An Optimal Torque Control Based on Effective Tracking Range for Maximum Power Point Tracking of Wind Turbines under Varying Wind Conditions

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Abstract: This paper focuses on the development of optimal torque (OT) control, which is a commonly used method for maximum power point tracking (MPPT). Due to the sluggish response of wind turbines with high inertia, conventional OT control was improved to increase MPPT efficiency by dynamically modifying the generator torque versus rotor speed curve. An idea that tracking a local interval of wind speed where the wind energy is primarily distributed rather than the total range of wind speed variation is applied in this paper. On this basis, an effective tracking range that corresponds to the local interval of wind speed with concentrated wind energy distribution is proposed and an improved OT control based on effective tracking range is developed. In this method, based on a direct relationship between effective tracking range and wind conditions, the torque curve can be quickly optimized so that higher and more stable MPPT efficiency can be achieved under varying wind conditions. Meanwhile, MPPT efficiency enhancement by reducing tracking range without increasing torque discrepancy leads to a low cost of generator torque fluctuation and drive train load. Finally, simulations based on FAST (Fatigue, Aerodynamics, Structures, and Turbulence) and experiments conducted on a wind turbine simulator are presented to verify the proposed method.

1. Introduction

Variable-speed wind power generation systems (WPGSs) [1]-[3] have received tremendous attention in recent decades because the variable-speed operation can provide more energy output and improved power quality compared to fixed-speed turbines. To maintain the optimal tip speed ratio (TSR) and achieve the maximum wind power at various wind speeds, variable-speed WPGSs need to adjust the rotor speed accordingly.

Previous research has focused on several types of maximum power point tracking (MPPT) control strategies, namely TSR control [4], [5], optimal torque (OT) control [4], [6]-[14], power signal feedback (PSF) control [4], [15], [16] and hill-climb searching (HCS) control [4], [17] (also called perturbation and observation control). Among these, OT and PSF controls are simple, fast, and commonly used methods [4], [5]. Because these two control methods are similar in performance and complexity of implementation [4], this paper only considers OT control.

Regardless of the dynamic behaviour of wind turbines and the dynamic process of MPPT, only steady state operation points at various wind speeds are defined by the generator torque versus rotor speed

curve (called torque curve for short) in conventional OT control. However, with the growing capacity of WPGSs, the rotor inertia of wind turbines can increase dramatically and lead to the turbines being unable to accelerate or decelerate quickly in response to wind speed changes and tracking losses [5]-[10], [12]-[14], [16]. To enhance MPPT efficiency, a number of improved OT controls have been proposed [8]-[13] to speed up the MPPT process of wind turbines under rapid wind variations by modifying the definition of the torque curve. They can be categorized into three types:

1) Compensation of generator torque, such as inertia compensation control (ICC) [18], optimally tracking rotor control [8], adaptive compensation control [13] and constant-bandwidth control [19]. By adding a term proportional to rotor acceleration into the torque demand, these methods increase the torque discrepancy to assist the acceleration and deceleration of wind turbines such that more wind energy can be captured. Because this compensation term leads to a higher-frequency component in torque demand, it is necessary to consider the compromise between wind energy capture and drive train load.

2) Modification of torque curve gain, such as decreased torque gain (DTG) control [9] and adaptive torque control (ATC) [11], [12]. Because the power available in the wind is proportional to the cube of wind speed, these control strategies only improve the acceleration ability of turbines by decreasing torque curve gain at the cost of the weakness of deceleration ability. Thus, wind energy harvested from wind lulls is partly discarded so as to capture more energy from wind gusts.

3) Reduction of tracking range. Considering that the MPPT process can also be accelerated by shortening the tracking distance, the variable range of rotor speed is reduced by simply raising the lowest rotor speed for power generation [20], which is totally different from increasing torque discrepancy utilized in the above two types.

Furthermore, the torque curve needs to be dynamically modified depending on varying wind conditions (including mean wind speed and turbulence intensity) [8], [9], [20]. Because the quantitative relationship between the optimal modification of the torque curve and wind conditions has not been thoroughly investigated, the torque curve is adjusted either without full consideration of wind conditions [8-9], [13], [15-16], [20] or only by complicated algorithms, such as ATC with adaptive algorithm [11], [12]. When ATC is applied, the iteration procedure is required and the fluctuation of wind conditions may also cause an incorrect search direction or even non-convergence of an iterative search [21].

Considering the sluggish dynamic response of large-inertia wind turbines to wind turbulence, it is feasible that tracking a local (conservative) interval of wind speed where the wind energy is primarily distributed, rather than the total range of wind speed variation, can effectively improve wind energy production. Meanwhile, MPPT efficiency enhancement by reducing tracking range without changing the

torque curve gain indicates an acceptable cost of generator torque fluctuation and drive train load. On this basis, this paper proposes an effective tracking range corresponding to the local interval of wind speed with concentrated energy distribution, by which a direct relation between reduced tracking range and wind conditions is established so that the torque curve can be rapidly and appropriately optimized. Then, an improved OT control based on effective tracking range is developed. This control strategy can stably improve MPPT efficiency under varying wind conditions at a low cost of drive train load.

The remainder of the paper is organized as follows. Section 2 briefly describes the wind turbine model. A review of OT control is provided in Section 3. In Section 4, the concept of effective tracking range and an improved OT control based on effective tracking range are proposed. In Section 5, simulations based on FAST (Fatigue, Aerodynamics, Structures, and Turbulence) [22] and experiments conducted on a wind turbine simulator (WTS) [23]-[25] are presented. Conclusions are drawn in Section 6.

2. Wind turbine modelling

The mechanical power that a wind turbine extracts from wind is calculated as:

$$P_m = 0.5\rho\pi R^2 v^3 C_P(\lambda,\beta) \tag{1}$$

where ρ is the air density, R is the radius of the wind turbine, and v is the wind speed. Considering the blade pitch angle β frequently remains constant in MPPT operation, the power coefficient C_P is a function of TSR λ , which is defined as

$$\lambda = \omega_r R / v \tag{2}$$

where ω_r is the rotor speed of the turbine. An optimum TSR, denoted by λ_{opt} , yields the maximum power coefficient C_p^{max} . The aerodynamic torque can then be given by

$$T_m = P_m / \omega_r \,. \tag{3}$$



Fig. 1. Block diagram of the wind turbine model and MPPT control system

As shown in Fig. 1, the mechanical characteristics of a wind turbine include a two-mass model of shaft dynamics [22] consisting of a rotor with large inertia J_r , a generator with small inertia J_g , and an ideal gearbox with gear ratio n_g :

$$\begin{cases} J_r \dot{\omega}_r = T_m - D_r \omega_r - T_{ls} \\ J_g \dot{\omega}_g = T_{hs} - D_g \omega_g - T_{em} \\ n_g = \omega_g / \omega_r = T_{ls} / T_{hs} \end{cases}$$
(4)

where D_r , D_g are rotor and generator external damping and T_{ls} , T_{hs} are low-speed and high-speed shaft torque, respectively. T_{em} is the electromagnetic torque of the generator. By combining the above three equations, the mechanical dynamics of a wind turbine can be further represented as a single lumped mass model:

$$J_t \dot{\omega}_r = T_m - D_t \omega_r - T_g \tag{5}$$

where $J_t = J_r + n_g^2 J_g$, $D_t = D_r + n_g^2 D_g$, and $T_g = n_g T_{em}$.

3. Review of optimal torque control

The MPPT control system based on the OT method is illustrated in Fig. 1 in the box outlined with the solid line. It consists of two cascading control loops [26]: 1) the MPPT control (outer loop) concerns the regulation of rotor speed and provides the generator torque reference as the input to the generator controller; and 2) the generator control (inner loop) regulates the electromagnetic torque. Because the electromagnetic response time is much faster than the mechanical response, it is reasonable to dissociate the designs of the MPPT and generator controls [26]. In this paper, the inner loop is assumed to be well controlled and the focus here is on the MPPT control.

3.1. Definition of optimum torque curve and OT control

 λ_{opt} implies that the maximum available wind power can be obtained only when the rotor speed is adjusted to an optimal value determined by

$$\omega_r^{opt} = \lambda_{opt} v / R.$$
(6)

Then, by connecting the maximum power points at various wind speeds, a maximum power curve of the wind turbine can be defined as

$$P_m^{opt}\left(\omega\right) = K_{opt}\omega_r^3, \quad \omega_r > \omega_r^{bgn} \tag{7}$$

where ω_r^{bgn} is the lowest rotor speed for power generation (called starting speed for short) and K_{opt} is the optimum torque curve gain, defined as

$$K_{opt} = \left(0.5\rho\pi R^5 C_P^{\max}\right) / \lambda_{opt}^3.$$
(8)

Then, the optimum torque curve for the two-mass model is derived as [26]

$$T_{em}^{opt}\left(\omega_{r}\right) = \frac{K_{opt}}{n_{g}}\omega_{r}^{2} - \left(n_{g}D_{g} + \frac{D_{r}}{n_{g}}\right)\omega_{r}, \quad \omega_{r} > \omega_{r}^{bgn}.$$
(9)

To achieve MPPT, the generator torque is adjusted along the optimum torque curve according to the measured rotor speed ω_r . This strategy is known as OT control. Note that when the rotor speed reaches ω_r^{bgn} , a PI controller is activated to maintain the rotor speed at ω_r^{bgn} [18].

The tracking losses due to the sluggish response of a rotor with high inertia to wind turbulence have been discussed [8], [14] and it has been recognized that the effect of the turbine inertia on wind energy capture and the tracking dynamic should be investigated to improve OT control method [12]-[14].

3.2. Compensation of generator torque

To speed up MPPT process by increasing the torque discrepancy, the inertia compensation control [18] introduces a torque term proportional to rotor acceleration and thus the torque curve (9) is modified as

$$T_{em}^{opt}\left(\omega_{r}\right) = \frac{K_{opt}}{n_{g}}\omega_{r}^{2} + K_{f}\dot{\omega}_{r} - \left(n_{g}D_{g} + \frac{D_{r}}{n_{g}}\right)\omega_{r}, \quad \omega_{r} > \omega_{r}^{bgn}.$$
(10)

Although increasing torque discrepancy can assist the acceleration and deceleration of wind turbines such that more wind energy can be harvested, the compensation term inevitably generates a higher-frequency component in torque demand and increase the short-term power variability. Therefore, K_f needs to be carefully selected to achieve a compromise between wind energy production and other considerations, such as drive train load and output power variation [9][18].

3.3. Modification of torque curve gain

Considering that the power available from wind is proportional to the cube of the wind speed, maximization of energy capture from wind gusts is much more important than wind lulls [9], [10], [12]. Based on this idea, DTG control uses a decreased torque curve gain K_d instead of K_{opt} [9]. However, there is a tradeoff determined by K_d between wind energy extraction from high and low wind speed. To

optimize K_d , an adaptive torque control is further developed [11], [12], as shown in the box outlined with the dashed line in Fig. 1. It dynamically adjusts the adaptive gain M that incorporates all of the terms in K_d except the slowly time-varying air density. Accordingly, the torque curve defined in (9) is rearranged to

$$T_{em}^{opt}(\omega_r) = \frac{\rho M}{n_g} \omega_r^2 - \left(n_g D_g + \frac{D_r}{n_g}\right) \omega_r, \quad \omega_r > \omega_r^{bgn}.$$
(11)

By calculating the change in the mean power coefficient \overline{C}_p between the adjacent adaptation periods, the adaptive gain M [11, 12] is determined by

$$M(k) = M(k-1) + \Delta M(k), \qquad (12)$$

$$\Delta M(k) = \gamma_{\Delta M} \cdot sgn[\Delta M(k-1)] \cdot sgn[\Delta \overline{C}_{P}(k)] \cdot \left| \Delta \overline{C}_{P}(k) \right|^{1/2},$$
(13)

$$\Delta \overline{C}_{p}(k) = \overline{C}_{p}(k) - \overline{C}_{p}(k-1).$$
(14)

Here, ΔM is the perturbation of M, $\gamma_{\Delta M}$ is the positive gain on adaptation law, and the mean power coefficient \overline{C}_P during an adaptation period $t = N\Delta t$ is numerically approximated as

$$\bar{C}_{P} = \frac{\sum_{i=1}^{N} P_{cap}(i)}{\sum_{i=1}^{N} 0.5 \rho \pi R^{2} v_{i}^{3} \cos^{3} \psi(i)} \quad i = 1, ..., N,$$
(15)

$$P_{cap}(i) = T_{em}(i)\omega_g(i) + J_t\omega_r(i)\dot{\omega}_r(i)$$
(16)

where $P_{cap}(i)$ is the captured wind power at the *i*-th step, Δt is the simulation or sampling step size, and ψ is the yaw error, which is assumed in this paper to be zero. The iteration continues in the above manner and M eventually converges towards the optimal gain. Because the algorithm used is similar to the HCS method, the iteration procedure is relatively time-consuming and the search direction or even convergence might also be affected by a change of wind conditions (as mentioned in Section 5).

3.4. Reduction of tracking range by increasing the starting speed

Considering that the acceleration of turbines with increasing wind speed can also be assisted by shortening the tracking distance, improved OT control based on reduction of the tracking range has been reported [20]. The optimum torque curve is also employed and the tracking range is reduced by simply increasing the starting speed, which is constant and usually set as the optimal rotor speed corresponding to

the cut-in wind speed in conventional OT control [27]. As illustrated in Fig. 2(a), the starting speed is increased by shifting the line section A-B to the right (to A'-B'). Note that section C-D remains unchanged.



Fig. 2. Illustration of tracking range reduction

(a) Optimum torque curve with increased starting speed for reduction of tracking range. (b) Improvement due to a reduction in the tracking range. ω_r^{bgn} and K_d are both optimized by the trial-and-error method.

To determine the improvement due to the reduction of the tracking range, a simulation was conducted on a two-mass model of a CART3 turbine [28] excited by a step variation of wind speed (from 3.5 to 6.0 m/s). Fig. 2(b) compares the simulated trajectories of rotor speed and aerodynamic power corresponding to the reduction of tracking range (solid line marked with °) vs. the decreased torque curve gain (solid line marked with *). The optimal rotor speed is also plotted as the dashed line. According to Fig. 2(b), the improvement due to the decreased torque curve gain can also be obtained by reducing the tracking range. During this simulation, the following were noted:

1) When the optimum torque curve with the increased starting speed is applied, the acceleration of the turbine during wind gusts can be rapidly accomplished because the tracking distance of the rotor speed is shortened.

2) Because the rotor speed can never drop below the starting speed, the turbine is unable to reach λ_{opt} at a low wind speed value. This implies that maximization of wind energy extraction for low wind speeds is partly discarded in exchange for more energy production from high wind speeds.

3) There is also a tradeoff, determined by the starting speed, between wind energy capture from high and low wind speeds. To illustrate this tradeoff, the energy captured from the high/low wind speeds (i.e., 6.0 and 3.5 m/s, respectively) and their total vs. different starting speeds is compared in Table 1. Note that

the optimal starting speed also varies with wind conditions. However, in [20], only the mean wind speed is considered when adjusting the starting speed and, additionally, an anemometer is required.

starting speed (rad/s)	1.2	1.4	1.6	1.8	2.0
energy captured from high wind speed of 6.0m/s (<i>MJ</i>)	3.550	3.781	3.810	3.756	3.614
energy captured from low wind speed of 3.5m/s (MJ)	0.720	0.687	0.625	0.537	0.429
total captured energy (MJ)	4.270	4.468	4.435	4.293	4.043

 Table 1
 Captured energy from high/low wind speeds (and the total) vs. different starting speeds

4. Effective tracking range and improved OT method

Based on the reduction of tracking range, in this section an effective tracking range corresponding to the local interval of wind speed with concentrated energy distribution is proposed. Then, an improved OT control method is developed in which the starting speed is rapidly optimized by effective tracking range.

4.1. Effective tracking range based on wind energy distribution

Because the variation of the rotor speed in response to wind turbulence is slow due to the high inertia of the turbine, it is reasonable that for a more efficient MPPT more attention should be paid to the local interval of wind speed, where the wind energy is primarily distributed, rather than the whole range of wind speed variation.

In the MPPT region, the available wind power for a specific wind turbine is proportional to the cube of the wind speed and can be expressed as

$$P_a(v) = 0.5 \rho \pi R^2 C_P^{\max} v^3.$$
 (17)

Then, the probability density of available wind power is defined using

$$f_{Pa}(v) = \frac{P_a(v)f_v(v)}{\overline{P}_a} = \frac{v^3 f_v(v)}{\int_0^\infty v^3 f_v(v) dv},$$
(18)

where $f_v(v)$ is the wind speed probability density function and \overline{P}_a is the mean available wind power, defined as

$$\overline{P}_{a} = \int_{0}^{\infty} P_{a}\left(v\right) f_{v}\left(v\right) dv = 0.5 \pi \rho R^{2} C_{P}^{\max} \int_{0}^{\infty} v^{3} f_{v}\left(v\right) dv .$$

$$\tag{19}$$

Obviously, $f_{Pa}(v)$ is very similar to the wind power density distribution [29, 30] with the exception of the air density, which is assumed to be constant in the scientific literature [30]. $f_{Pa}(v)$ describes the

distribution of the maximum wind power that can be absorbed by a turbine at different wind speeds. Due to tracking losses, however, only a portion of the maximum wind power can actually be obtained.

Based on the wind power density distribution models in [30], the occupation ratio of the available wind power contained in a wind speed interval $U_v = (v_t, v_u)$ can be calculated as

$$r = \int_{v_l}^{v_u} f_{P_a}(v) dv = \frac{\int_{v_l}^{v_u} P_a(v) f_v(v) dv}{\overline{P}_a} = \frac{\int_{v_l}^{v_u} v^3 f_v(v) dv}{\int_0^\infty v^3 f_v(v) dv}.$$
 (20)

Because the available wind energy extracted by a wind turbine operated in the MPPT region during the time period t is obtained using [25]

$$\overline{E}_{a} = t \int_{0}^{\infty} P_{a}(v) f_{v}(v) dv = \overline{P}_{a} \cdot t , \qquad (21)$$

r also can be computed using (22), which indicates the ratio occupied by a wind speed interval from the point of view of the available wind energy extraction:

$$r = \frac{E_a(U_v)}{\overline{E}_a} = \frac{\int_{v_l}^{v_a} P_a(v) \cdot t \cdot f_v(v) dv}{\overline{P}_a \cdot t}, \qquad (22)$$

where $E_a(U_v)$ is the available wind energy contained in the wind speed interval U_v during the time period t.



Fig. 3. Wind speed interval of the maximum energy carrier $U_v^m = (v_l^m, v_u^m)$

Using the occupation ratio, the wind speed interval of the maximum energy carrier $U_v^m = (v_l^m, v_u^m)$ is defined as that which provides the same occupation ratio *r* with the minimum interval width. As

illustrated in Fig. 3, this definition can be interpreted as the extension of the concept of wind speed of the maximum energy carrier (marked with Δ) [31] and denoted as the wind speed interval that provides the most concentrated wind energy.

According to the definition of the wind speed interval of the maximum energy carrier, its boundaries can be demonstrated to correspond to the same value of f_{Pa} , i.e.,

$$f_{Pa}\left(v_{l}^{m}\right) = f_{Pa}\left(v_{u}^{m}\right). \tag{23}$$

The proof is equivalent to solving a problem of extreme value with constraints, expressed as

$$Min F\left(v_l^m, v_u^m\right) = v_u^m - v_l^m \quad s.t. \int_{v_l^m}^{v_u^m} f_{Pa}\left(v\right) dv = const, \qquad (24)$$

which can be solved by Lagrange multipliers.

When both the concentrated distribution of wind energy and the tracking performance of the wind turbine are considered, the effective tracking range U_{ω}^{m} for a specific wind turbine is obtained by transforming the wind speed interval of the maximum energy carrier to the corresponding tracking range of rotor speed using (2), as follows:

$$U_{\omega}^{m} = \left\{ \left(\omega_{l}^{m}, \omega_{u}^{m} \right) \middle| \omega_{l}^{m} = \lambda_{opt} v_{l}^{m} \left(r_{t} \right) \middle| R, \ \omega_{u}^{m} = \lambda_{opt} v_{u}^{m} \left(r_{t} \right) \middle| R \right\}.$$

$$(25)$$

Note that the value of the occupation ratio r_t in (25) depends on the tracking performance of the specific turbine, which is approximated by the overall efficiency of the wind turbine operated by conventional OT control in turbulence and evaluated by simulations or experiments as follows:

$$r_{t} \approx \frac{\sum_{i=1}^{N} P_{cap}(i)}{\sum_{i=1}^{N} P_{a}(i) \cos^{3} \psi(i)}$$
(26)

where $P_a(i)$ is the available wind power at the *i*-th step.

In summary, the effective tracking range not only corresponds to the most energetic wind speed interval but also matches the limited dynamic behaviour of the turbine. Moreover, equation (25) provides a direct relation between effective tracking range and wind conditions. Therefore, the effective tracking range can be used to estimate the optimal starting speed so that the variable rotor speed range is reasonably shortened.

4.2. Estimation of effective tracking range and optimal starting speed

For a given wind dataset, this section presents two computational methods for estimating effective tracking range that are periodically invoked by improved OT control based on the effective tracking range proposed in Section 4.3. In these methods, the effective tracking range is first estimated and then the optimal starting speed is determined as its lower bound.

4.2.1 Method based on $f_{Pa}(v)$

For a wind dataset observed by the wind speed estimator, $f_v(v)$ is first estimated by the kernel method [32]. Then, based on (23), a series of auxiliary lines that are parallel to the x-axis and intersect the $f_{Pa}(v)$ curve in the $v - f_{Pa}$ plane are used to approach U_v^m , as illustrated in Fig. 4(a). Considering that $f_{Pa}(v)$ is usually unimodal because wind power density distribution functions commonly exhibit a near-Gaussian shape [30], it is assumed that there should be two and only two intersection points of the auxiliary line with the $f_{Pa}(v)$ curve during the search. For a specific wind turbine with known r_t , the procedure for searching for U_a^m is described below.

Step 1: Regard an observed wind dataset as samples and estimate its $f_{\nu}(\nu)$ by the kernel method, as follows:

$$f_{\nu}\left(\nu\right) = \frac{1}{Nh} \sum_{i=1}^{N} K\left(\frac{\nu - \nu_{i}}{h}\right),\tag{27}$$

$$K(v) = \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}}, \quad h = 1.06\hat{\sigma} N^{-1/5}$$
(28)

where K(v) is a Gaussian kernel function, N is the number of samples in the wind dataset, $\hat{\sigma}$ is the standard deviation of the distribution, and h is the bandwidth of the kernel-smoothing window.

Step 2: Calculate the $f_{Pa}(v)$ according to (18). Note that the intersection points of the horizontal line with the $f_{Pa}(v)$ curve are numerically calculated by the bisection method.

Step 3: Initial the trial wind speed interval U_{ν} by choosing the horizontal line with a zero value of f_{Pa} , i.e., coincident with the x-axis.

Step 4: Increase f_{Pa} by a step size of Δf_{Pa} (set to 0.01) and shift the horizontal line to the new value of f_{Pa} . Correspondingly, reset the trial U_v using the intersection points of the horizontal line with the $f_{Pa}(v)$ curve.

Step 5: Compute the occupation ratio r associated with U_{v} according to (20) by calculating the area enclosed by the $f_{Pa}(v)$ curve.

Step 6: Check whether the approach condition is satisfied: if $r < r_t$, define the wind speed interval of the maximum energy carrier U_v^m as U_v , and go to Step 7. Otherwise, go to Step 4.

Step 7: Define U_{ω}^{m} in accordance with U_{ν}^{m} using (25) and estimate ω_{r}^{bgn} as the lower bound of U_{ω}^{m} , i.e., ω_{l}^{m} .

Step 8: END.



Fig. 4. Illustration of the search procedure for U_v^m (a) the method based on $f_{Pa}(v)$ (b) the method based on bins

4.2.2 Method based on bins

To avoid the heavy computational burden of $f_{\nu}(\nu)$ estimation, an approximation approach to determine U_{ω}^{m} and ω_{r}^{bgn} based on summarizing wind data by bins is also presented. By dividing the total wind speed range into N_{B} intervals (known as bins) $U_{\nu}^{j} = (v_{l}^{j}, v_{u}^{j}), j = 1, 2, ..., N_{B}$ with the same width, the occupation ratio r^{j} corresponding to each bin U_{ν}^{j} is simply calculated using the following equation (its derivation from (22) is given in Appendix A):

$$r^{j} = \frac{E_{a}\left(U_{v}^{j}\right)}{\overline{E}_{a}} \approx \frac{\sum_{v_{i}^{j} \le v_{i} < v_{u}^{j}} P_{a}\left(v_{i}\right)}{\sum_{i=1}^{N} P_{a}\left(v_{i}\right)}, i = 1, \cdots, N, \quad j = 1, \cdots, N_{B}.$$
(29)

Then, U_v^m can be approximately defined as the connected set of bins which provides the specific occupation ratio with the minimum number of bins. Assuming r^j exhibits an unimodal distribution, U_v^m can be approximated by removing the boundary bins with small r^j from the total wind speed range (i.e., $\bigcup_{i=1}^{N_B} U_v^j$) till the sum of the remaining bins' r^j approaches to r_i , as shown in Fig. 4(b).

Step 1: Initialization.

Step 1.1: Divide the total wind speed range into N_B bins with the same width, such as 0.10 m/s.

Step 1.2: Distribute the wind data into each bin according to their values.

Step 1.3: Compute the occupation ratio r^{j} for each bin according to (29).

Step 1.4: Initial the trial wind speed interval U_v as the total range of wind speed variation, i.e., $U_v = \bigcup_{i=1}^{N_B} U_v^{j}$.

Step 2: Determine the bin with the minimum r^{j} from the two boundary bins and reset the trial U_{ν} by excluding this bin.

Step 3: Compute the occupation ratio r associated with U_v by summing the r^j of the bins included in U_v .

Step 4: Check whether the approach condition is satisfied: if $r < r_t$, define the wind speed interval of the maximum energy carrier U_v^m as U_v , and go to Step 5. Otherwise, go to Step 2.

Step 5: Define U_{ω}^{m} in accordance with U_{ν}^{m} using (25) and estimate ω_{r}^{bgn} as the lower bound of U_{ω}^{m} , i.e., ω_{l}^{m} .

Step 6: END.

4.3. Improved OT method based on effective tracking range

In this section, an improved MPPT method based on effective tracking range (ETR) is proposed, as illustrated in the block diagram in Fig. 5. In the ETR method:



Fig. 5. Block diagram of the ETR method

1) A wind speed estimator based on Newton-Raphson algorithm [33] is employed to obtain real-time wind data. As illustrated in Fig. 5, the estimator consists of two blocks, namely aerodynamic torque estimation and wind speed estimation. The simulation and experimental results (see Fig. 6(b) and Fig. 8(b)) show that the wind speed estimator provides a good estimate of wind speed;

2) ω_r^{bgn} is periodically refreshed. At the end of each refresh cycle, U_{ω}^m is calculated only according to the recent wind data collected in the current cycle and ω_r^{bgn} is updated by its lower bound ω_l^m . That is to say, the line section A-B of torque curve is shifted according to ω_r^{bgn} , and section C-D remains unchanged, as illustrated in Fig. 2(a);

3) The refresh cycle is usually set to 20 minutes to several hours, which mainly depends on the time scale of the variation of wind conditions.

The procedure of the ETR method is described as follows:

Step 1: Initialization.

Step 1.1: Set the refresh cycle of starting speed ΔT_r and the cycle of the wind speed estimator Δt_w

(e.g., 0.1 to 0.5 seconds).

Step 1.2: Initialize ω_r^{bgn} as the optimal rotor speed ω_r^{opt} corresponding to the cut-in wind speed.

Step 2: Start a new refresh cycle.

Step 2.1: Reset the timer of the refresh cycle, t_r , to zero.

Step 2.2: Clear the wind dataset that records the estimated wind speed \hat{v} in each refresh cycle.

Step 3: In a new estimation cycle, estimate wind speed \hat{v} , record it in the wind dataset, and increase

 t_r by Δt_w .

Step 4: Check whether the current refresh cycle is over. If $t_r \ge \Delta T_r$, go to Step 5. Otherwise, go to Step 3.

Step 5: Estimate the optimal ω_r^{bgn} according to the wind dataset by the methods described in Section 4.2, refresh ω_r^{bgn} as the estimated value, and go to Step 2.

4.4. Discussion on the improved MPPT method

Because effective tracking range is obtained through the statistical analysis on wind energy distribution, the ETR method has the following advantages:

1) A direct relationship between reduced tracking range and wind conditions is established, i.e., equation (25). Thus, the tracking range can be simply and rapidly optimized for wind turbines to track the local interval of wind speed where the wind energy is primarily distributed.

2) Since mean wind speed and turbulence intensity, which are highly correlated with wind energy distribution, can be represented by $f_{Pa}(v)$ curve, the wind conditions are fully considered in determination of effective tracking range.

3) The need for accuracy and real-time performance of wind speed observations in the ETR method is less than for TSR controls [5] because the starting speed is determined based on the statistics of wind speed over a period of time.

5. Simulation Studies and Experimental Validation

In this section, FAST-based simulations and WTS-based experiments on the National Renewable Energy Laboratory (NREL) CART3 wind turbine are presented to verify the ETR method. Its performance is compared to three baseline MPPT controllers: conventional OT control, ICC and ATC.

5.1. Comparison using FAST simulation

The ETR method is validated using simulations on the FAST model of the CART3 turbine [28]. Note that:

1) Two wind profiles with a time horizon of six hours are generated by two components: the longterm evolution of wind speed chosen from the measured 10-minute mean wind speed and the turbulent wind described by the Kaimal model [34] corresponding to the above mean wind speed.

2) According to the cycle used in ATC, which ranges from ten minutes to three hours [9][12], the cycles in ATC and ETR method are set to be equal and 20 minutes.

3) According to (26), the occupation ratio r_t for the CART3 turbine is set at approximately 0.8. It is intuitive that the smaller (roughly less than 0.9) the r_t , the more effective the ETR method.

4) The appropriate value selected for $\gamma_{\Delta M}$ is 0.002. This value corresponds to the highest efficiency determined by simulations on a number of trial values from 0.002~0.003 so that the comparison of the MPPT methods is not affected by $\gamma_{\Delta M}$.

5) In ICC, a differential-based acceleration observation with a low-pass filter is utilized and K_f is set to 10.0.

Verification tools	Wind profile	MPPT control	\overline{C}_{P} during 6 hours	Increase vs. OT control
FAST simulation	wind profile 1	conventional OT control	0.4046	_
		adaptive torque control	0.4120	1.83%
		inertia compensation control	0.4117	1.75%
		ETR method	0.4141	2.35%
	wind profile 2	conventional OT control	0.4060	_
		adaptive torque control	0.4026	-0.84%
		inertia compensation control	0.4129	1.70%
		ETR method	0.4135	1.85%
WTS-based test	wind profile 1	conventional OT control	0.4119	_
		adaptive torque control	0.4174	1.34%
		inertia compensation control	0.4160	1.00%
		ETR method	0.4196	1.87%
	wind profile 2	conventional OT control	0.4115	_
		adaptive torque control	0.4052	-1.53%
		inertia compensation control	0.4172	1.39%
		ETR method	0.4176	1.48%

	Table 2	Efficiency	comparison	of the	four methods	s
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The performance comparison is shown in Table 2 and Fig. 6(a). For the wind speed profile 1 (the fluctuation of wind conditions is small), it can be observed that although ATC, ICC and ETR method can capture over 1.5% more energy from the available wind energy as compared to the conventional OT control, the proposed method is more efficient. When wind speed profile 2 (the fluctuation of wind conditions is large) is applied, ICC and ETR method can still capture over 1.5% more wind energy than the conventional OT control. However, the \overline{C}_p during the six hours of ATC is considerably less than for conventional OT control, because the perturbation of adaptive gain *M* is affected by the large fluctuation in mean wind speed.

To evaluate the drive train load and output power variation due to the proposed method, the dynamic trajectories over a period of 5 minutes (chosen from the wind profile 2) are plotted in Fig. 6(b). For the ETR method, although a low-amplitude fluctuation of generator torque and electric power, resulted from constant speed control at ω_r^{bgn} , occurs more readily because of the increase of ω_r^{bgn} , the variation of generator torque and electric power above ω_r^{bgn} is almost the same as for the conventional OT control. By contrast, when applying ICC, a similar MPPT performance improvement is obtained by higher amplitude

and higher frequency control action. Therefore, K_f should be appropriately selected to achieve a compromise between wind energy production and drive train load or output power variation.



(a) Variation of \overline{C}_{P} during cycles

(b) Dynamic trajectories over a period of 5 minutes

5.2. Experimental validation of ETR method

To verify the ETR method, a 10 kW WTS-based WPGS test bench (Fig. 7(a)) that can replicate the dynamic behaviour of the CART3 turbine is established. As illustrated in Fig. 7(b), the test bench consists of three main parts:

1) WTS. It includes a direct current motor (DCM) driven by a DCM drive, a flywheel, and the simulation program running in a Beckhoff Programmable Logic Controller (PLC). The simulation program mainly contains the scaling transform for scaling down WPGS's operational quantities to the motor scale [23], the aerodynamic simulation algorithm for calculating aerodynamic torque, and the rotor inertia compensation algorithm for mimicking the slow mechanical dynamics of real wind rotors [25]. By combining the deployment of flywheel [24] and the simulation program, the aerodynamic behaviour and wind rotor dynamics of the CART3 turbine with large inertia can be simulated by the WTS.

2) Electrical part, which is the same as for the WPGS, consists of a permanent-magnet synchronous generator (PMSG) and its grid-connected convertor (including generator-side rectifier and grid-side

inverter). The rectifier receives the electromagnetic torque reference and controls the PMSG torque. Since the electric part and its control are not discussed in this paper, they are directly accomplished by industrial converters, as commonly implemented in practical WPGS.

3) MPPT controller based on PLC implements MPPT control algorithms and sends electromagnetic torque reference T_{em}^{ref} to the rectifier.





Fig. 7. 10kW WTS-based WPGS test bench

(a) Laboratory implementation for experimental testing

(b) Schematic diagram of WTS-based WPGS test bench

The comparison of four MPPT control algorithms with the same parameter settings as noted in Section 5.1 are conducted again through the test bench and summarized in Table 2. Moreover, the \overline{C}_{p} during cycles and the dynamic trajectories over the same period obtained from the test bench are plotted in Figs. 8. Experimental results similar to FAST simulation are observed. During these experiments, it is noted that: 1) The torque curve modification, including the decrease of torque curve gain and the reduction of tracking range, needs to be dynamically and appropriately adjusted. Otherwise, the improvement of MPPT efficiency cannot be maintained with varying wind conditions.

2) Because a direct relation of reduced tracking range to mean wind speed and turbulence intensity is established by the proposed effective tracking range, the ETR method can adjust the starting speed appropriately and achieve relatively stable MPPT performance.

3) The MPPT efficiency can be improved by the ETR method at a low cost of generator torque/electric power fluctuation and drive train load.



Fig. 8. Experimental results on the six-hour turbulent wind profiles (a) Variation of \overline{C}_{p} during cycles

(b) Dynamic trajectories over a period of 5 minutes

6. Conclusion

Because of the slow dynamic behaviour of wind turbines in response to turbulence due to their high inertia, the MPPT dynamic should be considered to improve conventional OT control. Considering the turbine is unable to accelerate or decelerate quickly, the local interval of wind speed with the concentrated distribution of wind energy, rather than the total range of wind speed variation, is valuable for tracking.

Following this idea, an effective tracking range is proposed, which corresponds to the wind speed interval where the wind energy is primarily distributed. Based on the effective tracking range, a direct quantitative relationship between the reduced tracking range and wind conditions is established. By combining conventional OT control and estimation of effective tracking range with a wind speed estimator, a simple and efficient OT control is presented.

The FAST-based simulations and WTS-based experiments are conducted to verify the ETR method. Using the proposed method, the torque curve (i.e., the starting speed) can be rapidly and conveniently optimized so that higher and more stable MPPT efficiency can be achieved under varying wind conditions. Additionally, due to reducing tracking range without changing the torque curve gain, MPPT performance is improved at a low cost of generator torque/electric power fluctuation and drive train load.

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9. Appendix A

The derivation of equation (29):

If the integration interval $U_v = (v_1, v_u)$ is divided into m tiny intervals, i.e.,

$$(v_l + k\Delta v, v_l + (k+1)\Delta v), \quad k = 0, 1, \dots, m-1 \text{ and } v_u = v_l + m\Delta v,$$
 (A.1)

then

$$E_{a}(U_{v}) = t \int_{v_{l}}^{v_{a}} P_{a}(v) f_{v}(v) dv$$

$$= \sum_{k=0}^{m-1} t \int_{v_{l}+k\Delta v}^{v_{l}+(k+1)\Delta v} P_{a}(v) f_{v}(v) dv$$
(A.2)

Because the value of Δv is very small, and can be defined as the measurement resolution of the anemometer, for example 0.01 m/s, the product of the time period t and the integration of $P_a(v)f_v(v)$ over the tiny wind speed interval $(v_l + k\Delta v, v_l + (k+1)\Delta v)$ is approximately calculated by discrete wind speeds, as follows:

$$t \int_{v_{l}+k\Delta v}^{v_{l}+(k+1)\Delta v} P_{a}(v) f_{v}(v) dv$$

$$\approx P_{a}(v_{l}+k\Delta v) \cdot \Pr(v = v_{l}+k\Delta v) \cdot t$$

$$= P_{a}(v_{l}+k\Delta v) \cdot F(v = v_{l}+k\Delta v) \cdot \Delta t_{w}$$
(A.3)

where $Pr(v = v_l + k\Delta v)$ and $F(v = v_l + k\Delta v)$ are the occurrence probability and occurrence number of the wind speed $v_l + k\Delta v$ during the time period t, respectively, and Δt_w is a constant sampling cycle of wind speed. Thus, according to equations (A.2) and (A.3), for a series of N wind speed observations $v_i, i = 1, ..., N$ during a time period $t = N\Delta t_w$, $E_a(U_v)$ and \overline{E}_a can be numerically approximated by sampled wind data as follows:

$$\begin{split} E_{a}\left(U_{v}\right) &\approx \sum_{k=0}^{m-1} P_{a}\left(v_{l} + k\Delta v\right) \cdot F\left(v = v_{l} + k\Delta v\right) \cdot \Delta t_{w} \\ &= \sum_{k=0}^{m-1} \left(\sum_{v_{i} = v_{l} + k\Delta v} \left(P_{a}\left(v_{i}\right)\Delta t_{w}\right)\right) \\ &= \sum_{v_{l} \leq v_{i} < v_{u}} \left(P_{a}\left(v_{i}\right)\Delta t_{w}\right) \qquad i = 1, \cdots, N \\ &\overline{E}_{a} \approx \sum_{i=1}^{N} \left(P_{a}\left(v_{i}\right)\Delta t_{w}\right) \qquad i = 1, \cdots, N . \end{split}$$
(A.5)

Finally, equation (22) can be numerically calculated as follows:

$$r = \frac{E_a(U_v)}{\overline{E}_a} \approx \frac{\sum_{v_i \le v_i < v_a} P_a(v_i)}{\sum_{i=1}^N P_a(v_i)} \qquad i = 1, \cdots, N.$$
(A.6)