obtained from phasor measurement units (PMUs). Firstly, a faulty line branch is selected to narrow down the suspected faulty area. Then, the faulty section and the exact fault location can be identified by calculating the normalized fault distance for each section on the selected faulty branch. Computational burden of the proposed analytical scheme is very low because it avoids iterative computations. Promising simulation results show that the proposed fault location technique can accurately locate the fault regardless of the fault type, fault resistance, fault location, pre-fault loading and line parameters inaccuracies.


## Nomenclature

Some notations used in this paper are shown as follows:

| $i$ | Index of transmission system nodes |
| :--- | :--- |
| $m$ | Index of line section |
| $N$ | Number of transmission system nodes |
| $M$ | Number of line sections for a specified line branch |
| $w_{i}, s_{i}$ and $e_{i}$ | Number of line sections of the line branch lying to the west, south and east of tap node $i$ respectively |
| $L$ | Length of line branch (km) |
| $l$ | Length of line section (km) |
| $Z_{c}$ and $\Gamma$ | Characteristic impedance and propagation constant |
| $Z$ and $Y$ | Impedance per unit length (Ohm/km) and admittance per unit length (S/km) |
| $V$ and $I$ | Voltage and current |
| $T$ | Phasor transformation matrix (PTM) for each line section |
| $J$ | Junction node |
| $F$ | Fault node |
| $K$ | A specified system node |
| $x$ | Fault distance (km) |
| $\lambda$ | Normalized fault distance |
| $I_{f}$ | Fault current injection |
| TVE | Total vector error |
| $A_{i}$ | Suspected fault area of the tap node $i$ |
| $\delta$ | Predefined threshold for TVE |

## 1. Introduction

With the advent of global positioning system (GPS), Phasor Measurement Units (PMUs) have become crucial elements of Wide-Area Situational Awareness (WASA) system, as they can significantly improve the performance of power system monitoring and control by offering fast acquisitions of time-synchronized phasor data [1, 2]. Applications of PMU as an accurate fault location technique have been widely developed to accelerate restoration, reduce crew repair costs and enhance reliability of delivery in power systems [35].

Initially, the development of fault-location technique mainly concentrated on two-terminal transmission line system [6-9]. A time-domain approach for fault-location was proposed in [6] by using synchronized voltage and current samples at two terminals, but the sampling rate required was up to 24 kHz . A numerical algorithm for two-terminal fault-location was developed in [7], based on positive and zero sequence components of post-fault voltages and currents. However, this work did not contain a long line model. The Newton-Raphson iteration was used to identify the fault-location for multi-section underground cables, but this method suffered from convergence and heavy computational burden issues [8]. To deal with this, an innovative fault-location technique was proposed for two-terminal multi-section nonhomogeneous transmission lines [9], which was suitable for any system and fault conditions. With the development of modern power systems, fault-location techniques for three-terminal and multi-terminal transmission lines have gradually been developed. A traveling-wave-based fault-location method for three-terminal transmission systems was developed by using discrete wavelet transformation (DWT) and support vector machines (SVMs) methods [10]. Meanwhile, incomplete three-terminal synchronized signals were used in [11] for fault-location, through this involved high computational burden. Then, the authors of [12] extended a two-terminal fault-location technique to an algorithm for three-terminal multi-section transmission lines. However, very few fault-location techniques have been investigated for multi-terminal transmission lines. A new scheme to locate a fault on a multi-terminal transmission line was developed in [13] by only using synchronized voltage measurements at all terminals. This algorithm helps eliminate current-transformer error in current measurements but the power source impedances have to be exactly known. A universal faultlocation method was proposed in [14] for $N$-terminal ( $N \geq 3$ ) transmission lines by calculating ( $N-1$ ) indices. The nodal current unbalance was defined and used as a fault-location index to locate the fault on a multiterminal transmission line [15]. However, none of these techniques [3-15] are suitable for fault-location in multi-terminal multi-section nonhomogeneous transmission lines that have been widely used in contemporary transmission system, e.g. Taipower, to prettify the city environment [12]. Therefore, there is a need to develop a new fault-location method.

The main difficulty of fault-location for multi-terminal transmission lines is to select the faulty branch [15]. Therefore, based on the voltage and current synchrophasors at all terminals, a novel faulty branch selector is first proposed in this paper to simplify the fault-location problem from multi-terminal multisection transmission lines to two-terminal multi-section compound transmission lines containing the faulty section. Then, the faulty section can be identified and the fault can be exactly located by calculating the normalized fault distance for each section on the selected faulty branch. The rest of the paper is organized as follows. Section II describes the proposed fault-location technique. The performance of the proposed algorithm is assessed and analysed under various scenarios in Section III. Section IV concludes the paper.

## 2. Fault Location Technique

### 2.1. Faulty Line Branch Identification for Multi-Terminal Multi-Section Nonhomogeneous Transmission Lines

Considering a multi-terminal N -node nonhomogeneous transmission line (Fig. 1), all nodes are classified into two types: terminal node $p(p=1,3,5, \cdots, N-3, N-1, N)$ and tap node $q(q=2,4,6, \cdots, N-6, N-4, N-$ 2). Suppose that every terminal node is equipped with a PMU. Thus, the voltage and current synchrophasors at all terminal nodes can be obtained. For the sake of simplicity, the nonhomogeneous characteristic of the transmission line and the PMUs installed at terminal nodes are not displayed in this figure. Here, the positivesequence quantities are used because they are applicable for all fault types. Each tap node directly connects three nodes which lie to its west, south and east respectively, e.g. the nodes $(i-2),(i+1)$ and $(i+2)$ lie to the west, south and east of tap node $i$, respectively. For tap node $i$, there are $w_{i}, s_{i}$ and $e_{i}$ line sections on the line branches $L_{i-1}, L_{i}$ and $L_{i+1}$ respectively; every line section length from nodes $(i-2),(i+1)$ and $(i+2)$ to tap node $i$ is $l_{1}^{W_{i}}, l_{2}^{W_{i}}, \cdots, l_{w_{i}}^{W_{i}}, l_{1}^{s_{i}}, l_{2}^{s_{i}}, \cdots, l_{i}^{s_{i}}$, and $l_{1}^{E_{i}}, l_{2}^{E_{i}}, \cdots, l_{i}^{E_{i}}$ respectively. Similarly, for tap node (i+2), there are $w_{i+2}, s_{i+2}$ and $e_{i+2}$ line sections on the line branches $L_{i+1}, L_{i+2}$ and $L_{i+3}$ respectively; every line section length from nodes $i,(i+3)$ and $(i+4)$ to tap node $(i+2)$ is $l_{1}^{W_{1+2}}, l_{2}^{W_{t+2}}, \cdots, l_{w_{t+2}}^{W_{1+2}}, l_{1}^{S_{12}}, l_{2}^{S_{t+2}}, \cdots, l_{s_{t+2}}^{s_{1+2}}$, and $l_{1}^{E_{t+2}}, l_{2}^{E_{t+2}}, \cdots, l_{e_{t+2}}^{E_{t+2}}$ respectively. It is obvious that: $e_{i}=w_{i+2}$ and $l_{m}^{E_{i}}=l_{e_{i}+1-m}^{W_{i+2}}$, where $m=1,2, \cdots, e_{i}$.


Fig. 1. Multi-terminal nonhomogeneous transmission lines.
The proposed faulty line branch selector for multi-terminal nonhomogeneous transmission lines can be developed into three steps as below:

Step 1: Transfer the Measured Data From South to Tap Node:

In Fig. 2, the junction nodes, i.e. $J_{1}^{W_{i}}, J_{2}^{W_{i}}, \cdots, J_{w_{i-1}}^{W_{i}}, J_{1}^{S_{1}}, J_{2}^{S_{i}}, \cdots, J_{s_{i-1}}^{S_{i}}$, and $J_{1}^{E_{i}}, J_{2}^{E_{i}}, \cdots, J_{e_{i-1}}^{E_{i}}$, and the tap nodes, i.e. nodes ( $i-2$ ), $i$ and ( $i+2$ ), are selected as reference points of sending or receiving ends. For example, the sending and receiving ends of the line section $l_{s_{i}^{S_{i}}}$ are junction node $J_{s_{i-1}}^{s_{i}}$ and tap node $i$, respectively.


Fig. 2. Specific parameters of line branches $L_{i-1}, L_{i}$ and $L_{i+1}$.
For line section $l_{1}^{s_{i}}$, the voltage and current at a distance of $x \mathrm{~km}$ away from junction node $J_{1}^{s_{i}}$ can be expressed as follows:

$$
\begin{gather*}
V_{x}=A \cosh \left(\Gamma_{1}^{S_{i}} x\right)+B \sinh \left(\Gamma_{1}^{S_{i}} x\right)  \tag{1}\\
I_{x}=\left(B \cosh \left(\Gamma_{1}^{S_{i}} x\right)+A \sinh \left(\Gamma_{1}^{S_{i}} x\right)\right) / Z_{c i}^{S_{i}} \tag{2}
\end{gather*}
$$

where $Z_{c 1}^{S_{i}}=\sqrt{Z_{1}^{S_{i}} / Y_{1}^{S_{i}}}$ and $\Gamma_{1}^{S_{i}}=\sqrt{Z_{1}^{S_{i}} Y_{1}^{s_{i}}}$ signify the characteristic impedance and the propagation constant of the line section $l_{1}^{S_{i}}$, respectively; $Z_{1}^{S_{i}}$ and $Y_{1}^{S_{i}}$ are its impedance and admittance, respectively. The constants $A$ and $B$ can be derived from Eqs. (3) and (4) respectively by using the boundary conditions of voltage and current, $\left(V_{i+1}, I_{i+1}\right)$, measured at terminal node $(i+1)$.

$$
\begin{align*}
& A=V_{i+1} \cosh \left(\Gamma_{1}^{S_{i} l_{1}^{S_{i}}}\right)-I_{i+1} Z_{c 1}^{S_{i}} \sinh \left(\Gamma_{1}^{S_{1}^{S}} l_{i}^{S_{i}}\right)  \tag{3}\\
& B=I_{i+1} Z_{c 1}^{S_{i}} \cosh \left(\Gamma_{1}^{\left.S_{i} l_{1}^{S_{i}}\right)-V_{i+1} \sinh \left(\Gamma_{1}^{S_{i}} l_{1}^{S_{i}}\right)}\right. \tag{4}
\end{align*}
$$

The voltage and current at junction point $J_{1}^{S_{i}},\left(V_{1}^{S_{i}}, I_{1}^{S_{i}}\right)$, can be calculated by substituting $x=0$ into Eqs. (1) ~ (2).

$$
\begin{align*}
& V_{1}^{S_{i}}=V_{i+1} \cosh \left(\Gamma_{1}^{S_{i} l_{1}^{S_{i}}}\right)-I_{i+1} Z_{c 1}^{S_{i}} \sinh \left(\Gamma_{1}^{S_{i} l_{1}^{S_{i}}}\right)  \tag{5}\\
& I_{1}^{S_{i}}=I_{i+1} \cosh \left(\Gamma_{1}^{\left.S_{i} l_{1}^{S_{i}}\right)-V_{i+1} \sinh \left(\Gamma_{1}^{S_{i}} l_{1}^{S_{i}}\right) / Z_{c 1}^{S_{i}}}\right. \tag{6}
\end{align*}
$$

Since the line branch $L_{i}$ has $s_{i}$ line sections, voltage and current at tap node $i$, $\left(V_{i}^{S}, I_{i}^{S}\right)$, can be obtained by a series of successive algebraic substitutions from the data sets $\left(V_{i+1}, I_{i+1}\right)$ as shown in Eq. (7).

$$
\left[\begin{array}{l}
V_{i}^{s}  \tag{7}\\
I_{i}^{s}
\end{array}\right]=T_{s_{i}}^{s_{i}} \cdot T_{s_{i}-1}^{s_{i}} \cdot \cdots \cdot T_{2}^{s_{i}} \cdot T_{1}^{s_{i}} \cdot\left[\begin{array}{l}
V_{i+1} \\
I_{i+1}
\end{array}\right]=\prod_{n=0}^{s_{i-1}} T_{s_{i}-n}^{s_{i}} \cdot\left[\begin{array}{l}
V_{i+1} \\
I_{i+1}
\end{array}\right], i=2,4,6 \cdots, N-2
$$

where $T_{1}^{S_{i}}, T_{2}^{s_{i}}, \cdots, T_{s_{i}}^{s_{i}}$ are defined as Phasor Transformation Matrices (PTMs) of line branch $L_{i}$ which lies to the south of tap node $i$. The subscript $1,2, \cdots, s_{i}$ denote utilization of parameters of line sections $l_{1}^{S_{i}}, l_{2}^{S_{i}}, \cdots, l_{s_{i}}^{S_{i}}$, respectively. The general form of $T_{m}^{S_{i}}$ is expressed as Eq. (8), where $m=1,2, \cdots, s_{i} ; Z_{c m}^{s_{i}}$ and $\Gamma_{m}^{S_{i}}$ are the characteristic impedance and propagation constant for the line section $l_{m}^{S_{i}}$.

## Step2: Transfer the Measured Data From West to Tap Node:

Based on the data processing in Step 1, data sets $\left(V_{i}^{W}, I_{i}^{W}\right)$ can be obtained by transferring the measured data from west to tap node $i$, as shown in Eq. (9), where $\left(V_{i-2}^{S}, I_{i-2}^{s}\right)$ can be obtained from Eq. (7).

$$
\left[\begin{array}{l}
V_{i}^{W}  \tag{9}\\
I_{i}^{W}
\end{array}\right]=\left\{\begin{array}{l}
\prod_{n=0}^{w_{i-1}-1} T_{w_{i}-n}^{W_{i}} \cdot\left[\begin{array}{l}
V_{1} \\
I_{1}
\end{array}\right], i=2 \\
\prod_{n=0}^{w_{i-1}} T_{w_{i}-n}^{W_{i}} \cdot\left[\begin{array}{c}
V_{i-2}^{S} \\
I_{i-2}^{W}+I_{i-2}^{S}
\end{array}\right], i=4,6, \cdots, N-2
\end{array}\right.
$$

The general form of $T_{m}^{W_{i}}$ is expressed as Eq. (10), where $m=1,2, \cdots, w_{i}, Z_{c m}^{W_{i}}$ and $\Gamma_{m}^{W_{i}}$ are the characteristic impedance and propagation constant for the line section $l_{m}^{W_{i}}$.

$$
T_{m}^{W_{i}}=\left[\begin{array}{cc}
\cosh \left(\Gamma_{m}^{W_{i}} l_{m}^{W_{i}}\right) & -Z_{c m}^{W_{i}} \sinh \left(\Gamma_{m}^{W_{i}} l_{i}^{W_{i}}\right)  \tag{10}\\
-\sinh \left(\Gamma_{m}^{W_{m}} l_{m}^{W_{i}}\right) / Z_{c m}^{W_{i}} & \cosh \left(\Gamma_{m}^{i} l_{m}^{W_{i}}\right)
\end{array}\right]
$$

Step3: Transfer the Measured Data From East to Tap Node:
The data sets $\left(V_{i}^{E}, I_{i}^{E}\right)$ can be calculated by transferring the measured data from east to tap node $i$, as shown in Eq. (11), where $\left(V_{i+2}^{S}, I_{i+2}^{S}\right)$ can be obtained from Eq. (7).

$$
\left[\begin{array}{c}
V_{i}^{E}  \tag{11}\\
I_{i}^{E}
\end{array}\right]=\left\{\begin{array}{l}
e_{n=0}^{e_{-1}-1} T_{e_{i}-n}^{E_{i}} \cdot\left[\begin{array}{c}
V_{N} \\
I_{N}
\end{array}\right], i=N-2 \\
\prod_{n=0}^{e_{i-1}} T_{e_{i}-n}^{e_{i}} \cdot\left[\begin{array}{c}
V_{i+2}^{S} \\
I_{i+2}^{E}-I_{i+2}^{S}
\end{array}\right], i=2,4,6, \cdots, N-4
\end{array}\right.
$$

The general form of $T_{m}^{E_{i}}$ is expressed as Eq. (12), where $m=1,2, \cdots, e_{i} . Z_{c m}^{E_{i}}$ and $\Gamma_{m}^{E_{i}}$ are the characteristic impedance and propagation constant for the line section $l_{m}^{E_{i}}$.

$$
T_{m}^{E_{i}}=\left[\begin{array}{cc}
\cosh \left(\Gamma_{m}^{E_{i}} I_{m}^{E_{i}}\right) & Z_{c m}^{E_{i}} \sinh \left(\Gamma_{m}^{E_{i}} I_{E_{i}}^{E_{i}}\right.  \tag{12}\\
\sinh \left(\Gamma_{m}^{E_{i}} l_{m}^{E_{m}}\right) / Z_{c m}^{E_{i}} & \cosh \left(\Gamma_{m}^{E_{m}} l_{m}^{E_{i}}\right)
\end{array}\right]
$$

If no fault occurs in the transmission system as shown in Fig. 1, for each tap node $i(i=2,4, \cdots, N-2)$, $V_{i}^{S}$ obtained from Eq. (7), $V_{i}^{W}$ derived from Eq. (9) and $V_{i}^{E}$ calculated from Eq. (11) are equal to each other. If a fault occurs on the transmission system, the data sets $\left(V_{i}^{W}, I_{i}^{W}\right),\left(V_{i}^{S}, I_{i}^{S}\right)$ and $\left(V_{i}^{E}, I_{i}^{E}\right)$ for each tap node
need to be modified. Here, we analyse the faulty line branch identification problem in the following two cases: a fault occurs on the main line branch ( $L_{1}, L_{3}, L_{5}$, etc.), and a fault occurs on the tapped line branch ( $L_{2}, L_{4}, L_{6}$, etc.).

## 1) Fault on the Main Line Branch

If a fault occurs on section $l_{j}^{W_{K}}$ of main line branch $L_{K-1}$ between tap nodes ( $K-2$ ) and $K$ (Fig. 3 (a)), fault node $F$ is treated as a fictitious node. The unknown variable $x$ is defined as fault distance from fault node $F$ to junction node $J_{j}^{W_{k}}$. For the sake of simplicity, the nonhomogeneous characteristic of the transmission line is only displayed for branch $L_{K-1}$. It is obvious that: $e_{K-2}=w_{K}$ and $l_{m}^{E_{K-2}}=l_{w_{K}+1-m}^{W_{K}}$, where $m=1,2, \cdots, w_{K}$.

(a)

(b)

Fig. 3. Fault occurring on (a) section $l_{j}^{W_{K}}$ of main line branch $L_{K-l}$ and (b) section $l_{j}^{S_{K}}$ of the trapped line branch $L_{K}$.

Due to the fault current injection, data sets $\left(V_{i}^{W}, I_{i}^{W}\right)$ need to be modified as Eq. (13), in which $T_{F 1}^{W}$ and $T_{F 2}^{W}$ can be expressed as $\left[\begin{array}{cc}\cosh \left(\Gamma_{j}^{W_{k}} y\right) & -Z_{c_{k}}^{W_{k}} \sinh \left(\Gamma_{j}^{W_{k}} y\right) \\ -\sinh \left(\Gamma_{j}^{W_{k}} y\right) / Z_{c j}^{W_{k}} & \cosh \left(\Gamma_{j}^{W_{k}} y\right)\end{array}\right]$ by respectively substituting $y=l_{j}^{W_{k}}-x$ and $y=x$ into it. Because there is no fault on the tapped line section, the data sets $\left(V_{i}^{s}, I_{i}^{s}\right)$ derived from Eq. (7) need no modification.

Similarly, the data sets $\left(V_{i}^{E}, I_{i}^{E}\right)$ should be modified as Eq. (14), in which $T_{F 1}^{E}$ and $T_{F 2}^{E}$ can be defined as $\left[\begin{array}{cc}\cosh \left(\Gamma_{j}^{W_{k}} y\right) & Z_{c j}^{W_{k}} \sinh \left(\Gamma_{j}^{W_{k}} y\right) \\ \sinh \left(\Gamma_{j}^{W_{k}} y\right) / Z_{c j}^{W_{k}} & \cosh \left(\Gamma_{j}^{W_{k}} y\right)\end{array}\right]$ by respectively substituting $y=x$ and $y=l_{j}^{W_{k}}-x$ into it.

Due to the large fault current $I_{f}$ when a fault occurs on the line section $l_{j}^{W_{K}}$ of main line branch $L_{K-1}$, for tap node $i(i=K, K+2, \cdots, N-2)$, there must be a substantial difference between the value $V_{i}^{W}$ calculated from Eq. (9) and its actual value as shown in Eq. (13). Meanwhile, for tap node $i(i=2,4, \cdots, K-2)$, there must be a significant difference between the value $V_{i}^{E}$ calculated from Eq. (11) and its actual value derived from Eq. (14).

## 2) Fault on the Tapped Line Branch

As shown in Fig. 3 (b), if a fault occurs on the line section $l_{j}^{s_{K}}$ of the tapped line branch $L_{K}$ between the tap node $K$ and the terminal node $(K+1)$, the fault node $F$ is treated as a fictitious node. The unknown variable $x$ is defined as the fault distance from fault node $F$ to the junction node $J_{j}^{S_{K}}$.

Due to the fault current injection, ( $V_{K}^{S}, I_{K}^{S}$ ) for tap node $K$ should be modified as Eq. (15), where $\left(V_{F}^{S}, I_{F 1}^{S}\right)$ can be derived from Eq. (16); while $\left(V_{i}^{S}, I_{i}^{S}\right)$ for other tap nodes ( $i=2,4, \cdots, K-2, K+2, \cdots, N-2$ ) can also be derived from Eq. (7).

$$
\begin{align*}
& {\left[\begin{array}{c}
V_{K}^{S} \\
I_{K}^{S}
\end{array}\right]={ }_{n=0}^{s_{K}-j-1} T_{s_{K}-n}^{s_{K}} \cdot T_{F 2}^{S} \cdot\left[\begin{array}{c}
V_{F}^{S} \\
I_{F 1}^{S}+I_{f}
\end{array}\right]}  \tag{15}\\
& {\left[\begin{array}{l}
V_{F}^{s} \\
I_{F 1}^{s}
\end{array}\right]=T_{F 1}^{s} \cdot \underset{n=s_{k}-j+1}{s_{s_{k}-1}^{1}} T_{s_{K}-n}^{s_{k}} \cdot\left[\begin{array}{l}
V_{K+1} \\
I_{K+1}
\end{array}\right]} \tag{16}
\end{align*}
$$

where $T_{F 1}^{s}$ and $T_{F 2}^{s}$ can be expressed as $\left[\begin{array}{cc}\cosh \left(\Gamma_{j}^{s_{K}} y\right) & -Z_{c j}^{s_{K}} \sinh \left(\Gamma_{j}^{S_{K}} y\right) \\ -\sinh \left(\Gamma_{j}^{S_{k}} y\right) / Z_{c j}^{s_{K}} & \cosh \left(\Gamma_{j}^{K_{j}} y\right)\end{array}\right]$ by substituting $y=l_{j}^{s_{K}}-x$ and $y=x$ into it respectively.

Because of the large fault current $I_{f}$, values of $\left(V_{K}^{S}, I_{K}^{S}\right)$ calculated from Eq. (7) must deviate far from their actual value, as shown in Eq. (15). Besides, the actual value of data sets $\left(V_{i}^{W}, I_{i}^{W}\right)$ and $\left(V_{i}^{E}, I_{i}^{E}\right)$ can also be derived from Eqs. (9) and (11) respectively, but ( $V_{K}^{S}, I_{K}^{S}$ ) used in these equations should be calculated from Eq. (15). Thus, the actual value of $V_{i}^{W}$ for tap nodes $(K+2),(K+4), \cdots,(N-2)$ and the actual value of $V_{i}^{E}$ for tap nodes $2,4, \cdots,(K-2)$ must respectively deviate far from the values derived from Eqs. (9) and (11) without using data set $\left(V_{K}^{S}, I_{K}^{S}\right)$ calculated from Eq. (15).

The suspected fault area for tap node $K$ in a multi-terminal $N$-node nonhomogeneous transmission line can be identified as Table 1. Ideally, the calculated values of $V_{K}^{S}, V_{K}^{W}$ and $V_{K}^{E}$ for tap node $K$ must equal to each other if no fault occurs in the transmission line system. However, due to the uncertainty of measurement and line parameters, there will be a slight deviation among these values. Here, total vector errors (TVE), i.e. $\operatorname{TVE}_{K}^{W S}, \operatorname{TVE}_{K}^{S E}$ and $\operatorname{TVE}_{K}^{E V}$, are defined as:

$$
\begin{gather*}
\mathrm{TVE}_{K}^{W S}=\left|V_{K}^{W}-V_{K}^{S}\right| / \max \left(\left|V_{K}^{W}\right|,\left|V_{K}^{S}\right|,\left|V_{K}^{E}\right|\right)  \tag{17}\\
\mathrm{TVE}_{K}^{S E}=\left|V_{K}^{S}-V_{K}^{E}\right| / \max \left(\left|V_{K}^{W}\right|,\left|V_{K}^{S}\right|,\left|V_{K}^{E}\right|\right)  \tag{18}\\
\mathrm{TVE}_{K}^{E W}=\left|V_{K}^{E}-V_{K}^{W}\right| / \max \left(\left|V_{K}^{W}\right|,\left|V_{K}^{S}\right|,\left|V_{K}^{E}\right|\right) \tag{19}
\end{gather*}
$$

If a fault occurs in the transmission line system, there is only one calculated voltage among ( $V_{K}^{s}, V_{K}^{W}, V_{K}^{E}$ ) that deviates far from its actual value; the absolute value is much smaller than that of the others because the large fault current $I_{f}$, shown in Eqs. (13) ~ (15), has not been considered during its calculation. Thus, the denominator in Eqs. (17) ~ (19), i.e. $\max \left(\left|V_{K}^{W}\right|,\left|V_{K}^{S}\right|,\left|V_{K}^{E}\right|\right)$, must be the actual voltage of tap node $K$, which can be an appropriate basis for TVE calculation. In Table 1, $\operatorname{TVE}_{K}^{\max }$ is the maximum value of $\left(\mathrm{TVE}_{K}^{W S}, \mathrm{TVE}_{K}^{S E}, \mathrm{TVE}_{K}^{E W}\right.$ ). If this value is less than or equal to a predefined threshold $\delta$ (Mode 1), the differences between $\left(V_{K}^{S}, V_{K}^{W}, V_{K}^{E}\right)$ are identified as the result of uncertainty of measurement and line parameters. Then, we can conclude that no fault occurs in the transmission line system or a fault occurs at/near to tap node $K$. Otherwise, one of the remaining three modes, i.e. Mode 2,3 and 4, can be identified according to $\mathrm{TVE}_{K}^{\min }$ that is the minimum value of $\left(\mathrm{TVE}_{K}^{W S}, \mathrm{TVE}_{K}^{S E}, \mathrm{TVE}_{K}^{E W}\right)$. For example, $\mathrm{TVE}_{K}^{\min }=\mathrm{TVE} K_{K}^{W S}$ means that $V_{K}^{W}$ and $V_{K}^{S}$ are the actual voltage of tap node $K$ due to the negligible difference between them; the line branches lying to the east of tap node $K$, i.e. $L_{K+1} \sim L_{N-1}$, are recognized as the suspected faulty area due to the large values of $\mathrm{TVE}_{K}^{S E}$ and $\mathrm{TVE}_{K}^{E W}$. Similarly, the suspected faulty areas of Mode 3 and 4 can be identified by the same logic. The threshold value $\delta$ may vary for different transmission line systems, which should satisfy the following two criteria simultaneously: no faulty line branch should be identified when the transmission line
system is in no-fault state, and any fault in the system should be successfully detected. Thus, Monte Carlo simulations are performed with different system parameters (e.g. pre-fault loading) and fault conditions (e.g. fault type and fault resistance) as illustrated in Figs. 12 and 13 to determine the appropriate threshold value which meets the above two criteria simultaneously.

Table 1 Fault branch location for tap node $K$ in a multi-terminal $N$-node nonhomogeneous transmission line system

| Mode | Faulty Branch Indication | Suspected Faulty Area |
| :---: | :---: | :---: |
| 1 | $\mathrm{TVE}_{K}^{\max } \leq \delta$ | No fault OR fault |
| 2 | $\left(\mathrm{TVE}_{K}^{\max }>\delta\right) \&\left(\mathrm{TVE}_{K}^{\min }=\mathrm{TVE}_{K}^{\text {ws }}\right)$ | at/near to tap node $K$ |
| 3 | $\left(\mathrm{TVE}_{K}^{\max }>\delta\right) \&\left(\mathrm{TVE}_{K}^{\min }=\mathrm{TVE}_{K+1}^{s \varepsilon}\right)$ | $L_{1} \sim L_{K-1}$ |
| 4 | $\left(\mathrm{TVE}_{K}^{\max }>\delta\right) \&\left(\mathrm{TVE}_{K}^{\min }=\mathrm{TVE}_{\kappa}^{E /}\right)$ | $L_{K}$ |

The proposed fault line branch identification scheme for multi-terminal multi-section nonhomogeneous transmission lines is summarized in Fig. 4.


Fig. 4. Faulty line branch identification for multi-terminal multi-section nonhomogeneous transmission lines.

After calculating $\mathrm{TVE}_{i}^{\text {max }}$ and $\mathrm{TVE}_{i}^{\text {min }}$ for each tap node $i$, the suspected faulty area $A_{i}$ of each tap node can be derived according to Table 1. If all tap nodes are in Mode 1, the transmission line system can be identified in non-fault state. Otherwise if no tap node is in Mode 1, the faulty line branch can be selected by getting an intersection of the suspected faulty area $A_{i}$ for all tap nodes, in Mode 2,3 or 4 . Furthermore, if only one tap node, e.g. tap node $K$, is in Mode 1 and others are in Mode 2 or 3, the fault can be located $\mathrm{at} /$ near to this tap node $K$. For this situation, we need to narrow the suspected fault area further, in preparation
for the exact fault location (described in the next sub-section). As shown in Fig. 4, the faulty line branch can be selected according to the value $\mathrm{TVE}_{K}^{\text {min }}$ converges to, for example, if $\mathrm{TVE}_{K}^{\text {min }}$ converges to $\mathrm{TVE}_{K}^{E W}$, the fault can be identified on line branch $L_{K}$. However, if $\mathrm{TVE}_{K}^{\text {min }}$ does not converge to any value and fluctuates among $\mathrm{TVE}_{K}^{W S}, \mathrm{TVE}_{K}^{S E}$ and $\mathrm{TVE}_{K}^{E W}$, the fault can be located at tap node $K$.

### 2.2. Exact Fault Location

Once the faulty line branch is identified, we can locate the exact fault point based on the voltage and current synchrophasors at both ends of the faulty line branch. For transmission lines shown in Fig. 1, there are four types of faulty branches, which are depicted in Fig. 5. For type (a), $\left(V_{1}, I_{1}\right)$ can be directly measured by a PMU installed at terminal node 1 ; the voltage and current of node 2 can be derived as $\left(V_{2}^{S}, I_{2}^{E}-I_{2}^{S}\right)$. For type (b), ( $V_{N}, I_{N}$ ) can be directly measured by a PMU installed at terminal node $N$; the voltage and current of node ( $N-2$ ) can be calculated as ( $V_{N-2}^{S}, I_{N-2}^{W}+I_{N-2}^{S}$ ). For type (c), the voltage and current of nodes (i-2) and $i$ can be obtained as $\left(V_{i-2}^{S}, I_{i-2}^{W}+I_{i-2}^{S}\right)$ and ( $V_{i}^{S}, I_{i}^{E}-I_{i}^{S}$ ) respectively. For type (d), $\left(V_{i+1}, I_{i+1}\right)$ can be directly measured by a PMU installed at terminal node $(i+1)$; the voltage and current of node $i$ can be calculated as $\left(V_{i}^{W}, I_{i}^{E}-I_{i}^{W}\right)$ or $\left(V_{i}^{E}, I_{i}^{E}-I_{i}^{W}\right)$.

(a)

(c)


(d)




Fig. 5. Four types of faulty branches: (a)~(c) a fault on the main line branch and (d) a fault on the tapped line branch.
Using the above analysis, the fault location for multi-terminal nonhomogeneous transmission lines can be converted into fault location for two-terminal nonhomogeneous transmission lines (Fig. 6). Assume that there are $M$ line sections on the line branch H-R. Every line section length from node H to node R is $l_{1}, l_{2}, \cdots, l_{M-1}$ and $l_{M}$. A fault occurs on the line section $l_{j}$ and locates $x$ km away from the junction node $J_{j}$.


Fig. 6. Equivalent two-terminal nonhomogeneous transmission lines.

Since $l_{1}, l_{2}, \cdots, l_{j-1}$ and $l_{j+1}, l_{j+2}, \cdots, l_{M}$ are healthy line sections, voltage and current phasors $\left(V_{j, \mathrm{H}}, I_{j, \mathrm{H}}\right)$ at junction node $J_{j-1}$ and $\left(V_{j, \mathrm{R}}, I_{j, \mathrm{R}}\right)$ at junction point $J_{j}$ can be likewise derived via a series of substitutions from the data sets $\left(V_{\mathrm{H}}, I_{\mathrm{H}}\right)$ and $\left(V_{\mathrm{R}}, I_{\mathrm{R}}\right)$ respectively, as follows:

$$
\begin{align*}
& {\left[\begin{array}{l}
V_{j, \mathrm{H}} \\
I_{j, \mathrm{H}}
\end{array}\right]=T_{j-1}^{\mathrm{H}} \cdot T_{j-2}^{\mathrm{H}} \cdot \ldots \cdot T_{2}^{\mathrm{H}} \cdot T_{1}^{\mathrm{H}} \cdot\left[\begin{array}{l}
V_{\mathrm{H}} \\
I_{\mathrm{H}}
\end{array}\right]}  \tag{20}\\
& {\left[\begin{array}{l}
V_{j, \mathrm{R}} \\
I_{j, \mathrm{R}}
\end{array}\right]=T_{j+1}^{\mathrm{R}} \cdot T_{j+2}^{\mathrm{R}} \cdot \ldots \cdot T_{M-1}^{\mathrm{R}} \cdot T_{M}^{\mathrm{R}} \cdot\left[\begin{array}{l}
V_{\mathrm{R}} \\
I_{\mathrm{R}}
\end{array}\right]} \tag{21}
\end{align*}
$$

where $T_{j-1}^{\mathrm{H}}, T_{j-2}^{\mathrm{H}}, \cdots, T_{2}^{\mathrm{H}}, T_{1}^{\mathrm{H}}$ and $T_{j+1}^{\mathrm{R}}, T_{j+2}^{\mathrm{R}}, \cdots, T_{M-1}^{\mathrm{R}}, T_{M}^{\mathrm{R}}$ are PTMs of nodes H and R respectively. Thus, the suspected faulty area is further narrowed down to a two-terminal homogeneous transmission line. The voltage at fault point $F$ can be expressed in terms of the two data sets $\left(V_{j, \mathrm{H}}, I_{j, \mathrm{H}}\right)$ and ( $V_{j, \mathrm{R}}, I_{j, \mathrm{R}}$ ) derived from Eqs. (20) and (21) respectively:

$$
\begin{array}{r}
V_{F, \mathrm{H}}=V_{j, \mathrm{H}} \cdot \cosh \left(\Gamma_{j} y\right)-\mathrm{Z}_{c \cdot} \cdot I_{j, \mathrm{H}} \cdot \sinh \left(\Gamma_{j} y\right) \\
V_{F, \mathrm{R}}=V_{j, \mathrm{R}} \cdot \cosh \left(\Gamma_{j} x\right)+\mathrm{Z}_{c j} \cdot I_{j, \mathrm{R}} \cdot \sinh \left(\Gamma_{j} x\right) \tag{23}
\end{array}
$$

where $y=l_{j}-x ; \Gamma_{j}$ and $Z_{c j}$ are the propagation constant and characteristic impedance of line section $l_{j}$ respectively. Using the relationship $V_{F, \mathrm{H}}=V_{F, \mathrm{R}}$ and equating Eqs. (22) and (23), the normalized distance variable $\lambda_{j}$ can be solved as follows [9]:

$$
\begin{equation*}
\lambda_{j}=\frac{x}{l_{j}}=\frac{\ln \left(P_{j} / Q_{j}\right)}{2 \Gamma_{j} l_{j}} \tag{24}
\end{equation*}
$$

$P_{j}$ and $Q_{j}$ are given by:

$$
\begin{gather*}
P_{j}=\left(V_{j, \mathrm{R}}-\mathrm{Z}_{c j} I_{j, \mathrm{R}}\right)-\left(V_{j, \mathrm{H}}-\mathrm{Z}_{c j} I_{j, \mathrm{H}}\right) \exp \left(\Gamma_{j} l_{j}\right)  \tag{25}\\
Q_{j}=\left(V_{j, \mathrm{H}}+\mathrm{Z}_{c j} I_{j, \mathrm{H}}\right) \exp \left(-\Gamma_{j} l_{j}\right)-\left(V_{j, \mathrm{R}}+\mathrm{Z}_{c j} I_{j, \mathrm{R}}\right) \tag{26}
\end{gather*}
$$

Based on Eq. (24), we can derive $M$ fault location indices $\lambda_{j}(j=1,2, \cdots, M)$. From $j=1$ to $M$, if the obtained $\lambda_{j}$ converges and falls within the interval $[0,1]$, then $l_{j}$ is recognized as the correct fault line section.

## 3. Performance Evaluation

In order to evaluate the proposed fault-location algorithm, a $345-\mathrm{kV} 50-\mathrm{Hz}$ transposed 5 -terminal nonhomogeneous line system (Fig. 7), consisting of three types of transmission lines, i.e. underground cable and two kinds of overhead lines with different parameters, was implemented in PSCAD software. Since the main focus of this paper is on the fault-location technique, it is assumed that the fundamental phasors from PMUs can be directly obtained without considering the decaying DC offset. The related parameters are depicted in Table 2. The fault-location error is defined as Eq. (27).

$$
\begin{equation*}
\text { Error\% } \%=\frac{\mid \text { Estimated location }- \text { Actual location } \mid}{\text { The length of faulty line section }} \times 100 \% \tag{27}
\end{equation*}
$$



Fig. 7. Simulation system consists of a 5-terminal nonhomogeneous line.

Table 2 Parameters of a five-terminal nonhomogeneous line system

| System voltage: 345 kV | System frequency: 50 Hz <br> Source impedance |
| :---: | :---: |
| Source |  |

$$
Z_{10}=1.784+\mathrm{j} 8.18
$$

Parameters of underground cable ( $l_{1}^{w_{2}}, l_{1}^{S_{2}}, l_{1}^{s_{4}}, l_{1}^{E_{c}}$ ):

$$
\begin{array}{ll}
Z_{1}=0.024+\mathrm{j} 0.0804(\Omega / \mathrm{km}) & Y_{1}=\mathrm{j} 143.768 \times 10^{-6}(\mathrm{~S} / \mathrm{km}) \\
Z_{0}=0.036+\mathrm{j} 0.1043(\Omega / \mathrm{km}) & Y_{0}=\mathrm{j} 143.768 \times 10^{-6}(\mathrm{~S} / \mathrm{km})
\end{array}
$$

Parameters of overhead line $1\left(l_{2}^{W_{2}}, l_{2}^{S_{2}}, l_{1}^{E_{2}}\right.$ or $l_{2}^{W_{S}}, l_{2}^{S_{4}}, l_{1}^{E_{\text {E }}}$ or $\left.l_{2}^{W_{+}}, l_{1}^{S_{S}}, l_{2}^{E_{*}}\right)$ :

$$
\begin{array}{ll}
Z_{1}=0.038+\mathrm{j} 0.2815(\Omega / \mathrm{km}) & Y_{1}=\mathrm{j} 4.083 \times 10^{-6}(\mathrm{~S} / \mathrm{km}) \\
Z_{0}=0.248+\mathrm{j} 0.8438(\Omega / \mathrm{km}) & Y_{0}=\mathrm{j} 2.238 \times 10^{-6}(\mathrm{~S} / \mathrm{km})
\end{array}
$$

Parameters of overhead line $2\left(l_{3}^{W_{2}}, l_{2}^{E_{5}}\right.$ or $l_{1}^{W_{5}}, l_{3}^{S_{S}}, l_{2}^{E_{4}}$ or $\left.l_{1}^{W_{5}}, l_{2}^{S_{4}}\right)$ :

| $Z_{1}=0.061+\mathrm{j} 0.2837(\Omega / \mathrm{km})$ | $Y_{1}=\mathrm{j} 3.936 \times 10^{-6}(\mathrm{~S} / \mathrm{km})$ |
| :--- | :--- |
| $Z_{0}=0.362+\mathrm{j} 1.0458(\Omega / \mathrm{km})$ | $Y_{0}=\mathrm{j} 2.119 \times 10^{-6}(\mathrm{~S} / \mathrm{km})$ |

Firstly, some typical cases are analysed to illustrate the proposed faulty branch localization mechanism. When an A-phase to ground fault (AG-fault) on the main line section $l_{1}^{E_{2}}$ occurs on the point which is $60 \%$ away from tap node 4 with a fault resistance of $20 \Omega$, Figs. 8 (a) $\sim$ (c) depict the calculated indices $\mathrm{TVE}_{i}^{W S}, \mathrm{TVE}_{i}^{S E}$ and $\mathrm{TVE}_{i}^{E W}$ for each tap node, in which negative values, zero and positive values in horizontal axis denote pre-fault cycles, fault occurring time and post-fault cycles, respectively. According to Table 1, it is obvious that tap node 2 is in Mode 2, while tap nodes 3 and 4 are in Mode 3. Besides, Fig. 8 (d) shows the selected faulty line branch by getting an intersection of the suspected faulty areas for each tap node, in which $L_{1} \sim L_{7}$ in vertical axis represent the seven line branches in Fig. 7, while 0 denotes no fault. Before

$$
\begin{aligned}
& E=1 \angle 30^{\circ} \\
& Z_{\text {11 }}=0.232+\mathrm{j} 5.87 \\
& E_{3}=1 \angle 20^{\circ} \quad Z_{31}=0.346+\mathrm{j} 6.23 \quad Z_{30}=2.134+\mathrm{j} 7.58 \\
& E_{7}=1 \angle 10^{\circ} \quad Z_{11}=0.198+\mathrm{j} 6.19 \quad Z_{70}=1.786+\mathrm{j} 7.53 \\
& E_{8}=1 \angle 0^{\circ} \quad Z_{71}=0.218+\mathrm{j} 5.95 \quad Z_{80}^{70}=1.592+\mathrm{j} 8.22 \\
& l_{1}^{w_{2}}=20, l_{2}^{w_{2}}=50, l_{3}^{w_{2}}=30 ; l_{2}^{E_{2}}=l_{1}^{w_{5}}=45, l_{1}^{E_{3}}=l_{2}^{w_{5}}=55 ; l_{2}^{s_{3}}=40 \text {, } \\
& l_{1}^{s_{s}}=40 ; l_{2}^{E_{s}}=l_{1}^{W_{x}}=60, l_{1}^{E_{s}}=l_{2}^{W_{x}}=40 ; l_{3}^{S_{s}}=25, l_{2}^{S_{s}}=30, l_{1}^{s_{x}}=25 ; \\
& l_{2}^{E_{S}}=48, l_{1}^{E_{S}}=52 ; l_{2}^{s_{S}}=35, l_{1}^{s_{S}}=45 \text {. }
\end{aligned}
$$

fault inception, the maximum TVE value of each tap node is smaller than the predefined threshold $\delta$ in Table 1 which is set as $0.2 \%$ in this paper. That means all the tap nodes are in Mode 1 and no fault is identified during this period. Due to the overshoot of $\mathrm{TVE}_{4}^{S E}$ caused by the fault, the selected faulty line branch fluctuates between $L_{3}$ and $L_{4}$ in the first half-cycle after fault inception. However, the selected fault line branch converges to $L_{3}$ in the next 3.5 -cycle, which can correctly identify the faulty branch. Furthermore, the normalized distance variables, i.e. $\lambda_{1}^{E_{2}}$ and $\lambda_{2}^{E_{2}}$, for each line section of faulty line branch $L_{3}$ are calculated to obtain the exact fault point by solving Eq. (24): $\lambda_{1}^{E_{2}}=0.5999, \lambda_{2}^{E_{2}}=-0.4765$. Line section $l_{1}^{E_{2}}$ is identified as the faulty section because $0 \leq \lambda_{1}^{E_{2}} \leq 1$, and the fault location error is $0.01 \%$, which is negligible.


Fig. 8. Faulty branch indices of (a) tap node 2, (b) tap node 4 and (c) tap node 6 , and (d) the selected faulty branch for a fault on the main line section $l_{1}^{E_{2}}$.
Figs. 9 (a) $\sim(c)$ show the calculated indices for each tap node when an AG-fault occurs on the tapped line section $l_{2}^{S_{4}}$ ( $30 \%$ away from junction node $J_{2}^{S_{4}}$ with a fault resistance of $50 \Omega$ ). With reference to Table 1, tap nodes 2, 4 and 6 are in Mode 2, 4 and 3 respectively. Thus, the faulty line branch (Fig. 9 (d)) can be correctly identified as $L_{4}$. Besides, we can obtain the exact fault point by solving Eq. (24): $\lambda_{1}^{S_{4}}=-2.7326$, $\lambda_{2}^{S_{4}}=0.3002, \lambda_{3}^{S_{4}}=1.3514$. Since $0 \leq \lambda_{2}^{S_{4}} \leq 1$, the corresponding line section $l_{2}^{S_{4}}$ is identified as the faulty section, and the fault location error is $0.02 \%$, which is pretty small.


Fig. 9. Faulty branch indices of (a) tap node 2, (b) tap node 4 and (c) tap node 6 , and (d) the selected faulty branch for a fault on the tapped line section $L_{2}^{S_{4}}$.

A temporary arcing fault case [16, 17] (BG-fault fault on line section $l_{1}^{E_{4}}, 50 \%$ away from tap node 6 with a fault resistance of $10 \Omega$ ) is implemented to further demonstrate the effectiveness of the proposed
fault-location technique. Figs. 10 (a) (c) describe the calculated indices for each tap node, which show that tap nodes 2 and 3 are in Mode 2, and tap node 4 is in Mode 3 within three cycles after fault inception. Therefore, the faulty line branch is identified as $L_{5}$ as shown in Fig. 10 (d). Due to the index overshoots resulted from the arcing fault and its clearance, the faulty line branches are wrongly selected as $L_{6}$ and $L_{4}$ at the very beginning of the first cycle after fault inception and in the last half-cycle before the all tap nodes return to Mode 1 respectively. However, the temporary arcing fault can be successfully identified at line branch $L_{5}$, because the wrong identification just takes up a small proportion of the fault period.


Fig. 10. Faulty branch indices of (a) tap node 2, (b) tap node 4 and (c) tap node 6, and (d) the selected faulty branch for a temporary arcing fault on the tapped line section $L_{1}^{E_{4}}$.

To assess the accuracy of the proposed faulty branch selector, Fig. 11 shows the value of $\mathrm{TVE}_{2}^{\text {min }}$ converges to when a three-phase short-circuit fault (ABC fault) occurs on the line section $l_{3}^{W_{2}}$ with a fault resistance of $30 \Omega$ in four fault-point scenarios, i.e. $0.5 \%, 0.3 \%, 0.1 \%$ and $0.03 \%$ away from tap node 2 . In all the four scenarios, tap node 2 is recognized in Mode 1 while tap nodes 4 and 6 are identified in Mode 3 according to Table 1. Thus, the fault can be located on/near to the tap node 2 with reference to faulty line branch identification logic summarized in Fig. 4. The suspected fault area is further narrowed down according to the value $\mathrm{TVE}_{2}^{\text {min }}$ converges to. For the first three scenarios (Figs. 11 (a) $\sim(\mathrm{c})$ ), the fault can be identified on the line branch $L_{1}$ as the value of $\mathrm{TVE}_{2}^{\min }$ converges to $\mathrm{TVE}_{2}^{S E}$ after 2.1-cycle, 2.4-cycle and 3.8cycle when fault occurs. It can be seen that the proposed faulty branch selector needs more time to identify the correct faulty line branch if a fault occurs closer to tap node. However, if a fault further approximates to the tap node, the fault location can only be identified on the tap node instead of its actual point. For example, the ABC fault occurs at a distance of $0.03 \%$ away from tap node 2 (Fig. 11 (d)), the value of $\mathrm{TVE}_{2}^{\min }$ fluctuates among $\mathrm{TVE}_{2}^{W S}, \mathrm{TVE}_{2}^{E W}$ and $\mathrm{TVE}_{2}^{S E}$, and the fault can only be located at tap node 2 by using the proposed scheme. Besides, further simulation shows that when a fault occurs on the tap node $2, \mathrm{TVE}_{2}^{\text {min }}$ cannot converge to any value just like Fig. 11 (d).


Fig. 11. Value of $\mathrm{TVE}_{2}^{\min }$ converges to with an ABC fault (fault resistance $=30 \Omega$ ) occurring on section $l_{3}^{W_{2}}$ for four fault-point scenarios: (a) $0.5 \%$, (b) $0.3 \%$, (c) $0.1 \%$ and (d) $0.03 \%$ away from tap node 2.

To validate the robustness of the proposed fault-location technique, 16 fault cases with various fault types and fault resistances on different line sections are simulated, as shown in the first two columns of Table 3. We can see from the third and fourth columns of Table 3, if a fault occurs on the tapped line branch, only one tap node is in Mode 4; other tap nodes are in Mode 2 or 3 . The faulty branch can be selected as the suspected faulty area of the tap node in Mode 4 . However, if a fault occurs on the main line branch $L_{K+1}$, Mode 2 for tap node $2,4, \cdots, K$ and Mode 3 for tap node $K+2, K+4, \cdots, N-2$ can be obtained. For example, when an A and C-phase to ground fault (ACG fault) occurs on $L_{3}$ (the seventh row of Table 3), tap node 2 is in Mode 2, while tap nodes 4 and 6 are in Mode 3. According to Table 1, faulty branch $L_{K+1}$ can be identified by getting an intersection of the suspected faulty areas for each tap node. With reference to the last two columns of Table 3, the exact fault point can be accurately located regardless of fault resistance, fault type and combinations of multi-terminal multi-section transmission line parameters.

The effect of variation of fault resistance in the algorithm's accuracy for four types of faults is shown in Fig. 12 (a) with the assumption that the fault occurs on the line section $l_{1}^{\omega_{4}}$ at a distance of $50 \%$ from junction node $J_{1}^{W_{4}}$. It can be easily seen that the proposed fault location algorithm is very accurate for all kinds of faults when the fault resistance is less than or equal to $10 \Omega$. However, fault-location errors for four types of faults increase by different degrees with the growth of fault resistance (from $10 \Omega$ to $1000 \Omega$ ). Besides, a fault with more severe condition can be located with higher fault-location precision. For example, the short circuit faults, i.e. BC and ABC faults, can be located more accurately than the ground faults, i.e. $A G$ and $A C G$ faults.

The effect of variation of fault location in the algorithm's accuracy for four types of faults is shown in Fig. 12 (b) with the assumption that the fault with $100 \Omega$ fault resistance occurs on the line section $l_{1}^{W_{4}}$. It can be observed that the fault location accuracy is very independent of the fault location.

Fig. 12 (c) illustrates the influence of the pre-fault loading on the algorithm's accuracy for AG, ACG, BC and ABC faults assuming that the fault with $100 \Omega$ fault resistance occurs on the line section $l_{1}^{\omega_{+}}$at a distance of $50 \%$ from junction node $J_{1}^{W_{4}}$. The pre-fault loading varies from 0.01 to 5 times its base case value ( $300 \mathrm{MW}+200 \mathrm{MVA}$ ). With reference to Fig. 12 (c), the fault-location errors for four types of faults present an increasing tendency with the growth of pre-fault loading, but the maximum error under various pre-fault loading conditions and different fault types is only $0.027 \%$, which is pretty small.

The effect of variation of three-phase voltage unbalance factor caused ONLY by the un-transposed transmission lines in the algorithm's accuracy for four types of faults is shown in Fig. 12 (d), with the same fault resistance and location assumption as Fig. 12 (c). For un-transposed transmission lines, the selfimpedance and mutual-impedance are obtained by averaging the diagonal and off-diagonal terms of the phase impedance matrix respectively [12, 18]. Then, the approximate positive sequence parameters, calculated through symmetrical components transformation, can be used to derive the fault location. As shown in Fig. 12 (d), the fault-location errors for four kinds of faults increase in different degrees with the growth of unbalance factor. For small unbalance factor, i.e. mildly un-transposed lines, the fault-location error introduced is insignificant which can still be acceptable. For large unbalance factor, i.e. highly untransposed lines, however, the fault-location error can be up to $3 \%$ for ABC fault with $0.5 \%$ unbalance factor, which needs further efforts in the future work.


Fig. 12. Effect of (a) fault resistance, (b) fault location, (c) pre-fault loading and (d) three-phase voltage unbalance factor on fault-location accuracy.

Table 3 Performance evaluation for different fault conditions

| Fault Location | Fault Type \& $\mathrm{R}_{\mathrm{F}}{ }^{\dagger}(\Omega)$ | Tap Node Mode |  |  | Faulty Branch | Fault location indices (p.u.) | Error <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 4 | 6 |  |  |  |
| $\begin{aligned} & l_{1}^{W_{2}}: 5 \% \\ & \text { from } J_{1}^{W_{2}} \end{aligned}$ | $\begin{gathered} \mathrm{AB}^{1} \\ 0.1 \end{gathered}$ | 3 | 3 | 3 | $L_{1}$ | $\begin{aligned} & \lambda_{1}^{W_{2}}=\mathbf{0 . 0 4 9 9 1} \\ & \lambda_{2}^{W_{2}}=1.00583 \\ & \lambda_{3}^{W_{2}}=2.63592 \end{aligned}$ | 0.009 |
| $\begin{aligned} & l_{2}^{W_{2}}: 12 \% \\ & \text { from } J_{2}^{W_{2}} \end{aligned}$ | $\begin{gathered} \mathrm{BG}^{2} \\ 10 \end{gathered}$ | 3 | 3 | 3 | $L_{1}$ | $\begin{aligned} & \lambda_{1}^{W_{2}}=-6.5135 \\ & \lambda_{2}^{W_{2}}=\mathbf{0 . 1 2 0 0 1} \\ & \lambda_{3}^{W_{2}}=1.19536 \end{aligned}$ | 0.001 |
| $l_{3}^{W_{2}}: 21 \%$ <br> from 2 | $\begin{gathered} \mathrm{ABC}^{3} \\ 2000 \end{gathered}$ | 3 | 3 | 3 | $L_{1}$ | $\begin{aligned} & \lambda_{1}^{W_{2}}=-8.2110 \\ & \lambda_{2}^{W_{2}}=-0.4704 \\ & \lambda_{3}^{W_{2}}=\mathbf{0 . 2 0 9 3 8} \end{aligned}$ | 0.062 |
| $l_{2}^{s_{2}}: 27 \%$ <br> from 2 | $\begin{gathered} \mathrm{BCG}^{4} \\ 1 \end{gathered}$ | 4 | 3 | 3 | $L_{2}$ | $\begin{aligned} & \lambda_{1}^{s_{2}}=-2.2768 \\ & \lambda_{2}^{s_{2}}=\mathbf{0 . 2 7 0 0 0} \end{aligned}$ | 0.000 |
| $\begin{aligned} & l_{1}^{s_{2}}: 33 \% \\ & \text { from } J_{1}^{s_{2}} \end{aligned}$ | $\begin{gathered} \mathrm{CG}^{2} \\ 50 \end{gathered}$ | 4 | 3 | 3 | $L_{2}$ | $\begin{aligned} & \lambda_{1}^{s_{3}}=\mathbf{0 . 3 3 0 1 4} \\ & \lambda_{2}^{s_{2}}=1.09699 \end{aligned}$ | 0.014 |
| $\begin{aligned} & l_{1}^{W_{s}}: 43 \% \\ & \text { from } J_{1}^{W_{s}} \end{aligned}$ | $\begin{gathered} \mathrm{ACG}^{4} \\ 100 \end{gathered}$ | 2 | 3 | 3 | $L_{3}$ | $\begin{aligned} & \lambda_{1}^{W_{s}}=\mathbf{0 . 4 2 9 9 5} \\ & \lambda_{2}^{W_{s}}=1.35922 \end{aligned}$ | 0.005 |
| $l_{2}^{W_{4}}: 51 \%$ <br> from 4 | $\begin{aligned} & \mathrm{AC}^{1} \\ & 200 \end{aligned}$ | 2 | 3 | 3 | $L_{3}$ | $\begin{aligned} & \lambda_{1}^{W_{+}}=-0.5824 \\ & \lambda_{2}^{W_{s}}=\mathbf{0 . 5 0 9 9 3} \end{aligned}$ | 0.007 |
| $\begin{aligned} & l_{3}^{s_{4}}: 59 \% \\ & \text { from } 4 \end{aligned}$ | $\begin{gathered} \mathrm{CG}^{2} \\ 500 \end{gathered}$ | 2 | 4 | 3 | $L_{4}$ | $\begin{aligned} & \lambda_{1}^{S_{1}}=-5.3366 \\ & \lambda_{2}^{S_{x}}=-0.3494 \\ & \lambda_{3}^{S_{s_{1}}}=\mathbf{0 . 5 9 0 8 2} \end{aligned}$ | 0.082 |
| $\begin{aligned} & l_{2}^{s_{4}}: 64 \% \\ & \text { from } J_{2}^{s_{4}} \end{aligned}$ | $\begin{gathered} \mathrm{ABC}^{3} \\ 3 \end{gathered}$ | 2 | 4 | 3 | $L_{4}$ | $\begin{aligned} & \lambda_{1}^{s_{4}}=-1.4336 \\ & \lambda_{2}^{s_{4}}=\mathbf{0 . 6 3 9 9 2} \\ & \lambda_{3}^{s_{4}}=1.74898 \end{aligned}$ | 0.008 |
| $\begin{aligned} & l_{1}^{s_{4}}: 66 \% \\ & \text { from } J_{1}^{s_{4}} \end{aligned}$ | $\begin{gathered} \mathrm{AB}^{1} \\ 50 \end{gathered}$ | 2 | 4 | 3 | $L_{4}$ | $\begin{aligned} & \lambda_{1}^{s_{4}}=\mathbf{0 . 6 6 0 7 7} \\ & \lambda_{2}^{s_{s}}=1.16083 \\ & \lambda_{3}^{s_{4}}=2.36058 \end{aligned}$ | 0.077 |
| $\begin{aligned} & l_{1}^{W_{c}}: 71 \% \\ & \text { from } J_{1}^{W_{c}} \end{aligned}$ | $\begin{aligned} & \mathrm{AG}^{2} \\ & 1000 \end{aligned}$ | 2 | 2 | 3 | $L_{5}$ | $\begin{aligned} & \lambda_{1}^{W_{s}}=\mathbf{0 . 7 0 9 7 8} \\ & \lambda_{2}^{W_{s}}=2.10383 \end{aligned}$ | 0.022 |
| $\begin{aligned} & l_{2}^{W_{\odot}}: 77 \% \\ & \text { from } 6 \end{aligned}$ | $\begin{gathered} \mathrm{BC}^{1} \\ 10 \end{gathered}$ | 2 | 2 | 3 | $L_{5}$ | $\begin{aligned} & \lambda_{1}^{W_{s}}=-0.1496 \\ & \lambda_{2}^{W_{s}}=\mathbf{0 . 7 7 0 0 0} \end{aligned}$ | 0.000 |
| $\begin{aligned} & l_{2}^{S_{1}}: 82 \% \\ & \text { from } 6 \end{aligned}$ | $\begin{gathered} \mathrm{AC}^{1} \\ 20 \end{gathered}$ | 2 | 2 | 4 | $L_{6}$ | $\begin{aligned} & \lambda_{1}^{S_{0}}=-0.1425 \\ & \lambda_{2}^{s_{0}}=\mathbf{0 . 8 2 0 0 1} \end{aligned}$ | 0.001 |
| $\begin{aligned} & l_{1}^{S_{6}}: 85 \% \\ & \text { from } J_{1}^{s_{6}} \end{aligned}$ | $\begin{aligned} & \mathrm{BG}^{2} \\ & 1000 \end{aligned}$ | 2 | 2 | 4 | $L_{6}$ | $\begin{aligned} & \lambda_{1}^{s_{0}}=\mathbf{0 . 8 4 9 4 1} \\ & \lambda_{2}^{s_{0}}=2.07797 \end{aligned}$ | 0.059 |
| $\begin{aligned} & l_{2}^{E_{c}}: 91 \% \\ & \text { from } J_{2}^{E_{s}} \end{aligned}$ | $\begin{gathered} \mathrm{ABC}^{3} \\ 100 \end{gathered}$ | 2 | 2 | 2 | $L_{7}$ | $\begin{aligned} & \lambda_{1}^{E_{i c}}=3.47748 \\ & \lambda_{2}^{E_{i}}=\mathbf{0 . 9 0 9 9 9} \end{aligned}$ | 0.001 |
| $l_{1}^{E_{i}}: 96 \%$ <br> from 8 | $\begin{gathered} \mathrm{ABG}^{4} \\ 20 \end{gathered}$ | 2 | 2 | 2 | $L_{7}$ | $\begin{aligned} & \lambda_{1}^{E_{i c}}=\mathbf{0 . 9 5 9 9 6} \\ & \lambda_{2}^{E_{0}}=-0.0128 \end{aligned}$ | 0.004 |

${ }^{\dagger}$ fault resistance; ${ }^{1}$ line-to-line fault; ${ }^{2}$ single-line-to-ground fault;
${ }^{3}$ three-phase fault; ${ }^{4}$ line-to-line-to-ground fault.

Fig. 13 shows the effect of line parameters inaccuracies on the algorithm's precision; blue is for low error and red is for high error. An AG fault occurs on a point of the line section $l_{1}^{W_{\star}}$ which is $50 \%$ away from
junction node $J_{1}^{W_{4}}$ with resistance $10 \Omega$. The maximum fault-location error (equals to $2.42 \%$ with $-5 \%$ line parameters inaccuracies in both line branches $L_{2}$ and $L_{3}$ ) demonstrates that the proposed method is not sensitive to system parameters.


Fig. 13. Effect of line parameters inaccuracies on fault-location accuracy.

## 4. Conclusions

A novel fault-location technique for multi-terminal multi-section nonhomogeneous transmission lines is presented. Basic principles and details of formulation are proposed. By only the derived indices TVE ${ }^{\max }$ and TVE ${ }^{\text {min }}$ for each tap node, the faulty line branch can be correctly identified and the considered faultlocation problem is simplified to the two-terminal nonhomogeneous transmission lines configuration. Besides, by calculating normalized fault distance for each section on the selected faulty branch, the fault section and the exact fault location can be correctly identified. Case studies verify the accuracy and robustness of the proposed technique for different degrees of fault resistance, fault type, fault location, prefault loading and line parameters inaccuracies.

## References

[1] Sodhi, R. and Sharieff, M. I.: 'Phasor measurement unit placement framework for enhanced wide-area situational awareness', IET Gener., Transm. \& Distrib., 2015, 9, (2), pp. 172-182
[2] Phadke, A.: ‘Synchronized phasor measurements in power systems', IEEE Comput. Applicat. Power, 1993, 6, (2), pp. 10-15
[3] He, Z., Mai, R. K., He, W., and Qian, Q. Q.: ‘Phasor-measurement-unit-based transmission line fault location estimator under dynamic conditions', IET Gener., Transm. \& Distrib., 2011, 5, (11), pp. 1183-1191
[4] Al-Mohammed, A. H. and Abido, M.: 'A fully adaptive PMU-based fault location algorithm for seriescompensated lines', IEEE Trans. Power Syst., 2014, 29, (5), pp. 2129-2137
[5] Salehi-Dobakhshari, A. and Ranjbar, A. M.: 'Robust fault location of transmission lines by synchronised and unsynchronised wide-area current measurements', IET Gener., Transm. \& Distrib., 2014, 8, (9), pp. 1561-1571
[6] Kezunović, M and Peruničić, B: ‘Automated transmission line fault analysis using synchronized sampling at two ends’, IEEE Trans. Power Syst., 1996, 11, (1), pp. 441-447
[7] Lee, C., Park, J., Shin, J., and Radojevié, Z.: 'A new two-terminal numerical algorithm for fault location, distance protection, and arcing fault recognition', IEEE Trans. Power Syst., 2006, 3, (21), pp. 1460-1462
[8] Yang, X., Choi, M. S., Lee, S. J., Ten, C. W., and Lim, S.-I.: 'Fault location for underground power cable using distributed parameter approach', IEEE Trans. Power Syst., 2008, 23, (4), pp. 1809-1816
[9] Liu, C. W., Lin, T. C., Yu, C. S., and Yang, J. Z.: 'A fault location technique for two-terminal multisection compound transmission lines using synchronized phasor measurements’, IEEE Trans. Smart Grid, 2012, 3, (1), pp. 113-121
[10] Livani, H. and Evrenosoglu, C. Y.: 'A fault classification and localization method for three-terminal circuits using machine learning', IEEE Trans. Power Deliv., 2013, 28, (4), pp. 2282-2290
[11] Izykowski, J., Rosolowski, E., Saha, M. M., Fulczyk, M., and Balcerek, P.: 'A fault-location method for application with current differential relays of three-terminal lines', IEEE Trans. Power Deliv., 2007, 22, (4), pp. 20992107
[12] Lin, T. C., Lin, P. Y., and Liu, C. W.: 'An algorithm for locating faults in three-terminal multisection nonhomogeneous transmission lines using synchrophasor measurements', IEEE Trans. Smart Grid, 2014, 5, (1), pp. 38-50
[13] Brahma, S. M.: 'Fault location scheme for a multi-terminal transmission line using synchronized voltage measurements', IEEE Trans. Power Deliv., 2005, 20, (2), pp. 1325-1331
[14] Liu, C. W., Lien, K. P., Chen, C. S., and Jiang, J. A.: 'A universal fault location technique for N-terminal transmission lines', IEEE Trans. Power Deliv., 2008, 23, (3), pp. 1366-1373
[15] Jiang, Q., Wang, B., and Li, X.: ‘An efficient PMU-based fault-location technique for multiterminal transmission lines', IEEE Trans. Power Deliv., 2014, 29, (4), pp. 1675-1682
[16] Lin, Y. H., Liu, C. W., and Chen, C. S.: ‘A new PMU-based fault detection/location technique for transmission lines with consideration of arcing fault discrimination-part I: theory and algorithms', IEEE Trans. Power Deliv., 2004, 19, (4), pp. 1587-1593
[17] Lin, Y. H., Liu, C. W., and Chen, C. S.: ‘A new PMU-based fault detection/location technique for transmission lines with consideration of arcing fault discrimination-part II: performance evaluation', IEEE Trans. Power Deliv., 2004, 19, (4), pp. 1594-1601
[18] Dobakhshari, A. S. and Ranjbar, A. M.: 'A wide-area scheme for power system fault location incorporating bad data detection', IEEE Trans. Power Deliv., 2015, 30, (2), pp. 800-808

