Equilibrium-Inspired Multiple Group Search Optimization with Synergistic Learning for Multiobjective Electric Power Dispatch

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***Abstract*—This paper proposes a novel multiple group search optimizer (MGSO) to solve the highly constrained multiobjective power dispatch (MOPD) problem with conflicting and competing objectives. The algorithm employs a stochastic learning automata based synergistic learning to allow information interaction and credit assignment among multi-groups for cooperative search. An alternative constraint handling, which separates constraints and objectives with different searching strategies, has been adopted to produce a more uniformly-distributed Pareto-optimal front (PF). Moreover, two enhancements, namely space reduction and chaotic sequence dispersion, have also been incorporated to facilitate local exploitation and global exploration of Pareto-optimal solutions in the convergence process. Lastly, Nash equilibrium point is first introduced to identify the best compromise solution from the PF. The performance of MGSO has been fully evaluated and benchmarked on the IEEE 30-bus 6-generator system and 118-bus 54-generator system. Comparisons with previous Pareto heuristic techniques demonstrated the superiority of the proposed MGSO and confirm its capability to cope with practical multiobjective optimization problems with multiple high-dimensional objective functions.**

***Index Terms*—Multiobjective power dispatch, Multiple group search optimizer, Synergistic learning, Nash equilibrium, Pareto-optimal front.**

NOMENCLATURE

a_i, b_i, c_i, d_i, e_i	Fuel cost coefficients for the ith generator with valve-point effects
$\alpha_i, \beta_i, \gamma_i, \zeta_i, \lambda_i$	Emission coefficients for the ith generator
$A_{\max}$	Maximum number of iterations of scanning
c_{rp}, $c_{r\mathrm{sl}}$	Constants for controlling ramping rate of the hyperbolic tangent functions
C_{sl}^k	Coefficient of synergistic components at the kth iteration
$C_{\mathrm{sl,max}}$, $C_{\mathrm{sl,min}}$	Upper and lower bounds of C_{sl}^k
C_p^k	Coefficient of leader component at the kth iteration
$C_{p,\max}$, $C_{p,\min}$	Upper and lower bounds of C_p^k
d_{cn}^k	Normalized crowding distance of the nth solution in the repository at the kth iteration
$D(\varphi)$	Polar to Cartesian transformation function
$f_g(\cdot)$	The gth objective function
f_{gj}	The gth objective's normalized fitness of the jth solution in the PF
h_{gj}	The gth objective's equilibrium value of the jth solution in the PF
H_g	Nash equilibrium solution of the gth objective
$iter_{L\max}$	Maximum number of iterations for space reduction
$iter_{R\max}$	Maximum number of iterations for adaptive ranger
$Iter_{\max}$	Maximum number of iterations
$L_{g\max}^k$	Maximum pursuit distance of the gth group at the kth iteration
LB	Lower bound for variable vector X_{gj}^k
L_{dn}^k	Normalized Euclidean distance in solution space between a given infeasible member and the nth element in the repository at the kth iteration
LF_{ij}	Apparent power flow from bus i to j
$LF_{k,\max}$	Maximum loading limit for the kth branch
M_{eq}	Number of equality constraints
M_{ineq}	Number of inequality constraints
M_{ing}^k	Number of infeasible members in the gth group at the kth iteration
M_{obj}	Number of MOPD objective functions
M_{p}	Population size of each searching group
M_{pf}	Maximum size of PF set in the repository
M_{rep}^k	Number of PF solutions to be selected for constraint handling in the repository at the kth iteration
M_{rg}^k	Number of rangers in the gth group at the kth iteration
M_{sg}^k	Number of scroungers in the gth group at the kth iteration
N_G	Number of generating units in the system
N_L	Number of branches in the system
N_{ng}^k	Number of members in set T_{ng}^k at the kth iteration
NP_i	Number of POZs for the ith generator
$prob_{gj}^k$	Selection probability of the jth member in the gth group at the kth iteration for synergistic learning
$prob_{Bn}^k$	Boltzmann distribution probability of the nth solution in the repository at the kth iteration
P_D	Total active power demand
P_{Gi}	Active power generation of generator i
$P_{Gi,\max}$	Maximum active power output of generator i
$P_{Gi,\min}$	Minimum active power output of generator i
$P_{Gi(j),\mathrm{lb}}$	Lower bound of the jth POZ for generator i
$P_{Gi(j),\mathrm{ub}}$	Upper bound of the jth POZ for generator i
P_{Loss}	Total power loss of transmission network
SP_{Gi}	Spinning reserve contribution of generator i
SP_R	System spinning reserve requirement

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T_{ng}^{k}	Set of members seeking out nondominated solutions in the gth group at the kth iteration
UB	Upper bound for variable vector X_{gj}^{k}
v_g	Upper expectation limit for the gth objective player
w	Temperature of Boltzmann distribution
X_{gj}^{k}	Position of the jth member in the gth group at the kth iteration
X_{gp}^{k}	Producer's position in the gth group at the kth iteration
$X_{m(\mathrm{sl})}^{k}$	Position of the member selected for synergistic learning from the mth group at the kth iteration
X_{pf}^{k}	Solution selected from the current PF for constraint handling at the kth iteration
η	Reinforcement factor
μ	Chaotic iteration factor
Ω	Set of generators with POZs
ω_g	Weight expressing relative importance of the gth objective function
$\theta_{\max}$	Maximum pursuit angle
$\psi_{\max}$	Maximum turning angle
φ_{gj}^{k}	Head angle of the jth member in the gth group at the kth iteration
φ_{gp}^{k}	Producer's head angle of the gth group at the kth iteration
$\%_{gr}^{k}$	Percentage of rangers for the gth group at the kth iteration
Δ_L	Predefined decrement size for $L_{g\max}^{k}$
Δ_R	Predefined increment size for $\%_{gr}^{k}$

I. Introduction

ELECTRIC power dispatch is an essential function required in modern energy management systems to determine the optimal steady-state operation of dispatchable generators with multiple contradictory objectives, such as fuel cost, emission reduction and energy saving, subjected to a set of operational and physical constraints [1]. Over the years, extensive research has been reported in the area of multiobjective power dispatch (MOPD). Most notably, many techniques have been presented to transform the MOPD into a single-objective optimization using the linear weighted aggregate method [2],[3]. However, many trials are required in those techniques to obtain a desired set of noninferior solutions by varying the weights, and they are not effective to handle problems with nonconvex Pareto-optimal fronts (PFs) [4]. Consequently, in recent years, various Pareto-based multiobjective stochastic optimization algorithms have been proposed to solve this MOPD problem.

So far the state-of-the-art in Pareto optimization algorithms, including multiobjective stochastic search technique (MOSST) [4], strength Pareto evolutionary algorithm (SPEA) [5], niched Pareto genetic algorithm (NPGA) [6], nondominated sorting genetic algorithm (NSGA) [6], NSGA-II [7], fuzzy clustering-based particle swarm optimization (FCPSO) [8], etc., have been successfully applied to the dual-objective environmental / economic dispatch (EED) problem on a small IEEE 30-bus 6-generator system to obtain a Pareto tradeoff between fuel cost and atmospheric emission. Those algorithms operate on a set of nondominated solutions with different search mechanisms in a single simulation trial. However, some important system constraints, like reserve constraints, have not been considered in the literatures yet. This work therefore is aimed to develop a novel multiobjective algorithm designed for highly constrained MOPD problems with high-dimensional objective functions.

Recently, a new optimization algorithm inspired by social group living and foraging behaviors of animals, called group search optimizer (GSO), was proposed based on the producer-scrounger model [9]. Previous application studies have been demonstrated that, compared to other evolutionary algorithms (EAs), the overall performance of the GSO exhibits superiority and high efficiency on the non-differential, high-dimensional and multimodal optimization problems [9]-[11], and hence it would be well suited for solving the highly constrained and nonlinear power system dispatch problems.

In this paper, a novel meta-heuristic multiple group search optimizer (MGSO) is further developed ingeniously to form a significantly improved multiobjective algorithm for large-scale MOPD applications. A new stochastic learning automata based reinforcement scheme is formulated to explicitly assign rewards among searching individuals for synergistic learning [12] which allows parallel groups to have information interaction and resource sharing in the cooperative search process. Furthermore, a dynamic search-space reduction strategy [13] is introduced in scanning mechanism to obtain the accurate and extreme vertex solutions in PF surface, and a chaotic sequence dispersion is adopted to improve population diversity and avoid entrapment into local optima. Meanwhile, the algorithm objectives and constraints are also handled separately based on the Boltzmann distribution [14] so as to direct infeasible members towards the sparsely populated regions of PF surface. Lastly, a novel Nash equilibrium-inspired decision making is proposed to extract the best compromise solution from the elitist PF set for decision maker (DM). The effectiveness and validity of the proposed MGSO algorithm have been thoroughly verified on the IEEE 30-bus and IEEE 118-bus test power systems.

II. Problem Formulation

A. Multiobjective Optimization

Real-world optimization problems often need to deal with two or more conflicting and incommensurable objectives [15], and such multiobjective optimization aims to find a family of Pareto-optimal solutions in which none of these solutions can outperform any other for all objectives. In mathematical terms, the Pareto optimization can be formulated as:

$$\begin{aligned} &\text{Min} \quad f_g(X) \qquad g = 1,2,\ldots, M_{\mathrm{obj}} \\ &\text{s.t.} \quad \begin{cases} y_j(X) \le 0, & j = 1,2,\ldots, M_{\mathrm{ineq}} \\ q_k(X) = 0, & k = 1,2,\ldots, M_{\mathrm{eq}} \end{cases} \end{aligned} \tag{1}$$

Based on Pareto optimality principle, there is a dominance relationship between the solution vector being considered and the others, and any solution which cannot be dominated by other solutions of a given set is called the nondominated solution. The solutions which are nondominated within entire feasible search space are known as Pareto-optimal solutions or Pareto set, and the front obtained by mapping these solutions to the fitness vectors in the objective space is the PF. The determination of the complete PF is extremely difficult and even infeasible due to computational complexity and memory constraint caused by the presence of infinite suboptimal PFs [6]. Consequently, the optimization goal of MGSO is to acquire a widely spread and well-distributed PF, in which the Pareto set can be diversified to cover the maximum possible regions of the solution space, within a limited repository. The following are three basic quality measure criteria for evaluating the PFs resulted from various multiobjective optimization algorithms [16].

1) The distance of the resulting nondominated PF set to the true PF should be minimized.
2) The PF should be as uniformly distributed as possible.
3) The extent of the obtained nondominated PF in objective space should be maximized.

B. MOPD Objectives

1) Economic Objective: The economic objective of MOPD is to minimize the total generation cost. The fuel cost of units with non-convexity caused by valve-point effects is modeled as the ripple curve [13], and the total fuel cost $F(P_G)$ in (\$/h) can be expressed with quadratic functions and sine components as:

$$F(P_G)=\sum_{i=1}^{N_G}\left(a_i P_{Gi}^2+b_i P_{Gi}+c_i+|\,d_i\sin[e_i(P_{Gi,\min}-P_{Gi})]\,|\right) \quad (2)$$

2) Emission Objective: The objective of emission dispatch is to minimize the atmospheric pollutants due to fossil-fueled thermal units, such as sulfur dioxides and nitrogen oxides, etc. [4]. The total emission $E(P_G)$ in (ton/h) can be represented as:

$$E(P_G)=\sum_{i=1}^{N_G}10^{-2}\left(\alpha_i P_{Gi}^2+\beta_i P_{Gi}+\gamma_i+\zeta_i\exp(\lambda_i P_{Gi})\right) \quad (3)$$

3) Transmission Loss Objective: The aim of energy-saving generation dispatch is to minimize power transmission losses, and the minimization of power loss in transmission lines can therefore be used as an objective of MOPD. The solution of P_{Loss} involves the calculation of load flow problem, which can be readily solved using Newton-Raphson method [1].

C. MOPD Constraints

1) Power Balance Constraint: Since the total power outputs of generators must equal to the sum of total load demand P_D plus power loss P_{Loss}, after the load flow calculation the active power output of the slack generator should be reassigned to satisfy the equality constraint (4).

$$\sum_{i=1}^{N_G}P_{Gi}-P_D-P_{\text{Loss}}=0 \quad (4)$$

2) Generation Constraints: The active power output of each generator should be within its lower and upper limits. For generator i with NP_i prohibited operating zones (POZs), its feasible operating zones can be described as a disjoint nonconvex set [17]:

$$\begin{cases} P_{Gi,\min}\le P_{Gi}\le P_{Gi(1),\text{lb}} & \\ P_{Gi(j-1),\text{ub}}\le P_{Gi}\le P_{Gi(j),\text{lb}} & j=2,3,\ldots,NP_i \\ P_{Gi(j),\text{ub}}\le P_{Gi}\le P_{Gi,\max} & j=NP_i \end{cases} \quad (5)$$

3) System Spinning Reserve Constraint: For reliable and secure operation, the spinning reserve demand [17] should be considered for contingency conditions. Since the POZs of the generators would severely limit their flexibility to regulate the system load, these generators cannot contribute to the system spinning reserve and the spinning reserve constraint with POZs considered can be expressed as below:

$$\sum_{i=1}^{N_G}SP_{Gi}\ge SP_R,\quad SP_{Gi}=\begin{cases}0 & \forall i\in\Omega \\ P_{Gi,\max}-P_{Gi} & \text{others}\end{cases} \quad (6)$$

4) Transmission Security Constraints: The apparent power flow through the kth transmission line should not excess its loading limit $LF_{k,\max}$ as follows so as to avoid any overloading:

$$\max\left[\left|LF_{ij}\right|,\ \left|LF_{ji}\right|\right]\le LF_{k,\max}\quad k=1,2,\ldots,N_L \quad (7)$$

III. Proposed Multiple Group Search Optimizer

A. Algorithm Framework

In the proposed MGSO, various stochastic global searching and probability selection techniques have been integrated for different types of swarm members. Firstly, the population of MGSO consists of multiple searching groups, and each group is designed based on producer-scrounger model [18] for each objective of the MOPD problem. For each searching group, there are four categories of swarm members for four different searching strategies as follows: 1) Producer: this member is designated to the member conferring the best single-objective fitness for each iteration, and it is the group leader which has a critical impact on the overall searching direction of the group. 2) Scroungers: except for the producer, 80% of the remaining feasible members are selected randomly as scroungers which constitute the main searching force in the algorithm. Thus, the update policy of this swarm should take all the objectives into account through social cooperative mechanism among groups. 3) Rangers: the remaining feasible members in the group are rangers, and they can move in an unpredictable dispersion to discover resource globally. 4) Infeasible members: the update policy of this swarm should be a constraint satisfaction process for handling the complex constraints. Also, every individual in the population has a current position $X\in\mathbb{R}^{N_G}$ and a head angle $\varphi\in\mathbb{R}^{N_G-1}$, and the search direction vector of head angle, $D(\varphi)=(d_1, d_2, \ldots, d_{N_G})\in\mathbb{R}^{N_G}$, can be calculated through a polar to Cartesian coordinate transformation [9].

B. Initialization

The population can be initialized by generating members of each group randomly within the boundary search patch as:

$$\begin{gathered} X_{gj}^{0}=LB+r_1\circ(UB-LB) \\ g=1,2,\ldots,M_{\text{obj}};\ \ j=1,2,\ldots,M_{\text{p}} \end{gathered} \quad (8)$$

where $r_1\in\mathbb{R}^{N_G}$ is a uniform random sequence in the range (0, 1); operator "$\circ$" calculates the entrywise product of two matrices. After the initialization of members in each group, the multi-objective fitness sequence for each member can be calculated. For each initial group, if there is no feasible solution that satisfies all problem constraints, members in this searching group will be reinitialized using (8) until there is at least one feasible member that can be used as the producer in each group. Furthermore, the initialized head angle for each group member is set to ($\pi/4$, … , $\pi/4$) [9].

C. Variable-size External Repository

The external repository is a bounded elite archive used for preserving the nondominated solutions found along the search process. After the initialization of each searching group, the initial repository is determined by the nondominated members obtained in the initial population. For each iterative step of the MGSO algorithm, each nondominated individual obtained in the new generation of the population is checked for dominance with the solutions in the current repository. The following is the dominance comparison strategy adopted for updating the archive: 1) In case the new solution obtained is infeasible or dominated by other members in the population, the solution will not be saved into the repository. 2) If a nondominated member in the population cannot be dominated by any solution in the current repository, the solution will be saved into the repository. 3) Any dominated solution in the current repository by this nondominated member will be removed from the repository.

Though a large-size memory of the elite repository tends to represent the better characteristics of the PF, it would lead to explosion in the computational burden due to the dominance comparisons [8]. Therefore, the number of PF solutions saved in the repository, i.e. the size of the PF, should be limited. In this paper, a variable-size external repository, which can be resized on demand, is adopted. After each iterative step of the algorithm, the repository will be resized to cover the entire nondominated set, including new nondominated members and the survival solutions in the repository. The resized repository could further be shrunk if the hierarchical clustering described in Section III-H was adopted.

D. Space Reduction-based Scanning Strategy for Producer

The producer employs the scanning strategy inspired from white crappies [9] to pursue new Pareto-optimal solutions. The scanning field can be characterized by maximum pursuit angle $\theta_{\max}\in\mathbb{R}^1$ and maximum pursuit distance $L_{\max}\in\mathbb{R}^1$, and the apex is the producer's current position X_p. In the gth group at the kth iteration, the producer will scan the N_G-dimensional hypercube field by randomly sampling three points: one point at the zero degree, two points in the left and right sides symmetrically as:

$$X_{gz} = X_{gp}^k + r_2 L_{g\max} D(\varphi_{gp}^k) \tag{9}$$

$$X_{gr} = X_{gp}^k + r_2 L_{g\max} D(\varphi_{gp}^k + r_3\,\theta_{\max}/2) \tag{10}$$

$$X_{gl} = X_{gp}^k + r_2 L_{g\max} D(\varphi_{gp}^k - r_3\,\theta_{\max}/2) \tag{11}$$

where $r_2\in\mathbb{R}^1$ is a normal distributed random number with mean 0 and standard deviation 1; $r_3\in\mathbb{R}^{N_G-1}$ is a uniform distributed random sequence in the range (0, 1).

After the scanning at each iteration, three objective fitness vectors are sampled by the producer in each group, and the corresponding nondominated solution will then be stored to the repository via dominance comparisons. Meanwhile, if the producer can find a better group single-objective fitness than its existing fitness, then it will move to this point; otherwise, it will stay at its current position and update its head angle as:

$$\varphi_{gp}^{k+1} = \varphi_{gp}^k + r_3\psi_{\max}, \quad \psi_{\max} = \theta_{\max}/2 \tag{12}$$

In case the producer cannot find a better position within $A_{\max}$ iterations, it will turn its head angle back to zero as below,

$$\varphi_{gp}^{k+A_{\max}} = \varphi_{gp}^k, \quad A_{\max} = round(\sqrt{N_G+1}) \tag{13}$$

In order to generate an accurate optimal solution for each objective as well as facilitate the convergence process, a space reduction strategy is added to adaptively adjust the maximum pursuit distance for better local exploitation in scanning space. This strategy will be activated when the performance of the producer in the gth group is not improved after a prespecified iteration period $iter_{L\max}$. In this case, the scanning space will be shrunk based on the pursuit distance at the kth iteration,

$$L_{g\max}^k = L_{g\max}^k(1-\Delta_L), \quad L_{g\max}^0 = \sqrt{\sum_{i=1}^{N_G}(P_{Gi,\max} - P_{Gi,\min})^2} \tag{14}$$

Furthermore, after each iterative cycle, the member found the best fitness value for the corresponding objective is chosen as the producer in this group. At the same time, if a better group single-objective fitness could be found from the member of external groups, the producer will move to the position of the member for the most promising resource.

E. Synergistic Learning for Scroungers

Here, a synergistic learning mechanism inspired from the stochastic learning automata [19] is proposed for extending the single objective GSO to cope with multiobjective problems. During the kth searching bout of the algorithm, the scroungers in the gth group use a special area copying behavior [9], which move across and learn from the promising resources found by its leader and external group members, to pursue PF solutions,

$$X_{gj}^{k+1} = X_{gj}^k + C_p^k r_{gp} \circ (X_{gp}^k - X_{gj}^k) + \sum_{m=1,\, m\neq g}^{M_{\text{obj}}} C_{\text{sl}}^k r_m \circ (X_{m(\text{sl})}^k - X_{gj}^k) \tag{15}$$
$$g = 1,2,\ldots,M_{\text{obj}}; \quad j = 1,2,\ldots,M_{sg}^k$$

where r_{gp}, $r_m\in\mathrm{R}^{N_G}$ are uniform random vectors in the range (0, 1); the second and third terms in (15) are referred as the leader component and synergistic components.

Since the selection of member from external groups for the interactive cooperation is important to the performance of the synergistic learning, a new linear reinforcement scheme based probability distribution selection [14] is proposed such that all nondominated solutions found in the search process are taken as the social achievement of these searching groups, and the reinforcement [20] can be assigned to each member in terms of the good solutions found by this member. Initially, a uniform probability distribution for members in each group is adopted. Then, the $prob_{gj}^k$ can be updated as follows:

$$prob_{gj}^{k+1} = \begin{cases} prob_{gj}^k + \eta(1 - \sum_{t\in T_{ng}^k} prob_{gt}^k) & j\in T_{ng}^k \\ prob_{gj}^k(1-\eta N_{ng}^k) & \text{otherwise} \end{cases} \tag{16}$$

where η ($0<\eta<1/N_{ng}^k$) is a small constant in this paper. It can be found that the member which can find most nondominated solutions, e.g. the scanning strategy-based producer, has high selection probability for the synergistic component in (15).

Simulation studies indicated that the search performance of MGSO can be significantly enhanced through fine-tuning the coefficients in (15). In the initial stage of the search process, since most of group members have not sought out the promising areas, scroungers should give priority to following the producer in its own group. On the other hand, along with the convergence of the groups, scroungers can learn from other groups for the PF solutions, and the leader component should decrease while synergistic components increase in the search process. After comparative studies, the coefficients at the *k*th iteration are set using hyperbolic tangent functions as below:

$$C_p^k = \frac{(C_{p,\max} - C_{p,\min})}{2}\left[-\tanh\left(c_{rp}(k - \frac{Iter_{\max}}{2})\right) + 1\right] + C_{p,\min} \quad (17)$$

$$C_{\mathrm{sl}}^k = \frac{(C_{\mathrm{sl},\max} - C_{\mathrm{sl},\min})}{2}\left[\tanh\left(c_{r\mathrm{sl}}(k - \frac{Iter_{\max}}{2})\right) + 1\right] + C_{\mathrm{sl},\min} \quad (18)$$

F. Chaotic Sequence Dispersion for Rangers

A special random walk dispersion is employed by rangers to improve the population diversity and the global exploration for dispersive PF resources over the entire search space as:

$$\begin{gathered} X_{gj}^{k+1} = X_{gj}^k + r_{\mathrm{sign}} r_{gL}^k L_{g\max}^0 D(\varphi_{gj}^k + r_{g\varphi}^k 2\pi) \\ g = 1,2, \ldots, M_{\mathrm{obj}}; \quad j = 1,2, \ldots, M_{rg}^k \end{gathered} \quad (19)$$

where r_{sign} represents a randomly generated sign equal to −1 or 1. Previous work showed that global searching performance can be enhanced with the use of chaotic sequences instead of random sequences [21]. Therefore, here, $r_{gL}{}^k \in \mathbb{R}^1$, $r_{g\varphi}{}^k \in \mathbb{R}^{N_G\text{-}1}$ are time series generated by chaotic logistic functions rather than random number generators, and the logistic map based chaotic sequence iterator in (19) can be expressed as follows:

$$r_{gL}^{k+1} = \mu r_{gL}^k (1 - r_{gL}^k), \quad r_{g\varphi}^{k+1} = \mu \circ r_{g\varphi}^k \circ (1 - r_{g\varphi}^k) \quad (20)$$

where μ is a control factor determining the time series to be constants, oscillate within limits, or behave chaotically in an unpredictable pattern. Here, sequences (20) are deterministic and will display chaotic behaviors when μ=4.0, and the initial values of the chaotic sequences shall not contain any members of the following {0, 0.25, 0.50, 0.75, 1.0} [21].

Furthermore, GSO is not sensitive to most of its parameters except for the percentage of rangers, and the recommended percentage in [9] is 20%. In order to motivate the individuals to explore the global search space when getting stuck into the local PF, an adaptive strategy for the ranger percentage $\%_{gR}^k$ is adopted. This strategy will be activated once no nondominated solutions can be sought to improve the PF in a given iteration period $iter_{R\max}$. Thereafter, the percentage of rangers at the *k*th iteration can be increased as follows:

$$\%_{gR}^k = \%_{gR}^k + \Delta_R \quad (21)$$

G. Constraint Handling Strategy

Firstly, in order to restrict the groups to search within their generation constraints (5), the following strategy is placed to cope with the bounded search patch: when a member moves outside the search patch, it will be turned back to the search patch by setting the violated dimensional variables of this member to its previous values.

For the sake of effective constraint handling for the highly complex constrained search space, infeasible members will be separated from the population in order to guide them towards feasible space for pursuing new Pareto-optimal solutions. Here, this is achieved through the following policy:

$$\begin{gathered} X_{gj}^{k+1} = X_{gj}^k + r_{\mathrm{pf}} \circ (X_{\mathrm{pf}}^k - X_{gj}^k) \\ g = 1,2, \ldots, M_{\mathrm{obj}}; \quad j = 1,2, \ldots, M_{ing}^k \end{gathered} \quad (22)$$

where $r_{\mathrm{pf}} \in \mathbb{R}^{N_G}$ is a uniform random vector in the range (0, 1). Also, in order to direct the infeasible swarm towards sparsely populated regions for a uniformly-distributed PF, a Boltzmann distribution based on crowding distance [22] is used to form the probability distribution for selecting X_{pf}, as follows:

$$prob_{Bn}^k = e^{d_{cn}^k / wL_{dn}^k} \Big/ \sum_{n=1}^{M_{\mathrm{rep}}^k} e^{d_{cn}^k / wL_{dn}^k} \quad (23)$$

The crowding distance is a measure of PF density with the neighborhood. For the *g*th objective, the boundary solutions, with the smallest and largest fitness values, are assigned to 1 as the crowding distances in (23), and the normalized crowding-distance calculation for all other intermediate solutions is detailed in [22]. Consequently, the infeasible members can be hauled towards the nearby preferable feasible PF regions, and thereby the border of feasible search space can also be readily located to seek the Pareto-optimal solutions.

H. Pruning Pareto Set

After each searching bout, new nondominated solutions will be found and saved in the variable-size repository. When the number of repository elements exceeds the prespecified size of the PF, M_{pf}, an average linkage-based hierarchical clustering [23] is utilized to prune the PF set to a desirable size with its trade-off characteristics preserved. This clustering method is to iteratively classify and join the repository solutions into the required number of clusters. Then, the nearest individual to the centroid of each cluster can be extracted as the representative to form the elitist PF [5].

I. Nash Equilibrium-inspired Decision Making

The best compromise solution should be identified from the resulting PF to simulate the DM's preference. Previous MOPD algorithms generate the best compromise solution using fuzzy logic theory in which a simple fuzzy membership function is defined based on the experiences without considering the PF's trade-off characteristics [5],[6],[8]. In the MGSO, the competing objectives are considered as noncooperative decision making players, and the PF's objective fitness can be modeled as the players' set of actions for Nash equilibria of game theory [24]. Consequently, an alternative multi-criteria decision making is proposed on the basis of the Nash equilibrium to extract an individual with the best joint actions.

Based on the concept of Pareto optimality, this equilibrium selection problem with several noncooperative objectives can be modeled and transformed to find a Nash equilibrium point [24] of multiobjective players, which involves an optimization problem with probability and rationality constraints to yield the joint probability distribution over the PF's action space as:

$$
\begin{aligned}
\text{Max}\quad & Nash(H_1, H_2, \ldots H_g, \ldots H_{M_{\text{obj}}}, \nu_1, \ldots \nu_g, \ldots \nu_{M_{\text{obj}}}) \\
&= \sum_{g=1}^{M_{\text{obj}}} \left(\sum_{j=1}^{M_{\text{pf}}} (-\omega_g f_{gj}) (\prod_{g=1}^{M_{\text{obj}}} h_{gj}) \right) - \sum_{g=1}^{M_{\text{obj}}} \nu_g \\
\text{s.t.}\quad & \sum_{j=1}^{M_{\text{pf}}} h_{gj} = 1, \quad g = 1,2, \ldots, M_{\text{obj}} \qquad (24) \\
& h_{gj} \geq 0, \quad g = 1,2, \ldots, M_{\text{obj}}, \quad j = 1,2, \ldots, M_{\text{pf}} \\
& \sum_{j=1}^{M_{\text{pf}}} (-\omega_g f_{gj}) h_{gj} \leq \nu_g, \quad g = 1,2, \ldots, M_{\text{obj}}
\end{aligned}
$$

where $H_g = [h_{g1}, h_{g2}, \ldots, h_{gj}, \ldots, h_{gM_{\text{pf}}}]$ represents a probability distribution over the PF's fitness. Here, ω_g is set to 1 for the unbiased preference of DM. The optimization problem (24) is a standard constrained nonlinear programming (NLP) solved in this paper by sequential quadratic programming (SQP) [25], a highly effective and matured method for the NLP. As a result, a list of equilibrium values will be provided for the DM, and the best compromise solution can then be derived from the best joint equilibrium which represents the highest payoff outcome obtained from this joint action, as follows:

$$
\max[\prod_{g=1}^{M_{\text{obj}}} h_{g1}, \prod_{g=1}^{M_{\text{obj}}} h_{g2}, \ldots \prod_{g=1}^{M_{\text{obj}}} h_{gj}, \ldots \prod_{g=1}^{M_{\text{obj}}} h_{gM_{\text{pf}}}] \qquad (25)
$$

J. Procedures for MGSO

To sum up, the flowchart of the equilibrium-inspired MGSO with synergistic learning is depicted in Fig. 1.

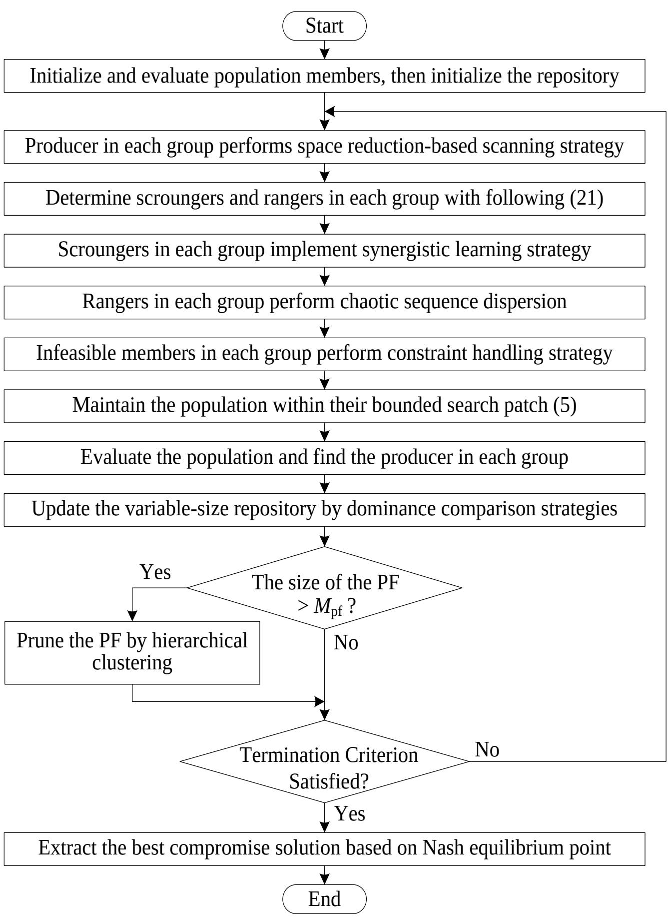

Fig. 1. Flowchart of the proposed algorithm for MOPD problems

IV. Simulation Studies

A. EED Studies on IEEE 30-bus System

For comparison with previously published algorithms and results, the proposed MGSO is tested on the IEEE 30-bus 6-generator system to solve the dual-objective MOPD problems. While the detailed system data are given in [2],[6], the fuel cost and emission coefficients in (2) and (3) are available in [8]. Since the overall performance of the algorithm is not sensitive to most of its parameters [9], the setting guidelines in [9] were adopted such that the maximum pursuit angle $\theta_{\max}$ is set to $\pi/(A_{\max})^2$ and termination criterion $Iter_{\max}$ is fixed to 300. The iteration periods $iter_{L\max}$ and $iter_{R\max}$ in (14) and (21) were set to 35 and 16, respectively. Since the producer in (9)-(11) requires three function evaluations, the population size of each group M_{p} is set to 28 such that the total number of function evaluations in a generation is 60. For all the optimization runs, the size of the PF, M_{pf}, is fixed to 50 [5]. Meanwhile, the settings for other parameters in this paper are heuristically well-tuned as shown in Table I through a large amount of comparative studies and simulations [26]. In the case studies, the problem constraints include only (4), (5) and (7), and the following three cases are considered.

TABLE I
PARAMETER SETTINGS OF MGSO FOR EED OF IEEE 30-BUS SYSTEM

Δ_L	Δ_R	$C_{p,\max}$	$C_{p,\min}$	$C_{\text{sl},\max}$	$C_{\text{sl},\min}$	c_{rp}	$c_{r\text{sl}}$	η	w
0.03	0.01	1.20	0.30	1.05	0	0.015	0.015	0.03	0.15

1) Case 1

Ten independent runs of MGSO were carried out. The PFs of the best runs from all algorithms were selected to compare the algorithm performances [6]. Tables II and III detailed the best solutions for fuel cost and emission obtained by the extreme vertices in the resulting PF of MGSO and other algorithms published in [5],[6],[8]. In addition, the PF solutions obtained from MGSO were plotted in Fig. 2 and compared with those from the well-know NSGA-II [22]. The results indicate that MGSO performs well with two better outer solutions and compares well with other EED algorithms, and its frontier is well-distributed and dispersedly covered the entire PF of NSGA-II.

TABLE II
COMPARISON OF BEST FUEL COST FOR CASE 1 OF IEEE 30-BUS SYSTEM

Case 1	NSGA [6]	NPGA [6]	SPEA [5]	FCPSO [8]	SPEA2 [27]	NSGA-II [16]	MGSO
P_{G1}	0.1358	0.1127	0.15975	0.1596	0.1543	0.1646	0.1775
P_{G2}	0.3151	0.3747	0.35339	0.3535	0.4066	0.3757	0.3588
P_{G3}	0.8418	0.8057	0.79600	0.7974	0.7270	0.7108	0.7448
P_{G4}	1.0431	0.9031	0.97176	0.9719	0.5916	0.5861	0.5913
P_{G5}	0.0631	0.1347	0.08684	0.08624	0.5925	0.6229	0.5996
P_{G6}	0.4664	0.5331	0.49709	0.49609	0.3873	0.3995	0.3870
$F(P_G)$	620.87	620.46	620.165	620.18	620.0149	619.8271	**619.6269**
$E(P_G)$	0.2368	0.2243	0.22826	0.2283	0.20330	0.20289	0.20346

TABLE III
COMPARISON OF BEST EMISSION FOR CASE 1 OF IEEE 30-BUS SYSTEM

Case 1	NSGA [6]	NPGA [6]	SPEA [5]	FCPSO [8]	SPEA2 [27]	NSGA-II [16]	MGSO
P_{G1}	0.4403	0.4753	0.47975	0.47969	0.4033	0.4188	0.4102
P_{G2}	0.4940	0.5162	0.52868	0.5287	0.4602	0.4457	0.4630
P_{G3}	0.7509	0.6513	0.67109	0.67116	0.5436	0.5424	0.5436
P_{G4}	0.5060	0.4363	0.53174	0.5318	0.3990	0.4069	0.3896
P_{G5}	0.1375	0.1896	0.12571	0.1257	0.5441	0.5465	0.5438
P_{G6}	0.5364	0.5988	0.53010	0.5299	0.5146	0.5045	0.5147
$F(P_G)$	649.24	657.59	651.633	651.62	643.9420	643.4741	645.1976
$E(P_G)$	0.2048	0.2017	0.20470	0.2047	0.19419	0.19423	**0.19418**

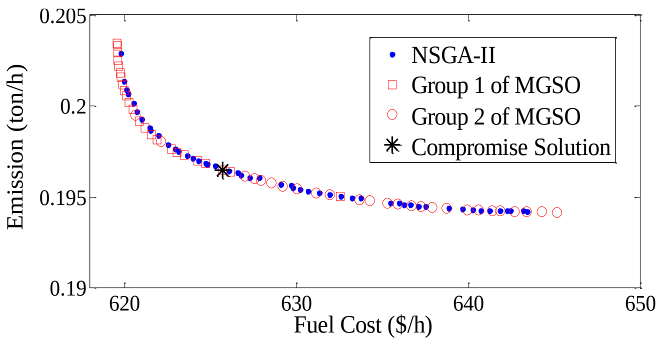

Fig. 2. Comparison of the PFs obtained for Case 1

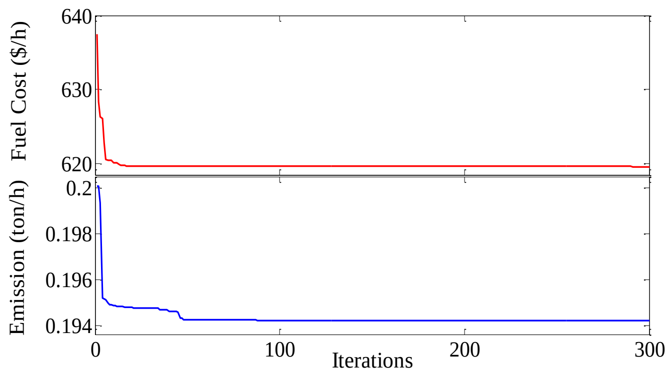

Fig. 3. Producer’s convergence of each group objective in Case 1

For the best run of MGSO in Fig. 2, the convergence process of each objective function is shown in Fig. 3. Furthermore, as shown from the distribution of solutions which were found by multi-groups and marked differently in Fig. 2, synergistic learning for diverse regions of the PF indeed can maintain the diversity characteristics and full extent of the PF over the tradeoff surface.

In order to estimate the spread of the PFs, the span metric in [16] is used to measure the normalized distance in the objective space between the PF’s two boundary solutions. The averages of this metric over ten different runs for different algorithms presented in Table IV confirm that the proposed MGSO outperforms the other algorithms. Besides, compared with NSGA, NPGA, SPEA and NSGA-II in previous findings [8], MGSO and FCPSO require much fewer function evaluations to form the optimal front. Coupled with its high exploratory capability, the proposed MGSO can give the best performance but with less number of fitness evaluations.

TABLE IV
RESULTING STATISTICS OF SPAN MEASURE OF DIFFERENT ALGORITHMS

NSGA [6]	NPGA [6]	SPEA [5]	FCPSO [8]	NSGA-II	SPEA2	MGSO
0.85539	0.81312	0.85363	0.85358	0.87961	0.88526	**0.90133**

2) Case 2

In this case, a dual-objective dispatch is investigated to optimize the fuel cost and power loss. For further comparison and discussion, two advanced Pareto optimization algorithms, NSGA-II and improved SPEA (SPEA2) [27], were considered and implemented. Table V listed the best results obtained for this case, and the corresponding PF solutions were plotted in Fig. 4. It can be found from Fig. 4 that, with the help of space reduction-based scanning, MGSO can find solutions with better fitness on each objective compared to other algorithms. This confirms the Pareto optimality of the proposed algorithm and its potential to find solutions covering the entire true PF.

TABLE V
COMPARISON OF BEST SOLUTIONS FOR CASE 2 OF IEEE 30-BUS SYSTEM

Case 2	Best Fuel Cost			Best Power Loss		
	SPEA2	NSGA-II	MGSO	SPEA2	NSGA-II	MGSO
P_{G1}	0.1638	0.1828	0.1741	0.1632	0.1511	0.1596
P_{G2}	0.3671	0.3559	0.3581	0.3160	0.3305	0.3061
P_{G3}	0.7639	0.7584	0.7492	0.9826	0.9793	0.9848
P_{G4}	0.5931	0.5925	0.5914	0.3936	0.4430	0.3927
P_{G5}	0.6191	0.6069	0.6001	0.6568	0.6157	0.6638
P_{G6}	0.3516	0.3622	0.3861	0.3444	0.3370	0.3497
$F(P_G)$	619.8322	619.7626	**619.6288**	637.8233	633.7917	637.9634
P_{Loss}	0.024695	0.024787	0.024977	0.022561	0.022608	**0.022558**

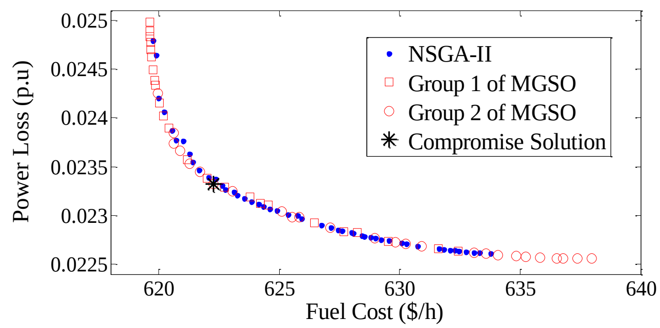

Fig. 4. Comparison of the PFs obtained for Case 2

3) Case 3

This case is to study a dual-objective MOPD with the objectives of emission and power loss. From the results presented in Table VI and Fig. 5, MGSO performs well with outstanding diversity and spanning of the PF.

TABLE VI
COMPARISON OF BEST SOLUTIONS FOR CASE 3 OF IEEE 30-BUS SYSTEM

Case 3	Best Emission			Best Power Loss		
	SPEA2	NSGA-II	MGSO	SPEA2	NSGA-II	MGSO
P_{G1}	0.4100	0.4122	0.4154	0.1517	0.1529	0.1590
P_{G2}	0.4754	0.4471	0.4632	0.3138	0.2978	0.3070
P_{G3}	0.5507	0.5535	0.5438	0.9864	0.9884	0.9850
P_{G4}	0.3838	0.4032	0.3896	0.3993	0.4280	0.3873
P_{G5}	0.5341	0.5592	0.5439	0.6563	0.5945	0.6674
P_{G6}	0.5109	0.4892	0.5091	0.3491	0.3951	0.3509
$E(P_G)$	0.19420	0.19426	**0.19418**	0.21365	0.21289	0.21355
P_{Loss}	0.030804	0.030344	0.030927	0.022560	0.022683	**0.022557**

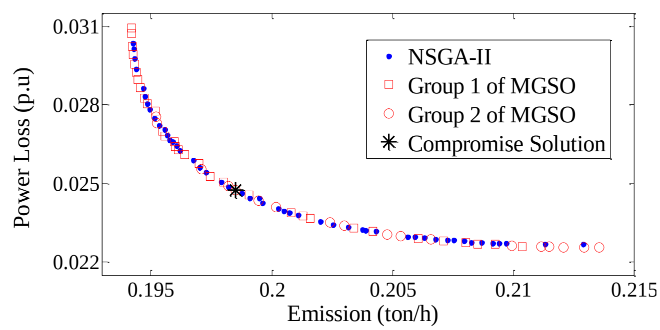

Fig. 5. Comparison of the PFs obtained for Case 3

TABLE VII
COMPROMISE SOLUTIONS OF MGSO FOR DIFFERENT DECISION MAKING

	Case 1		Case 2		Case 3	
	Fuzzy	Equilibrium	Fuzzy	Equilibrium	Fuzzy	Equilibrium
P_{G1}	0.3150	0.3150	0.1435	0.1435	0.3057	0.3313
P_{G2}	0.4159	0.4159	0.3032	0.3032	0.3743	0.3662
P_{G3}	0.5538	0.5538	0.9300	0.9300	0.7696	0.7541
P_{G4}	0.5639	0.5639	0.6052	0.6052	0.4016	0.3748
P_{G5}	0.5612	0.5612	0.5500	0.5500	0.5746	0.5990
P_{G6}	0.4538	0.4538	0.3254	0.3254	0.4331	0.4337
Fitness	625.7582 0.196505	625.7582 0.196505	622.2655 0.023321	622.2655 0.023321	0.19848 0.024743	0.19802 0.025057

Moreover, the best compromise solutions of MGSO solved by the proposed Nash equilibrium point were marked with ∗ in Fig. 2,4 and 5, and the compromise solutions obtained with fuzzy decision making [8] and the proposed method were listed in Table VII. It is interesting to note that the results for Case 3 are different while the results in Case 1 and 2 are the same. This confirms that the equilibria-based decision making method is able to provide a reasonable bargaining solution for power system dispatchers.

B. *Computational Studies on IEEE 118-bus System*

For in-depth investigation of the proposed method on larger power systems, a modified 54-generator 118-bus power system [28] is used for a tri-objective MOPD optimization including fuel cost, emission and energy saving objectives. For unit 1-24, a POZ is set in the middle of the generation capacity patch for each unit, and the POZ interval is 10% of the unit's original feasible operating zone. In this study, all system constraints listed in Section II-C are considered, and SP_R is set to 38 pu. Besides, the settings of MGSO for the IEEE 118-bus system can be obtained following the guidelines outlined in Section IV-A and are tabulated in Table VIII. As the problem complexity for this case study is much higher with larger system size and more variables and constraints, $Iter_{\max}$, M_{p} and M_{pf} are set to 2000, 98 and 150, respectively. On the other hand, numerous trials have been carried out to determine the optimum settings for NSGA-II and SPEA2. In addition, their population size and maximum number of generations are set to 300 and 2000, respectively, such that the number of function evaluations for the EAs is the same as MGSO for fair comparisons. Moreover, the probabilities of crossover and mutation are set to 0.9 and 0.01, respectively [6].

TABLE VIII
PARAMETER SETTINGS OF MGSO FOR MOPD OF IEEE 118-BUS SYSTEM

Δ_L	Δ_R	$C_{p,\max}$	$C_{p,\min}$	$C_{\mathrm{sl},\max}$	$C_{\mathrm{sl},\min}$	c_{rp}	$c_{r\mathrm{sl}}$	η	w
0.03	0.01	1.20	0.30	1.05	0	0.025	0.025	0.01	0.10

The PFs resulted from the above three MOPD algorithms should be assessed systematically with performance measures derived from the three basic quality criteria stated in Section II-A and the true PF of the MOPD problem. However, since the true PF is difficult to determine and guarantee for the problems with high-dimensional and highly complex search space, a pseudo PF, named as reference PF [29], is used instead as the true PF to compare the PFs generated by various algorithms. Here, the reference PF is formed by 50 independent runs of all MOPD algorithms, NSGA-II, SPEA2 and MGSO, i.e. 150 sets of PFs. All the 150 sets of PF solutions were then combined and ranked by the dominance comparisons. As a result, the new population, which consists of 1725 nondominated solutions, is the reference PF, in which approximately 69.86% and 30.14% of the solutions are contributed by MGSO and SPEA2, respectively. It can be observed that all the solutions found with NSGA-II are covered by those with MGSO and SPEA2, and MGSO has contributed the majority of reference PF solutions. This also reveals that the solutions of MGSO are closer to the true Pareto set. Meanwhile, the PF of the overall best run in the 50 runs of each algorithm is selected for further analysis. Table IX lists the best solutions for fuel cost, emission, and power loss obtained from the best run of each algorithm. It can be found that, in terms of the best results for each objective, MGSO can find more optimal outer solutions to maintain a widespread Pareto set over the entire true PF region.

TABLE IX
COMPARISON OF BEST SOLUTIONS FOR COST, EMISSION AND SYSTEM LOSS

	NSGA-II			SPEA2			MGSO		
	Best Cost	Best Emission	Best Loss	Best Cost	Best Emission	Best Loss	Best Cost	Best Emission	Best Loss
P_{G1}	0.72173	0.76199	0.70488	0.75703	0.76673	0.65630	0.79389	0.77471	0.69754
P_{G2}	0.73771	0.65878	0.43894	0.76224	0.81038	0.40696	0.74791	0.79572	0.59249
P_{G3}	0.44926	0.62338	0.42939	0.75586	0.77147	0.42817	0.75325	0.76807	0.85366
P_{G4}	0.77448	0.84211	0.79662	0.73924	0.77614	0.80210	0.75467	0.77642	0.69375
P_{G5}	0.98092	0.52309	0.60977	1.00706	0.39282	0.81436	0.01160	0.42353	0.01819
P_{G6}	1.09317	0.73307	1.48669	0.49696	0.55485	1.40722	0.50197	0.54468	1.75926
P_{G7}	0.65785	0.70475	0.81324	0.97751	0.77939	0.70993	0.98957	0.76611	0.99996
P_{G8}	0.80228	0.66317	0.44580	0.58385	0.78275	0.58037	0.96314	0.73736	0.66563
P_{G9}	0.77893	0.76393	0.82309	0.95651	0.79377	0.82214	0.97826	0.78596	0.78492
P_{G10}	0.75292	0.72547	0.23790	0.94955	0.75319	0.39684	0.98398	0.77855	0.59552
P_{G11}	1.15408	1.04424	0.33425	0.75750	1.28962	0.46604	0.74600	1.37738	0.00136
P_{G12}	0.88530	0.65124	0.23569	0.89259	0.61387	0.30727	0.91078	0.68242	0.00100
P_{G13}	0.85460	0.74357	0.71436	0.98936	0.87079	0.68996	0.96614	0.81381	0.62676
P_{G14}	0.88577	0.71981	0.79245	0.86347	0.59690	0.82249	0.85305	0.62000	0.73927
P_{G15}	0.93039	0.84767	0.77682	0.89189	0.80287	0.81963	0.89597	0.83377	0.82216
P_{G16}	0.90418	0.90416	0.90018	0.86265	0.87498	0.89455	0.88205	0.91320	0.97097
P_{G17}	0.92126	0.87521	0.93866	0.89383	0.89483	0.90842	0.90140	0.91888	0.91020
P_{G18}	0.95221	0.94356	0.96827	0.90763	0.98433	0.95022	0.88826	0.97425	0.99692
P_{G19}	0.88275	0.93224	0.95870	0.92972	0.99551	0.93865	0.90450	0.98081	0.99631
P_{G20}	0.85610	0.84480	0.90154	0.85233	0.65568	0.89298	1.18878	0.65467	0.85525
P_{G21}	0.74622	0.62417	1.98481	0.74734	0.63205	1.90454	1.12491	0.56249	1.86667
P_{G22}	0.64131	0.81922	1.08317	1.43396	0.50959	1.04511	1.47025	0.50199	1.40251
P_{G23}	0.74873	0.91233	0.98645	0.76689	0.87363	0.94238	0.77110	0.88130	0.83522
P_{G24}	0.80231	0.94115	0.99171	0.78098	0.91534	0.81012	0.80278	0.86603	0.97481
P_{G25}	1.15279	1.51391	2.15846	1.47571	1.45494	1.83209	1.49572	1.44090	2.29563
P_{G26}	0.93228	1.12075	1.37003	1.03391	0.93109	1.36289	1.01507	0.83258	0.90776
P_{G27}	0.71643	0.78851	0.71322	0.85855	0.85812	0.67920	0.75854	0.85283	0.99533
P_{G28}	1.27803	0.92673	0.97124	1.50905	0.68337	1.16498	2.17752	0.60144	1.32547
P_{G29}	1.40964	1.46552	0.97493	0.74968	1.40593	0.83725	0.05950	1.43433	0.76845
P_{G30}	0.02693	0.90302	0.36969	0.00653	0.94975	0.76350	0.04171	1.00482	0.27964
P_{G31}	0.23952	0.80366	0.51854	0.04576	0.84294	0.37394	0.06338	0.88686	0.46747
P_{G32}	0.09128	0.59800	0.29906	0.03870	0.90200	0.22116	0.00506	0.87758	0.02999
P_{G33}	0.11316	0.75677	0.26399	0.02122	0.88151	0.24564	0.02321	0.88413	0.03644
P_{G34}	0.67900	0.73683	0.74611	0.99796	0.90314	0.71788	0.99544	0.90619	0.98794
P_{G35}	0.63257	0.79890	0.87603	0.97556	0.96946	0.76831	0.99925	0.90754	0.98240
P_{G36}	0.88149	0.85558	0.93063	0.98961	0.99535	0.91323	0.99312	0.99583	0.99978
P_{G37}	1.21877	0.87279	2.80108	2.04168	0.67283	2.56623	2.04236	0.66046	2.79243
P_{G38}	0.76777	0.79579	0.70509	0.82205	0.81776	0.74408	0.75501	0.82751	0.95005
P_{G39}	0.41465	0.44914	0.19342	0.04985	0.42581	0.29801	0.03853	0.42570	0.09411
P_{G40}	0.97630	1.02684	1.01993	0.88626	1.01118	0.93242	0.90297	0.99215	0.96017
P_{G41}	0.72530	0.70676	0.83877	0.79854	0.72233	0.75973	0.76411	0.70055	0.99682
P_{G42}	0.73647	0.58912	0.42754	0.75838	0.66047	0.64023	0.78433	0.66182	0.21507
P_{G43}	0.83822	0.83467	0.96992	0.75697	0.70865	0.83990	0.75272	0.67808	0.96677
P_{G44}	0.75673	0.71566	0.73817	0.75342	0.67425	0.71376	0.74150	0.68847	0.74161
P_{G45}	0.94380	1.09169	0.97317	0.90037	1.33429	0.92563	0.88861	1.35972	1.37848
P_{G46}	0.84489	0.51758	0.28708	0.03608	0.37129	0.40727	0.00949	0.37292	0.08100
P_{G47}	0.89287	0.80195	0.68979	0.89057	0.82506	0.69729	0.90119	0.85349	0.49493
P_{G48}	0.87476	0.59212	0.30922	0.85788	0.79681	0.28432	0.01488	0.77265	0.44628
P_{G49}	0.83149	0.66800	0.64611	0.89339	0.63813	0.70726	0.89016	0.72345	0.61843
P_{G50}	0.89014	0.67858	0.58328	0.00328	0.68032	0.62070	0.86860	0.72147	0.55969
P_{G51}	0.64005	0.39074	0.20380	0.32419	0.32488	0.27389	0.29227	0.33549	0.00333
P_{G52}	0.84295	0.50473	0.45103	0.89109	0.52492	0.56915	0.87988	0.57440	0.46647
P_{G53}	0.87936	0.62738	0.29869	0.89873	0.57696	0.44271	0.89031	0.59374	0.07533
P_{G54}	0.89432	0.83860	0.88388	0.92523	0.63988	0.87228	0.89933	0.61370	0.97590
$F(P_G)$	66781.4	77327.5	72453.5	63863.3	80888.8	70493.7	**63166.7**	81109.9	75201.9
$E(P_G)$	2.94914	2.68451	3.55349	3.24115	2.61526	3.38979	3.42424	**2.61030**	4.04161
P_{Loss}	0.61645	0.35346	0.14520	0.32615	0.45485	0.17165	0.30795	0.49290	**0.12346**

In this paper, three typical performance metrics were used to compare and analyze the solution quality of PFs obtained from various MOPD algorithms. The first is the convergence metric adopted to measure the degree of closeness between the resulting PF and the reference PF [22]. For each PF solution obtained, the Euclidean distance from it to the nearest solution of reference PF in the objective space is first calculated, and this metric can then be obtained using the average of these distances. Secondly, the distribution uniformity of PF solutions can be assessed by spacing metric [15] which is calculated as the relative crowding distance between consecutive solutions in the resulting PF set. The desired value for this metric is 0, which means the elements of PF solutions can be equidistantly spaced. Thirdly, as explained in Section IV-A, the extent of the PF can be assessed with the normalized Euclidean distance of the extreme solutions for the three MOPD objectives.

Furthermore, the performance metrics of the overall best run of each algorithm are listed in Table X, and the statistical results on the convergence, spacing and span measures over the 50 optimization runs are tabulated in Table XI-XIII, respectively. The resulting statistics demonstrate that, with the same number of fitness function evaluations, MGSO can markedly outperform the two earlier methods, and provides satisfactory performance on various measures, especially on the convergence and span metrics. The simulations also confirmed that the different types of functional members do facilitate the developed algorithm to effectively propagate the search towards the well-scattered and diverse PF.

TABLE X
PERFORMANCE MEASURES OF THE BEST RUN OF EACH ALGORITHM

Algorithms	Convergence	Spacing metric	Span metric
NSGA-II	3.057125	**0.013010**	0.721867
SPEA2	1.797026	0.019365	0.969229
MGSO	**0.980039**	0.023083	**0.991228**

TABLE XI
RESULTING STATISTICS OF CONVERGENCE MEASURES IN 50 RUNS

Algorithms	Best	Worst	Average	Variance	Std. Dev.
NSGA-II	2.156753	3.849242	2.795033	0.127215	0.356672
SPEA2	1.126984	3.057916	1.925615	0.248221	0.498218
MGSO	**0.809761**	2.078176	**1.192056**	**0.065518**	**0.255965**
MGSO1	0.847567	**1.936765**	1.228693	0.077964	0.279221
MGSO2	0.906680	2.564372	1.381657	0.156229	0.395258

TABLE XII
RESULTING STATISTICS OF SPACING MEASURES IN 50 RUNS

Algorithms	Best	Worst	Average	Variance	Std. Dev.
NSGA-II	**0.008862**	**0.059555**	**0.013219**	**4.0870E-5**	**0.006393**
SPEA2	0.017403	0.089156	0.030587	4.1440E-4	0.020357
MGSO	0.017849	0.077575	0.028133	2.2716E-4	0.015072
MGSO1	0.018132	0.077043	0.029576	2.8264E-4	0.016812
MGSO2	0.018956	0.089222	0.028883	2.7403E-4	0.016554

TABLE XIII
RESULTING STATISTICS OF SPAN MEASURES IN 50 RUNS

Algorithms	Best	Worst	Average	Variance	Std. Dev.
NSGA-II	0.721867	0.187814	0.470604	0.018573	0.136284
SPEA2	0.969229	0.496084	0.782732	0.028724	0.169483
MGSO	**0.991228**	**0.660227**	**0.832738**	**0.012285**	**0.110838**
MGSO1	0.955163	0.609305	0.809921	0.015886	0.126040
MGSO2	0.980816	0.642971	0.830181	0.014417	0.120071

In particular, from the statistical comparative experiments, it should be pointed out that MGSO can effectively find the nondominated solutions in the separated feasible islands so as to guarantee the diversity of the PF. This validates its superior efficiency of solution searching for complex nonlinear constrained problems with high-dimensional search space. Thus, the spacing metrics of MGSO are greater than those of NSGA-II and SPEA2. Besides, it can be found that the PFs from NSGA-II are more uniformly-spaced, but perform worst on the other two metrics. Also, the variance and standard deviation values of these measures indicate the stable performance of MGSO for the resulting Pareto set.

For the investigation of contribution of the space reduction strategy for producer and the adaptive ranger percentage with chaotic sequence, statistical results were collected over 50 runs for MGSO with fixed $L_{g\max}$ (referred as MGSO1) and MGSO with the fixed $\%_{gR}$ and random number sequence [9] (referred as MGSO2), and tabulated in Table XI-XIII. It is worth noting that the span metrics of the PFs can be statistically improved with the help of the space reduction-based scanning strategy. Furthermore, referring to the MGSO2, it also verified that the proposed chaotic sequence dispersion strategy can statistically enhance the overall performance of MGSO, especially on the convergence metric.

The ultimate goal of any Pareto-based algorithm is to identify a unique solution with the best compromise among multiple objectives. In the MGSO, the solution having the maximum joint equilibrium value will be chosen as the PF's best compromise solution. In this case study, the fitness of the compromise solution obtained with the Nash equilibrium-based decision making is (70674.1, 2.96582, 0.17269), as compared to (67819.5, 3.05191, 0.19393) obtained by the fuzzy method [8]. It can be seen that the two compromise solutions are quite different in this study because the proposed method takes into account the objectives' trade-off of the PF solutions and its solution model is based on the compromise between the gain of one objective and the degradation in other objectives [30] with solid technical foundation based on the non-cooperative game theory [24].

From the investigations and analysis above, it can be found that, though the performance improvement of the proposed MGSO is moderate compared with other algorithms on the small IEEE 30-bus system, the proposed MGSO has exhibited its superior capability to 1) provide largely improved solution for the larger 118-bus system with limited number of function evaluations, 2) significantly enhance the searching ability, 3) ensure the quality of PF solutions, and 4) efficiently manage the highly complex power system constraints in solving high-dimensional MOPD problems with more objectives [31].

A comparative study of the average run time per generation over 50 optimization runs for each of the MOPD algorithms is given in Table XIV. All the algorithms were implemented in Matlab 7.6 and ran on a personal computer with 3.2 GHz Intel Core 2 Quad CPU and 4GB RAM. It is quite evident that the computation time of MGSO is less than that of the other two techniques.

TABLE XIV
RUN TIME PER GENERATION OF DIFFERENT ALGORITHMS

	NSGA-II	SPEA2	MGSO
Run time (s)	7.877	8.305	**7.650**

As the main searching force members, scroungers perform the proposed synergistic learning strategy and their behaviors are mainly determined by reinforcement factor η which is crucial to the credit assignment and information interaction in the cooperative searching process. In order to investigate the tuning rules for design parameters on the search performance of MGSO, the sensitivity of η was studied over a range from 0.001 to 0.01 in step of 0.001. Table XV provides the statistical results of the performance metrics for MGSO with various reinforcement factors over 50 optimization runs. It can be seen that a large value of η can effectively enhance the interactive cooperation between searching groups and hence can improve the average performance of the proposed algorithm. Therefore, the overall best PF results can be achieved when η was set to 0.01. Similarly, the other algorithm parameters can also be heuristically fine-tuned, as listed in Table I and VIII, using this cut-and-try approach [26].

TABLE XV
REINFORCEMENT FACTOR EFFECTS ON PERFORMANCE OF MGSO

η	Convergence	Spacing metric	Span metric

	Average	Std. Dev.	Average	Std. Dev.	Average	Std. Dev.
0.001	4.390579	**0.219971**	0.042959	0.018695	0.516577	0.257505
0.002	3.937078	0.576131	0.043629	0.019949	0.657509	0.235598
0.003	3.126982	0.428249	0.040692	0.021899	0.638330	0.192542
0.004	3.359306	0.271617	0.033698	0.023583	0.549821	0.221789
0.005	2.791334	0.386273	0.026564	0.019275	0.618557	0.171531
0.006	2.561396	0.362625	**0.022615**	0.017922	0.715560	0.142848
0.007	2.323595	0.315735	0.027596	0.018787	0.787631	**0.106881**
0.008	1.622543	0.325889	0.031324	0.020455	0.810356	0.152771
0.009	1.465805	0.309156	0.034161	0.016727	0.806117	0.124352
0.01	**1.192056**	0.255965	0.028133	**0.015072**	**0.832738**	0.110838

However, it shall also be noted that, according to the No Free Lunch theorem, "for any search algorithm, any elevated performance over one class of problems would be exactly paid for in performance over another class" [32]. This implies that though the proposed MGSO algorithm is more superior in this class of power system dispatch problems, it may not be necessary as well performed in other class of problems.

V. Conclusions

In this paper, a novel Pareto optimization algorithm, MGSO, is developed to solve highly nonlinear constrained and large-scale MOPD problems. The following are main advantages of the proposed approach: 1) Four categories of group members in association with the searching strategies are designed in the algorithm for effective formation and exploration of the PF front and improving the extension, convergence, diversity and uniformity of the Pareto solutions. 2) A synergistic learning mechanism based on the stochastic learning automata is first proposed for the credit assignment and information interaction among multiple groups to achieve the cooperative search for Pareto set. 3) A new decision making criterion based on Nash equilibrium point is presented to identify a more reasonable compromising solution from the resulting PF with multiple contradictory objectives.

The proposed MGSO has been successfully applied to the dual-objective EED problem on the IEEE 30-bus system and a tri-objective MOPD problem on the IEEE 118-bus system. In-depth numerical simulation studies have confirmed the superior efficiency of MGSO for solution searching and the effectiveness of the proposed new searching strategies. Compared with the previously published Pareto algorithms on various performance measures, the proposed MGSO is very competitive in small EED problems and clearly superior in the high-dimensional MOPD problems with complex constraints and objectives.

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