Optimal Energy Flow for Integrated Energy Systems Considering Gas Transients

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***Abstract* — This letter presents an optimal energy flow (OEF) for integrated energy systems considering the transient process of natural gas flows, whose main dynamic features are efficiently approximated in our setup. To achieve this, new discretization criteria are proposed to divide the gas flow equations into reasonable temporal and spatial segments to accurately model the gas flow dynamics in a set of difference equations. Simulation results validate the effectiveness of the OEF based on the proposed discretization criteria and its advantages over existing methods.**

***Index Terms* — Integrated energy system, optimal energy flow, transient gas flow.**

I. Introduction

INTEGRATED energy systems (IESs), especially integrated electric power and natural gas systems, are a promising way to achieve the target of energy transition around the world. Similar to the optimal power flow in power system studies, the optimal energy flow (OEF) is of great importance to an IES. When optimizing the operation of a power system from an economic perspective, it is reasonable to neglect the fast dynamics of electrical transients. However, natural gas flow may have much slower dynamics and become more difficult to solve because it is generally described by a set of partial differential equations (PDEs). As a result, it is reasonable to approximate and simplify the PDEs to ease the computational burden while at the same time keeping track of the gas flow transients.

This letter considers the one-dimensional transient gas flow through horizontal pipelines under isothermal conditions [1]. The following PDEs are used to describe the gas pipe dynamics:

$$\partial_t p + \frac{4c^2}{\pi D^2}\partial_x m = 0 \tag{1}$$

$$\partial_x p + \frac{4}{\pi D^2}\partial_t m + \partial_x(\rho u^2) = -\frac{8fc^2}{\pi^2 D^5}\frac{m|m|}{p} \tag{2}$$

where ∂_t is the partial derivative against the time variable t and ∂_x against the spatial variable x, and $p = p(x,t)$, $m = m(x,t)$, $\rho = \rho(x,t)$, and $u = u(x,t)$ respectively denote the pressure (Pa), mass flow rate (kg/s), gas density (kg/m^3), and gas flow velocity (m/s). Finally, c is a constant representing the speed of sound (m/s), D is the pipe diameter (m), and f is a constant representing the friction factor. Note that the above equations also assume a constant compressibility factor (i.e., $p = c^2\rho$) [1].

Several approaches have aimed to simplify and linearize (1) and (2). For instance, the steady-state gas flow that ignores $\partial_t p$, $\partial_t m$, and $\partial_x(\rho u^2)$ is employed in [2]. Both $\partial_t m$ and $\partial_x(\rho u^2)$ are neglected in [3] and [4], but only $\partial_x(\rho u^2)$ is neglected in [5] and [6]. In these studies, the PDEs are linearized and discretized for ease of computation. However, the temporal and spatial discretization in these studies is not clearly discussed and may only be feasible for specific cases [6]. Especially for OEF problems where the emphasis is usually placed on energy production and consumption with large time intervals (e.g., 1 h for day-ahead and 5 or 15 min for real-time), existing works do not address concerns regarding how to discretize the transient gas flows in both time and space for a given time interval.

In this context, the main work of this letter is to develop temporal and spatial discretization criteria to integrate gas flow transients into OEF problems. The transient gas flow is approximated based on a finite difference method and Talyor series. The discretization criteria are derived from the transfer function of gas pipelines to divide gas flow equations into segments in spatiotemporal coordinates to enhance the accuracy. Based on the approximated transient gas flow, the OEF model is developed and verified by numerical simulations.

II. Approximation of Transient Gas Flow

A. Finite Difference Approximation of PDEs

By omitting the convective term $\partial_x(\rho u^2)$ but keeping the intertia term $\partial_t m$ in (2), the partial differential equations (1) and (2) can be processed by numerical integration methods. The effectiveness of neglecting $\partial_x(\rho u^2)$ has been validated by [1].

The finite difference method is adopted and the time derivative is approximated by the implicit trapezoidal rule. Other numerical methods such as pseudospectral approximation and lumped element approximation to discretize PDEs can be found in [7] and will not be further discussed. A diagram of the time and spatial grid can be found in [1]. The numerical formulations for a natural gas pipe segment between spatial grid points i and i+1 and from time grid points t to t+1 can be presented as:

$$\frac{p_{i+1}^{t+1}-p_{i+1}^{t}}{2\Delta t} + \frac{p_{i}^{t+1}-p_{i}^{t}}{2\Delta t} + \frac{4c^2(m_{i+1}^{t+1}-m_{i}^{t+1})}{\pi D^2 \Delta x} = 0 \tag{3}$$

$$\frac{p_{i+1}^{t+1}-p_{i}^{t+1}}{\Delta x} + \frac{2(m_{i+1}^{t+1}-m_{i+1}^{t})}{\pi D^2 \Delta t} + \frac{2(m_{i}^{t+1}-m_{i}^{t})}{\pi D^2 \Delta t} = -\frac{8fc^2}{\pi^2 D^5}\frac{m|m|}{p} \tag{4}$$

where Δt and Δx denote the time and spatial steps, respectively.

Note that the friction force term on the right-hand side of (4) is still non-linear. The following steps are taken to linearize (4):

1) The direction of gas flow rarely changes for pipes with slow dynamics; therefore, $m|m|$ is rewritten as m^2.
2) The first-order Taylor series is employed to approximate the non-linear term in (4) based on a reference point (m_0, p_0). The references m_0 and p_0 are further replaced by the reference gas flow velocity (typically around 10 m/s [6]). The friction force term in (4) is linearized as:
$$\frac{8fc^2}{\pi^2 D^5}\frac{m^2}{p} \approx \frac{8fc^2}{\pi^2 D^5}\left(\frac{2m_0}{p_0}m - \frac{{m_0}^2}{{p_0}^2}p\right) = \frac{4fu_0}{\pi D^3}m - \frac{f{u_0}^2}{2c^2 D}p \tag{5}$$
3) For each segment, m and p can be approximated as the average mass flow and pressure of its two terminal points.

Finally, the non-linear equation (4) is linearized as (6). Then, (3) and (6) are the final linearized equations employed in this letter to approximate the PDEs (1) and (2).

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$$\frac{p_{i+1}^{t+1}-p_i^{t+1}}{\Delta x}+\frac{2(m_{i+1}^{t+1}-m_{i+1}^{t})}{\pi D^2\Delta t}+\frac{2(m_i^{t+1}-m_i^{t})}{\pi D^2\Delta t}=\frac{f{u_0}^2}{4c^2D}(p_{i+1}^{t+1}+p_i^{t+1})-\frac{2fu_0}{\pi D^3}(m_{i+1}^{t+1}+m_i^{t+1}) \quad (6)$$

Note that the accuracy of finite difference methods depends on the step sizes of Δt and Δx. Smaller step sizes can improve the accuracy but at the cost of larger computational burdens. Because OEF problems focus on energy production and consumption, larger step sizes may be acceptable as long as the error can be constrained to a satisfactory level.

B. Temporal Discretization Criteria

To achieve satisfactory accuracy with the finite difference method, the temporal discretization should consider both the speed of gas flow transients and the studied time interval of IES optimization. The gas flow transient speed can be analyzed through a transfer function as described in [4]-[6]. In short, equations (3) and (6) can be written in matrix form to derive the transfer function $H(s)$ of input p_i^{t+1} or m_{i+1}^{t+1} to output p_{i+1}^{t+1} or m_i^{t+1}. Using the first-order series expansion, the approximated transfer functions are of the following type:

$$H(s)=\frac{k_1+k_2 s}{1+\tau s} \quad (7)$$

where k_1, k_2, and τ are parameters.

According to the approximated $H(s)$, the gas flow transient speed is mainly decided by τ, which can be calculated in terms of pipeline parameters as shown in (8):

$$\tau=\frac{L}{u_0}ye^{\frac{y}{2}}\left(1-\frac{y}{6}+\frac{y^2}{24}\right)\ ,\ y=\frac{fu^2L}{2Dc^2}\approx\frac{f{u_0}^2L}{2Dc^2} \quad (8)$$

Thus, the slow dynamics of gas flow can be approximated by an exponential decay function $e^{-t/\tau}$ $(\tau>0)$ [5]. For a given gas pipeline, the length L, diameter D, and friction factor f are known and u_0 and c can be assumed to be constant. Thus, τ can be approximated by (8) prior to executing the OEF. Assume the parameters D, f, u_0, and c are respectively 1 m, 0.01, 10 m/s, and 380 m/s, and τ is 0.35, 35.0, and 3900.1 s for pipelines with respective lengths of 1, 10, and 100 km.

Considering an exponential decay function e^{-t}, we say the function is *sufficiently decayed* with regards to an arbitrary tolerance $\varepsilon\in\mathbb{R}_+$, $\exists t^*\in\mathbb{R}_+$ such that $e^{-t}\le\varepsilon, \forall t\ge t^*$. Provided the ε, we get t^* equals $\tau\ln(\varepsilon)$. Assuming ε=0.05, the calculation then shows that e^{-t} will not sufficiently decay before $t^*\approx$1.05 s and 11700 s for Δx=1 km and 100 km, respectively. If the time interval of OEF is 1 h, the gas transients can be ignored for a 1 km pipe but should be properly addressed in the model for a 100 km pipeline. Hence, the discretization requirements for the time rely on τ, the predefined optimization time interval ΔT, and the discretization error tolerance ξ.

With any given ΔT, the exponential decay function $e^{-t/\tau}$ can be tranformed into a non-dimensional term $e^{-t\Delta T/\tau}$. The local truncation error E of the trapezoidal rule with a non-dimensional step size $h=\Delta t/\Delta T$ over the time span $[0,-\tau\ln(\varepsilon)/\Delta T]$ can then be estimated by (9) and the maximum step size Δt to accommodate the error tolerance ε can be calculated by (10).

$$E=\frac{h^2}{12}\frac{-\tau\ln(\varepsilon)}{\Delta T}\frac{\Delta T^2}{\tau^2}e^{-\tilde{t}\Delta T/\tau}\le\frac{-\ln(\varepsilon)\Delta T}{12\tau}h^2\le\xi \quad (9)$$

$$\Delta t=h\Delta T\le\sqrt{-12\xi\tau\Delta T/\ln(\varepsilon)} \quad (10)$$

where $\tilde{t}\in[0,-\tau\ln(\varepsilon)/\Delta T]$.

An interesting phenomenon can be observed from (10). Take ΔT=3600 s and ξ=0.05 as an example. When dealing with short pipelines with small L (e.g., L=10 km), no additional time discretization is required because the transient period is much shorter than ΔT. However, for a very long pipeline (e.g. $L\ge$200 km in this example), additional time discretization is also not necessary because the time interval ΔT is sufficient to approximate the slow dynamics of long gas pipes. The following criteria are summarized with respect to discretizing time steps:

(i) If $\tau\le-\Delta T/\ln(\varepsilon)$, ΔT need not be discretized.
(ii) If $\Delta t\ge\Delta T$, i.e., ΔT is sufficiently small, temporal discretization is not required.
(iii) If neither criteria (i) nor (ii) is satisfied, each time interval ΔT should be discretized into N_T segments as shown in (11). In (11), $\lceil\ \ \rceil^+$ denotes the ceiling function.

$$N_T=\lceil h^{-1}\rceil^+\ge\sqrt{-\ln(\varepsilon)\Delta T/12\xi\tau} \quad (11)$$

C. Spatial Discretization Criteria

In addition to temporal discretization, spatial discretization may be required to guarantee the eligibility of the transfer function approximation in (7). For a given pipeline length L, the reasonable number of spatial segments, denoted N_X, should be considered a function Ψ with respect to length L, the OEF time interval ΔT, and other parameters of the pipeline. All the variables of Ψ can be regrouped into two ancillary variables, z_1 and z_2, as shown in (12).

$$N_X=L/\Delta x=\Psi(z_1,z_2) \quad (12)$$

where $z_1=\frac{fu_0L}{2Dc}$ and $z_2=\frac{L}{2c\Delta T}$

Based on the empirical diagram simulated in [6], Ψ can be approximated by (13). The base of the logarithm is 10.

$$\log N_X\approx(\beta_1\log z_1+\beta_2)\log z_2+\beta_3\log z_1+\beta_4 \quad (13)$$

where parameters β_1, β_2, β_3, and β_4 are approximated as −0.154, 0.779, 0.461, and 0.279, respectively.

An alternative approximation is also developed to intuitively illustrate the influences of pipeline parameters and temporal interval on the spatial discretization. Because the typical value of $\beta_1\log z_1+\beta_2$ ranges from 0.5 to 0.8, the mean value 0.65 is employed and the alternative approximation is shown in (14).

$$N_X\approx 0.9f^{0.46}{u_0}^{0.46}D^{-0.46}c^{-1.11}L^{1.11}\Delta T^{-0.65} \quad (14)$$

Take a 100 km pipeline as an example. If ΔT is 3600, 900, and 300 s (i.e., 1 h, 15 min, and 5 min), N_X is estimated at 0.84, 1.94, and 3.78 by (13) and 0.74, 1.82, and 3.72 by (14), respectively. Thus, the 100 km pipeline should be divided into 1, 2, and 4 spatial segments when ΔT equals 3600, 900, and 300 s, respectively.

D. OEF Model with Transient Gas Flow

Taking the daily optimal operation of IESs as an example, a generalized OEF can be expressed as follows:

$$\underset{\mathbf{P},\mathbf{m}}{\text{Minimize}}\quad \sum_t C(\mathbf{P},\mathbf{m})$$

$$\text{s.t.}\quad g_p(\mathbf{P})=\mathbf{0}\,,\, g_n(\mathbf{m})=\mathbf{0}\,,\, g_{pn}(\mathbf{P},\mathbf{m})=\mathbf{0}$$

where $C(\mathbf{P},\mathbf{m})$ denotes the cost function and $\mathbf{P}$ and $\mathbf{m}$ respectively denote the variables in power and gas systems. g_p, g_n, and g_{pn} denote the constraints of the power system, gas system, and coupling constraints of these two systems, respectively. Note that the OEF model is expressed in slack form, where the non-equivalent constraints in g_p, g_n, and g_{pn} are converted to equality constraints by introducing non-negative slack variables.

III. Case Study

In this section, the values of parameters D, f, u_0, c, ε, and ξ in Section II are employed. The dynamic features of gas flow are then simulated by varying the length L and time interval ΔT. The simulations are carried out on a desktop computer with an Intel i7-7700 processor and 12 GB RAM.

A. Validation of Approximated Transient Gas Flow

Gas flows through three gas pipelines with respective lengths of 25, 100, and 200 km are simulated. The inlet pressure is kept at 60 bar and the outlet gas flow is decreased from 200 to 150 kg/s at time t=0 and ΔT=3600 s. According to the proposed discretization criteria, N_T (N_X) are 1 (1), 2 (1), and 1 (2) for pipelines with respective lengths of 25, 100, and 200 km. Equations (3) and (6) with Δx=100 m and Δt=10 s are solved to obtain the benchmark results. The results of steady-state gas flow [2], non-discretized gas flow [3], and the proposed discretized gas flow are compared in Table I. The non-discretized method and the proposed method for a single pipeline can be solved in less than 0.01 s, while the computational burden of the benchmark gas flow of the 200 km pipeline is 2574.6 s.

All three methods perform well for the 25 km gas pipe, where τ equals 222.8 s and is much smaller than ΔT. Fig. 1 further compares the gas flows for the 100 and 200 km gas pipes. The simulation results validate that the proposed discretization method keeps the maximum error within the acceptable range of 5% and is more effective at approximating the transient gas flows in pipelines of various lengths compared to the non-discretized gas flow and steady-state flow.

TABLE I
MAXIMUM RELATIVE ERROR OF APPROXIMATED TRANSIENT GAS FLOWS

Pipeline length (km)		Steady-state gas flow (%)	Non-discretized method (%)	Proposed method (%)
25	m_i	0.0	1.13	1.13
	p_{i+1}	0.0	0.04	0.04
100	m_i	4.18	10.37	3.97
	p_{i+1}	0.48	0.74	0.17
200	m_i	20.98	10.80	3.82
	p_{i+1}	9.09	1.39	1.11

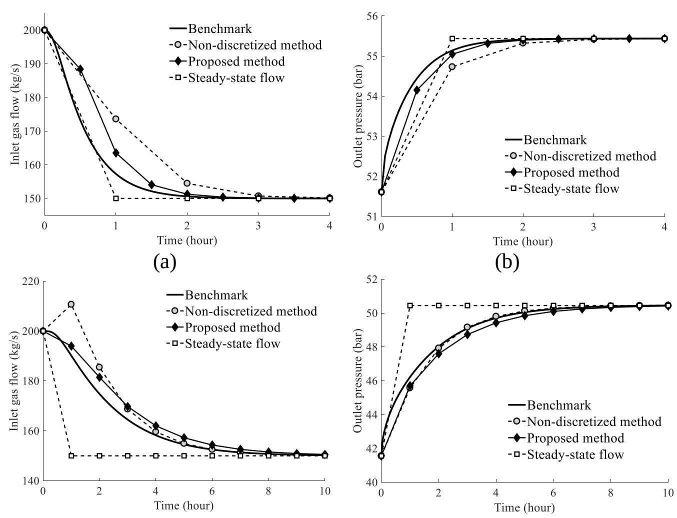

Fig. 1. Performance of approximated transient gas flows. (a) Inlet gas flow and (b) outlet pressure for the 100 km gas pipe. (c) Inlet gas flow and (d) outlet pressure for the 200 km gas pipe.

B. OEF Simulation Results

The IES with three electrical buses and three natural gas buses discussed in [2] is employed to verify the proposed OEF scheme with a time interval of 15 min (ΔT=900 s). The simulated system has one gas well, one coal-fired generator (G_1), and one gas-fired generator (G_2). Using the proposed discretization method, only the 90 km pipeline 1-2 should be divided into two spatial segments, i.e., $\Delta x = L/2$ =45 km. In addition, each optimization interval ΔT is discretized into two segments according to the proposed temporal discretization criteria, i.e., Δt=450 s. The OEF results over 24 consecutive operating hours are provided in Table II.

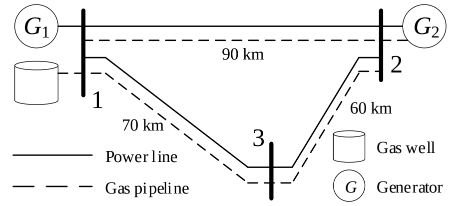

Fig. 2. An example IES for simulation.

TABLE II
SIMULATED DAILY OEF RESULTS BASED ON DIFFERENT METHODS

	Steady-state	Non-discretized	Proposed
Total cost (×10^6 m.u.*)	21.18	20.41	20.44
G1 generation (×10^4 MWh)	12.94	11.98	12.08
G2 generation (×10^4 MWh)	6.80	7.76	7.67
Lowest nodal pressure (bar)	48.52	51.73	51.75
Number of 15-minute intervals in which the gas supply limit is reached	40	29	34
CPU time (s)	0.95	0.97	1.02

* m.u. stands for the monetary unit.

The steady-state gas flow ignores the slow dynamics of gas flow and leads to a conservative operating strategy, and thus the steady-state flow based OEF has the highest cost. The non-discretized method and the proposed method share similar operational results and computational efficiency. Compared to the proposed method, the non-discretized method concludes that more gas can be consumed but the gas supply limit is less likely to be reached. Thus, the non-discretized method may lead to unrealistic operating strategies. The proposed method is validated to achieve an efficient OEF result while maintaining a satisfactory accuracy of transient gas flow approximation.

IV. CONCLUSION

This letter integrates transient gas flow equations into an OEF model for IESs through finite difference approximation. Temporal and spatial discretization criteria are developed based on gas pipelines parameters and the desired optimization time intervals of OEF to enhance the approximation accuracy. The results validate that the proposed discretization criteria and OEF are applicable to gas pipelines and IESs of varying parameters and optimization time intervals while maintaining a satisfactory degree of accuracy.

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