Chapter IV. Sheath physics.

One of the most distinct processes at the edge of plasma devices is a plasma flow along the magnetic field lines to the material surface (target) where plasma is neutralized. Since for the edge plasma conditions, the electron thermal speed greatly exceeds the ion one, to maintain ambipolarity of plasma flow (for simplicity, here we assume no electric current) such plasma flow is accompanied by the formation of so-called sheath region in the vicinity of the target. This region is characterized by a rather strong electrostatic electric field, which, usually repels most of free streaming electron flux and, as a result, establishes ambipolarity of plasma flow (see Fig. IV.1).

Therefore, in the absence of strong electron emission from the material surface, a monotonic electrostatic potential, $\varphi(z)$, is built up between the plasma interior and the material surface (here the z coordinate goes perpendicular to the surface that is considered to be flat). The magnitude of the sheath potential drop φ_{sh} (see Fig. IV.1) is determined either from the condition of ambipolarity of the plasma flow or by the value of the electric current flowing through the plasma-surface interface. The electric field in the sheath causes energy exchange between electrons and ions and for the target potential negative with respect to the plasma, the energy of the ions impinging on the surface can significantly exceed the thermal ion energy at the entrance to the sheath. This effect can boost erosion and degradation of the plasma-facing components and cause unwanted plasma contamination with impurities.

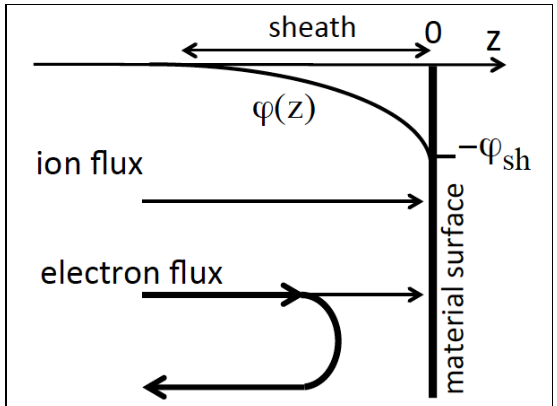

Fig. IV.1. Electrostatic potential $\varphi(z)$ within the sheath plugs most of the free streaming electron flux to maintain ambipolar plasma flow.

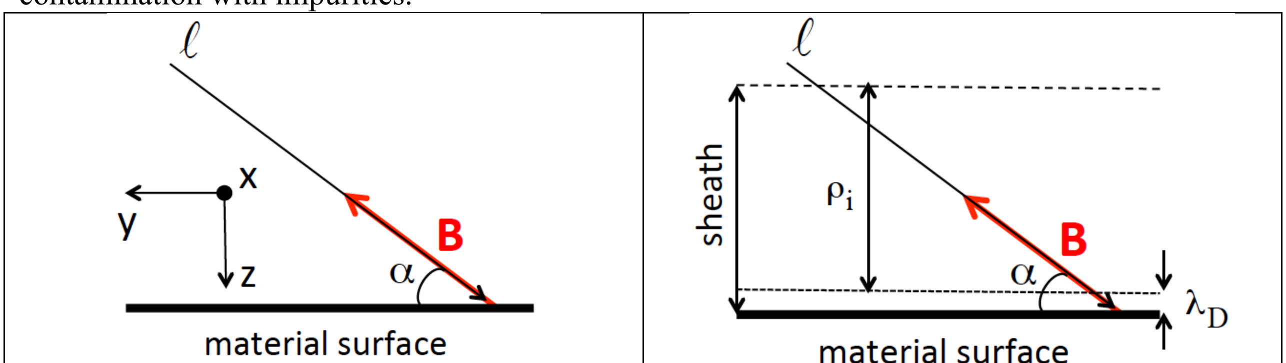

Fig. IV.2. (left) Direction of the magnetic field and the coordinates which will be used. (right) Magnetic presheath, with the width $\sim \rho_i$, and Debye sheath with the width $\sim \lambda_D$ for the case $\alpha << 1$.

The structure of the sheath depends on the angle, α, between the magnetic field, $\vec{B}$, and the target (see Fig. IV.2; we notice that the direction of $\vec{B}$ is arbitrary, whereas the "parallel" coordinate, ℓ, goes toward the material surface). For the case of no magnetic field or the magnetic field being perpendicular to the surface, the thickness of the sheath, Δz_{sh}, is of the order of the Debye length, $\lambda_D = \sqrt{T_e / 4\pi n_{sh} e^2}$, where T_e is the electron temperature, n_{sh} is the plasma density in the sheath and e is the elementary charge.

For $\alpha \neq \pi/2$ the thickness of the sheath starts to depend on the ion gyro-radius, $\rho_i = (Mc/eB)\sqrt{T_i/M}$ (here T_i is the ion temperature, c is the speed of light, B is the strength of the magnetic field, and M is the ion mass) and for $\alpha << 1$ $\Delta z_{sh} \sim \rho_i$, since in the edge plasma, the Debye length is usually small in comparison with ρ_i. In this case, one can split the sheath into the magnetic presheath and the Debye sheath with the thicknesses $\sim \rho_i$ and $\sim \lambda_D$ respectively.

What is important for many aspects of edge plasma physics is that for smooth transition of electrostatic potential from the sheath region into the plasma interior, the averaged ion velocity along the magnetic field lines for such plasma flow relatively far away from the surface, $V_{||}^{(-\infty)}$, is limited by the so-called Bohm-Chodura [1], [2] sheath criterion:

$$V_{||}^{(-\infty)} \geq V_{crit}, \qquad \text{(IV.1)}$$

where the critical velocity V_{crit} is determined by plasma parameters (see also [3]).

Interestingly, inequality (IV.1) can be obtained from an analysis of the asymptotic behavior of the solution of the Poisson equation with proper perturbation of the electron and ion density by the sheath potential at $z \to -\infty$. To demonstrate this, we consider an idealized case assuming no magnetic field and monoenergetic ions streaming to the target with the velocity component perpendicular to the target at $z \to -\infty$ equal to $V_{-\infty}$. We also assume that at $z \to -\infty$, electrons with $v_z > 0$ are described by the Maxwellian distribution function $f_e(v_z > 0) \propto \exp(-mv_z^2/2T_e)$.

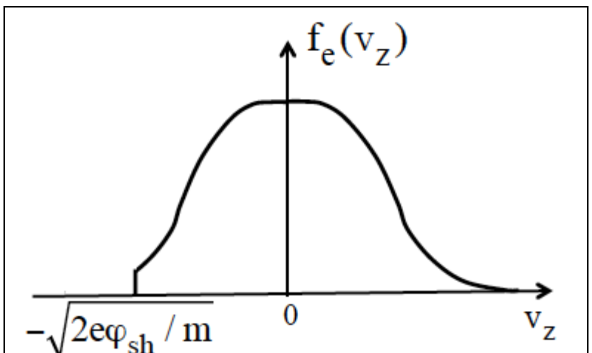

Fig. IV.3. Sketch of electron distribution function $f_e(v_z)$ at $z \to -\infty$.

Considering the case $V_{-\infty} << \sqrt{T_e/m}$ and taking into account that at $z \to -\infty$ the plasma is quasi-neutral, we conclude that in order to maintain ambipolarity of the plasma flow to the target, the majority of the electrons reaching the sheath region must be reflected back by the electrostatic potential. As a result, the electron distribution function at $z \to -\infty$ is almost symmetric with respect to the sign of v_z, with the only exception related to the cut-off of the tail of the reflected electrons (see Fig. IV.3), which manifests the absorption of the electrons that could penetrate through the potential barrier $e\varphi_w$ at the target (recall Fig. IV.1).

From ion energy conservation and the ion continuity equation we find the following expression for the ion density at $z \to -\infty$, assuming that $MV_{-\infty}^2/2 >> e|\varphi(z)|$:

$$n_i(z) = n_{sh}\left(1 + \frac{e\varphi(z)}{MV_{-\infty}^2}\right), \tag{IV.2}$$

where n_{sh} is the plasma density at the entrance to the sheath (in our case at $z \to -\infty$). Neglecting the impact of the cut-off of the tail of the electron distribution function at $z \to -\infty$, we can take the Boltzmann relation for the electron density, which for $T_e >> e|\varphi(z)|$ gives: $n_e(z) = n_{sh}\left(1 + e\varphi(z)/T_e\right)$. Substituting both the ion and electron densities into the Poisson equation, we find

$$\frac{d^2\varphi(z)}{dz^2} = \frac{1}{\lambda_D^2}\left(1 - \frac{T_e}{MV_{-\infty}^2}\right)\varphi(z). \tag{IV.3}$$

From Eq. (IV.3) one sees that in accordance with the expression (IV.1), a "smooth" (exponentially decaying at $z \to -\infty$) variation of the electrostatic potential is only possible for

$$V_{-\infty} \geq V_{crit} \equiv C_s = \sqrt{T_e / M}. \tag{IV.4}$$

We should notice that such a flow of monoenergetic ions through almost Maxwellian electrons results in the instability associated with excitation of the sound waves (e.g. see [4], [5]). However, this instability has a convective nature and is stabilized by even a small broadening of the ion distribution function [4].

Since in practice, the ion velocity distribution at $z \to -\infty$ is far from being monoenergetic, the simple expression (IV.1) for the Bohm-Chodura criterion should be altered to allow for the finite spread of the ion velocity distribution function, $f_i^{(-\infty)}(\vec{v})$, at $z \to -\infty$. For the case of no magnetic field, in Ref. [6] it was shown that the expression (IV.4) can be generalized as follows:

$$\int_0^\infty \frac{f_i^{(-\infty)}(v_z)}{n_{sh} v_z^2} dv_z \leq \frac{M}{T_e}, \tag{IV.5}$$

which for the case of monoenergetic ions reduces to the expression (IV.4). As we see from Eq. (IV.5), the integral expression on the left-hand side converges only for the case where $f_i^{(-\infty)}(v_z \to +0)$ approaches zero fast enough.

The Bohm-Chodura limitation on the velocity of the plasma flow onto the material surface is often viewed as a result of "pure" plasma effects, based solely on electron-ion coupling through the ambipolar electric field. However, this is not the case. The constraint similar to Eq. (IV.1) does also exist for the velocity of collisional neutral gas flowing onto an absorbing surface (e.g. see [7]). Another example is the constraint on the speed of the gas flow into a standing shock wave where such speed must be supersonic. As a matter of fact, all these features have deep physical meaning. Indeed, both the plasma and neutral gas flows onto absorbing targets resemble the gas flow into a standing shock. However, the stability of the 1D standing shock wave is ensured by the fact that the gas flow velocity into it is supersonic, or, in other words, there is no wave propagating upstream away from the shock [8]. Interestingly, in [9] it was demonstrated that the expression (IV.5) means that the ion sound waves cannot propagate in the direction away from the target. Therefore, the Bohm-Chodura constraint can be viewed as an extension of the Landau stability criterion, with respect to the ion sound waves, to the collisionless plasma flow onto an absorbing target.

Although the expression (IV.5) removes the limitation of the monoenergetic ion velocity distribution and goes beyond the simple constant (IV.4), it still does not describe the effect of the magnetic field line tilting with respect to the target (see Fig. IV.2), even though this feature is ubiquitous in edge plasma. To address both these issues, we need to consider kinetic equations for ions and electrons, which in the stationary 1D limit can be written as follows:

$$\vec{v}\cdot\nabla f_{i/e}(\vec{v},z) \pm e\frac{\vec{v}\times\vec{B}}{Mc}\cdot\nabla_{\vec{v}} f_{i/e}(\vec{v},z) \mp \frac{e}{M}\frac{d\varphi(z)}{dz}\vec{e}_z\cdot\nabla_{\vec{v}} f_{i/e}(\vec{v},z) = 0\,, \qquad \text{(IV.6)}$$

where $f_{i/e}(\vec{v},z)$ is the ion/electron velocity distribution function and $\vec{e}_z$ is the unit vector along the z-coordinate. A similar equation can be used for the electrons. Incorporating the solutions of the ion and electron kinetic equations into the Poisson equation, we can find all the necessary information. However, in practice, to find the constraint similar to Eq. (IV.4), (IV.5), we do not need to have the full solution of these equations. As we have found in the course of the derivation of Eq. (IV.4), the Bohm-Chodura constraint comes from the asymptotic behavior of the solution of the Poisson equation, recall Eq. (IV.3), at $z\to-\infty$ where the electrostatic potential is low. Therefore, instead of solving the complex nonlinear system of the kinetic and Poisson equations, we can consider their linearized versions. As a result, we come to the problem similar to that of finding the dielectric constant of the plasma, $\varepsilon(\omega,\vec{k})$. Usually, it is determined as a function of the frequency, ω, and the wavenumber, $\vec{k}$, which characterize the plasma wave. However, in our case, we are looking for the conditions of the formation of a stationary evanescent electrostatic potential that links the sheath region with the plasma interior. Therefore, we should consider the plasma dielectric constant with zero frequency and the wavenumber having the imaginary part ensuring that $\varphi(z\to-\infty)\to 0$.

Following [4] we take $f_{i/e}(\vec{v},z) = f_{i/e}^{(-\infty)}(\vec{v}) + f_{i/e}^{(1)}(\vec{v},z)$, where $f_i^{(1)}(\vec{v},z)$ is a small correction. Moreover, by analogy with Eq. (IV.3), we will assume and justify *a posteriori* that $\varphi(z) = \varphi_0\exp(z\hat{k}_z)$ and $f_{i/e}^{(1)}(\vec{v},z) = \tilde{f}_{i/e}^{(1)}(\vec{v})\exp(z\hat{k}_z)$, where $\hat{k}_z$ is an adjustable parameter playing the role of the wavenumber of evanescent wave and which can be found from the solution of the Poisson equation. For "smooth" transition from the sheath to $z\to-\infty$ we need $\mathrm{Re}(\hat{k}_z) > 0$.

We notice that $f_{i/e}^{(-\infty)}(\vec{v})$, being the solution of the stationary kinetic equation with $\varphi(z)=0$, should be expressed in terms of the integrals of motion, which gives: $f_{i/e}^{(-\infty)}(\vec{v}) \equiv F_{i/e}(v_{\|},\varepsilon_\perp)$ where $\varepsilon_\perp = \vec{v}_\perp^2/2$ whereas $\vec{v}_\perp$ and $v_{\|}$ are the velocity components perpendicular and parallel to the magnetic field. Then, from [4] we have

$$\hat{k}_z^2 = -\sum_{i/e}\frac{4\pi e^2}{m_{i/e}}\sum_{j=-\infty}^{\infty}\left\langle \frac{J_j^2\left(-i\cos(\alpha)\dfrac{\hat{k}_z v_\perp}{\Omega_{B_{i/e}}}\right)}{j\Omega_{B_{i/e}} - i\sin(\alpha)\hat{k}_z v_{\|}}\left(j\Omega_{B_{i/e}}\frac{\partial F_{i/e}}{\partial\varepsilon_\perp} - i\sin(\alpha)\hat{k}_z\frac{\partial F_{i/e}}{\partial v_{\|}}\right)\right\rangle, \qquad \text{(IV.7)}$$

where $J_j(x)$ are the Bessel functions and $\langle ... \rangle = \int (...) d\varepsilon_\perp dv_\parallel$. We notice that the normalization of $F_{i/e}(v_\parallel, \varepsilon_\perp)$ assumes $\langle F_{i/e}(v_\parallel, \varepsilon_\perp) \rangle = n_{sh}$.

First, we consider the case of the normal incidence of the magnetic field lines onto the target, $\alpha = \pi/2$. In this case, Eq. (IV.7) is reduced to

$$\hat{k}_z^2 = -\sum_{i/e} \frac{4\pi e^2}{m_{i/e}} \left\langle \frac{1}{v_z} \frac{\partial F_{i/e}}{\partial v_z} \right\rangle. \qquad \text{(IV.8)}$$

Since we assume that the ions, striking the target, are "absorbed" by the material surface, we have $F_i(v_z) \propto H(v_z)$, where $H(x)$ is the Heaviside function, $H(x>0)=1$ and $H(x<0)=0$. For electrons, as we discussed above, we largely have symmetric distribution $F_e(v_z) = F_e(|v_z|)$. As a result, for Maxwellian $F_e(v_z)$, from Eq. (IV.8) we find

$$\hat{k}_z^2 = \frac{1}{\lambda_D^2} \left(1 - \frac{T_e}{M n_{sh}} \left\langle \frac{F_i}{v_z^2} \right\rangle \right). \qquad \text{(IV.9)}$$

As we see, the condition $\mathrm{Re}(\hat{k}_z) \sim \lambda_D^{-1} > 0$ can only be satisfied for the case where the generalized Bohm-Chodura criterion Eq. (IV.5) is valid. From Eq. (IV.9) one finds also that, in agreement with Eq. (IV.3), the characteristic scale of $\hat{k}_z$ is of the order of λ_D^{-1}.

Next, we consider the case

$$1 > \alpha > \lambda_D / \rho_i \text{ and } \lambda_D / \rho_e > 1, \qquad \text{(IV.10)}$$

where ρ_e and ρ_i are the electron and ion gyro-radii. For comparable electron and ion temperatures, which is rather typical for the plasma near the targets in high recycling conditions, these inequalities impose the following restriction on the angle α: $1 > \alpha > \sqrt{m/M}$. We assume that $\hat{k}_z \sim \lambda_D^{-1}$ and, taking into account that $\lambda_D / \rho_e > 1$ and $\alpha << 1$, we conclude due to the conservation of both the electron adiabatic invariant $v_\perp^2 / B = \text{const.}$ and the total electron energy, the electron dynamics in the sheath region is virtually adiabatic, so that $m v_\parallel^2 / 2 - e\varphi = \text{const.}$ [10]. Therefore, similar to our previous cases, neglecting the small tail cut-off, we can assume an almost symmetric electron distribution function $F_e(v_\parallel) = F_e(|v_\parallel|)$. We also assume complete "absorption" of the ions on the target, which gives $F_i(v_\parallel) \propto H(v_\parallel)$. Then, taking into account inequalities (IV.10) and recalling that $\sum_{j=-\infty}^{\infty} J_j^2(\varsigma) = 1$ for $\arg|\varsigma| < \pi$, from Eq. (IV.7) we find that it can be reduced to

$$\hat{k}_z^2 = \frac{1}{\lambda_D^2} \left(1 - \frac{T_e}{M n_{sh}} \left\langle \frac{F_i}{v_\parallel^2} \right\rangle \right) \ge 0 \to \frac{1}{n_{sh}} \left\langle \frac{F_i}{v_\parallel^2} \right\rangle \le \frac{M}{T_e}, \qquad \text{(IV.11)}$$

which justifies our assumption $\hat{k}_z \sim \lambda_D^{-1}$. For the case of monoenergetic ion distribution function along the magnetic field, the expression (IV.11) is reduced to the Chodura inequality (IV.1) [2].

Once the ion flux onto the target is defined by $f_i^{(-\infty)}(\vec{v})$, the potential drop between the plasma away from the target and the target can be found by equilibrating the electric currents in plasma and through the sheath. For the case of ambipolar plasma flow assuming that the ion flow to the target along the magnetic field is $\approx n_{sh} C_s$, ignoring other effects that could alter the ion flux to the target (e.g. drifts), and adopting the Maxwellian electron distribution function, we find $\sim e\varphi_{sh} \approx \Lambda_{sh} T_e$, where $\Lambda_{sh} \sim \ell n(M/m) \sim 3 \div 4$ [3].

In [5] the constrains (IV.5) and (IV.11) were criticized, in particular, from the point of view of "unphysical" v_z^{-2} ($v_{\|}^{-2}$) moment of the ion distribution function and the omission of the impact of the collision operator for small v_z ($v_{\|}$) in the course of the derivation of Eq. (IV.5), (IV.11) from the corresponding Vlasov equations. A replacement of Eq. (IV.5) based on positive powers of the velocity moments of the distribution function, which do not diverge at v=0, was suggested (see [5] for details). But in [11] it was argued that the inequality (IV.5) holds also in the presence of collisions provided that Eq. (IV.5) is applied at the entrance to the sheath and the sheath is considered in the limit of $\lambda_D \to 0$. Although this discussion is important from an academic point of view, for the practical application of the Bohm constraint as the boundary condition for edge plasma flow to the target it becomes rather meaningless. This is because in the most relevant, from the point of view of the reduction of the power loading of the target, high recycling regimes of divertor operation, strong plasma-neutral interactions become ubiquitous. As a result, in this case, all the models suggest virtually the same constraint on the plasma flow velocity $\langle v_z \rangle \approx C_s$ (for the case of normal incidence of the magnetic field lines onto the target) with some correction for the finite ion temperature. The difference between various models is within the error bar imposed by the boundary conditions used for the ion and electron heat fluxes to the target, the calculation of which should invoke spatial variation of both the electrostatic potential and the electron and ion distribution functions at the entrance to the sheath (e.g. see [12]).

In our considerations, we assume that the electron distribution function along the magnetic field lines for $v_{\|} > 0$ (in the direction towards the target) is Maxwellian, whereas for $v_{\|} < 0$ the tail of the distribution function is cut at velocities below $-\sqrt{e\varphi_{sh}/m}$. For the case of the ambipolar plasma flow, φ_{sh} can be found by equilibrating the electron and ion fluxes. However, such approximation is only applicable for the case where electron collisionality in the SOL plasma is relatively high and the tail can be replenished by Coulomb collisions before the electrons reach the target. In the opposite case, depletion of the tail of the electron distribution function can drive the whistler waves, which are capable of scattering the electrons and effectively populating the "gap" in the electron distribution function (see [13] and the references therein). However, in practice, in the SOL plasmas, the electrons with the energies $\sim e\varphi_{sh} \approx \Lambda_{sh} T_e$ are weakly collisional, which makes it difficult to make quantitative estimates of the shape of the tail of the electron distribution function and of the subsequent impact of developing of the whistler wave instability. Usually, in edge plasma transport codes it is assumed that electrons impinging onto the target have the Maxwellian distribution.

So far we assumed that the plasma flowing onto the target is completely collisionless within some proximity to the target. In practice, this is not the case and collisions are always present. In particular, in the detached divertor regime, the neutral density becomes high and ion-neutral collisions result in a very short ion-neutral collision mean free path, λ_{i-N}. However, in a ballpark, they do not change one of the key conclusions of the sheath physics: for the case of $\lambda_{i-N} > \lambda_D$, the plasma flow velocity at the distance $\sim \lambda_{i-N}$ from the target should about C_s (see [3] for details).

As an illustration, we consider an ambipolar flow of weakly ionized plasma on a material surface. We assume a constant electron temperature T_e, Boltzmann electrons, no plasma sink or source, and no magnetic field. Then, the plasma flow is governed by the following equations

$$Mj^2 \frac{d(n_i^{-1})}{dx} = -e\frac{d\varphi}{dx} n_i + M\nu j, \tag{IV.12}$$

$$-\frac{d^2\varphi}{dx^2} = 4\pi e\left(n_i - n_e\right), \tag{IV.13}$$

where $j = \text{const.}$ is the plasma particle flux, $\nu = \text{const.}$ is the ion-neutral collision frequency, $n_e = n_e^w \exp(e\varphi / T_e)$ is the electron density, and n_e^w is the electron density at the target where, for convenience, unlike Fig. IV.1, we take zero electrostatic potential. To maintain ambipolarity of the plasma flow we adopt the following boundary condition at the target: $j = \beta n_e^w (T_e / m)^{1/2}$, where $\beta \sim 1$.

We notice that Eq. (IV.12) describes two regimes of the ion flow: i) dynamic ion acceleration, corresponding to the case where the terms on the left-hand side and the first term on the right-hand side dominate (this case describes standard acceleration of ions in a collisionless sheath); and ii) diffusive ion flow corresponding to the case where the terms on the left-hand side are small. In the latter case, assuming plasma quasi-neutrality from Eq. (IV.12) and the Boltzmann relation, we find

$$n_e \cong n_i \propto -x, \text{ and } \varphi \propto \ell n(-x). \tag{IV.14}$$

It is more convenient to switch in Eq. (IV.12), (IV.13) from the variable φ and coordinate x to the variable $f(\phi) = n_i(\phi) / n_e^w \exp(\phi)$ and the coordinate $\eta = (m/M)^{1/2} \beta^{-1} \exp(\phi)$ (we notice that $\eta \ge \eta_{min} \equiv (m/M)^{1/2} \beta^{-1}$). As a result, from Eq. (IV.12), (IV.13) we find

$$-\frac{1}{2}\frac{d}{d\eta}\left(f\eta - (f\eta)^{-1} - f^{-2} df / d\eta\right)^{-2} = P\left(f - 1\right), \tag{IV.15}$$

where $P = 4\pi e^2 j / MC_s \nu^2$. To shed the light on the physical meaning of the parameter P, let us calculate the ratio of the ion-neutral collision mean-free path, λ_{iN}, and local Debye length, λ_D. After simple algebra we find $\xi \equiv \lambda_{iN} / \lambda_D = f^{-1}\sqrt{P/\eta}$. We will see below that at $f(\eta \sim 1) \sim 1$, the ion velocity is close to the sound speed. As a result, we find $\xi \approx \sqrt{P}$. So at $P >> 1$, Λ is large and ions in the vicinity of the target are moving virtually in dynamic regime. In what follows we will assume $P >> 1$.

When the plasma is close to quasi-neutrality ($f \cong 1$), from Eq. (IV.15) we find the following correction ($\sim P^{-1} << 1$) to f:

$$f \approx f_{\infty}(\eta) \equiv 1 + \eta(\eta^2 + 1) / P(\eta^2 - 1)^3 . \qquad \text{(IV.16)}$$

From the Poisson equation, it is easy to show that the expression (IV.16) gives correct asymptotic dependence $\varphi \propto \ell n(-x)$ for quasi-neutral plasma (recall Eq. (IV.14)). However, at $\eta \to 1$ function $f_{\infty}(\eta)$ diverges.

On the other hand, the dynamic regime of the ion flow is described by the expression in the brackets on the left-hand side and in dynamics regime, it should be close to zero. This can only be for the case where the right-hand side of Eq. (IV.15) is large for $f \gtrsim 1$, which holds for $P >> 1$. Then, from the equation $f\eta - (f\eta)^{-1} - f^{-2} df / d\eta = 0$, we find $f = \eta^{-1}\{1 - 2\ell n(\eta)\}^{-1/2}$ where, keeping in mind the expression (IV.16), we take the boundary condition $f(\eta = 1) = 1$. However, this boundary condition implies that the ion velocity reaches C_s at the entrance to the domain with the dynamic acceleration of ions (the Debye sheath). Once $f(\eta)$ is known, the spatial dependence $\varphi(x)$ can be deduced from the Poisson equation. Keeping in mind our assessment of the physical meaning of P, from the Poisson equation we find that the spatial domain occupied by quasi-diffusive, quasi-dynamic ion transport corresponding to $f \sim 1$ is about few λ_{iN} from the target. On the other hand, the dynamic acceleration of ions occurs at the scale $\sim \lambda_D << \lambda_{iN}$ just in front of the material surface.

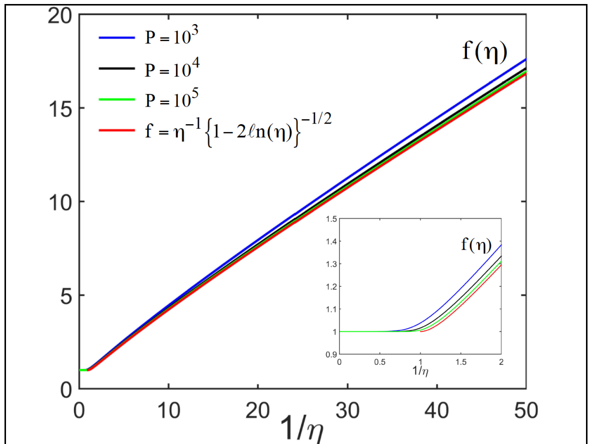

Fig. IV.4. Comparison of numerical solutions of Eq. (IV.15) for different parameters P with the asymptotic analytic solution corresponding to $P \to \infty$.

Numerical solution of Eq. (IV.15) shows good agreement with the results of the analytic analysis presented here (see Fig. IV.4). As we see from Fig. IV.4, there is a continuous transition of $f(\eta)$ and, therefore, $\varphi(x)$ from the ion diffusion-limited to the ion dynamic-limited regimes.

We notice that our fluid equation-based consideration, predicting that the ion flow velocity becomes $\sim C_s$ at the entrance to the Debye sheath agrees, in a ballpark, with the solution of the one-dimensional kinetic equation from [14], [15] where the ion-neutral collisions were described via the charge exchange process whereas the electrons were assumed to follow the Boltzmann relation with a constant temperature. The ion distribution, $f_i(\hat{\varepsilon}_i)$ (where $\hat{\varepsilon}_i = Mv^2 / 2T_e$), at the entrance to the sheath, found from the corresponding kinetic equation, is shown in Fig. IV.5. We notice that at small $\hat{\varepsilon}_i$ we have $f_i(\hat{\varepsilon}_i) \propto \hat{\varepsilon}_i$, so there is no divergence in Eq. (IV.5) [14].

Since in the edge plasma of fusion devices, rather strong ion-neutral collisions in the vicinity of the divertor target are ubiquitous, these results justify the widely used boundary condition for the plasma flow velocity, $V_{\parallel} \approx C_s$ at the target, which, otherwise, is ill-defined by the inequalities (IV.4), (IV.11).

So far we assumed that the only electric field present in our analysis of the Bohm-Chodura constraint of the plasma flow onto the target is due to the sheath effects. However, in the edge plasmas of fusion devices, a cross-field electric field parallel to the target, $\vec{E}_p$, can also be present (e.g. due to the electric currents or electron temperature variation along the target, etc.). As we will see, the impact of this electric field can significantly alter the Bohm-Chodura constraint. For the case where one can ignore the spatio-temporal variation of both the magnetic field and $\vec{E}_p$, the impact of $\vec{E}_p$ on the Bohm-Chodura constraint can be easily found by a transition to the moving frame [16]. Indeed, considering the sheath in a slab approximation (see Fig. IV.2, where $\vec{E}_p$ is in x-direction), we can recall that the transition from the laboratory frame to the frame moving with nonrelativistic velocity $\vec{V}_f$ results in the following transformation of the electric field: $\vec{E}'_p = \vec{E}_p + (\vec{V}_f \times \vec{B})/c$ (where $\vec{E}'_p$ is the electric field in the moving frame), whereas the magnetic field in the moving frame remains virtually equal to the magnetic field in the laboratory frame. Since we assume that both the magnetic field and $\vec{E}_p$ are constants, with a proper choice of $\vec{V}_f$ we can have $\vec{E}'_p = 0$. As a result, in the moving frame we can use the standard Bohm-Chodura constraint (e.g. given, in the simplest case, by Eq. (IV.4), $V'_{\parallel} > C_s$). Then, making the backward transformation into the laboratory frame, we find the impact of $\vec{E}_p$ on the Bohm-Chodura constraint:

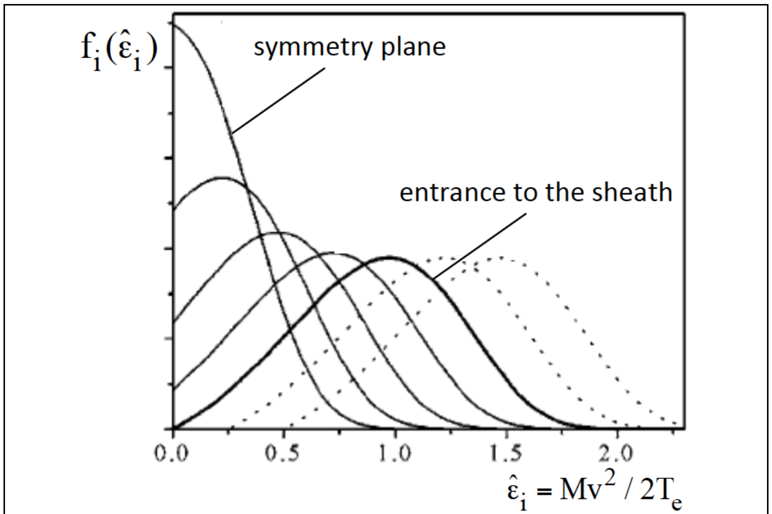

Fig. IV.5. Ion distribution function found from the solution of one-dimensional kinetic equation accounting for ion-neutral charge exchange process and assuming Boltzmann electrons with constant temperature T_e. Reproduced with permission from [15], © AIP Publishing 2006.

$$\vec{V}_{-\infty} = V'_{\parallel}\vec{B}/B + c\left(\vec{E}_p \times \vec{B}_{\perp}\right)B_{\perp}^{-2}, \qquad \text{(IV.17)}$$

where $\vec{B}_{\perp}$ is the component of the magnetic field perpendicular to the target.

The sheath properties impose important boundary conditions for such quantities as the plasma flow velocity to the target, the electric current from the plasma to the material surface and the electron and ion heat fluxes to the target (e.g. see [3], [17], [18], [12] and the references therein). These boundary conditions are used as the closures for the differential fluid plasma equations at the target in 2D fluid plasma transport codes such as SOLPS and UEDGE.

For example, in the simplest case of normal incidence of the magnetic field onto the surface, the electric current flowing through the plasma to the target, j_z^{tar}, has the following relation to the electrostatic potential drop φ_{sh}:

$$j_z^{tar} = en_{sh}\left(V_{-\infty} - (1-\gamma_{see})\sqrt{T_e / 2\pi m}\exp(-e\varphi_{sh} / T_e)\right), \tag{IV.18}$$

where $\gamma_{see} < 1$ is the effective coefficient of secondary electron emission that includes also thermionic electron emission. Taking $V_{-\infty} = \sqrt{T_e / M}$, from Eq. (IV.18) it follows that for ambipolar plasma flow to the target and $\gamma_{see} << 1$ we have $e|\varphi_{sh}| \sim \ell n(M / m)T_e$.

For the electron and ion heat fluxes to the material surface, one can find $q_z^{(e)} = \gamma_e j_z^{(e)} T_e$ and $q_z^{(i)} = \gamma_i j_z^{(i)} T_i$, where $j_z^{(e)}$ and $j_z^{(i)}$ are correspondingly the electron and ion particle fluxes to the target, whereas γ_e and γ_i are the so-called heat transmission coefficients, which in general case of tilted magnetic field depend on the ion distribution function $F_i(v_{\|}, \varepsilon_{\perp})$, secondary electron emission, drifts, and potential drop φ_{sh}. However, the expressions for the fluid closures at the target (including those for the electron and ion heat fluxes) become much more cumbersome if we include grazing magnetic field, particle drifts and time dependence of the plasma parameters related, for example, to the SOL plasma turbulence. A discussion of these issues goes beyond the scope of this chapter. More details can be found in [19], [18], [12] and the references therein.

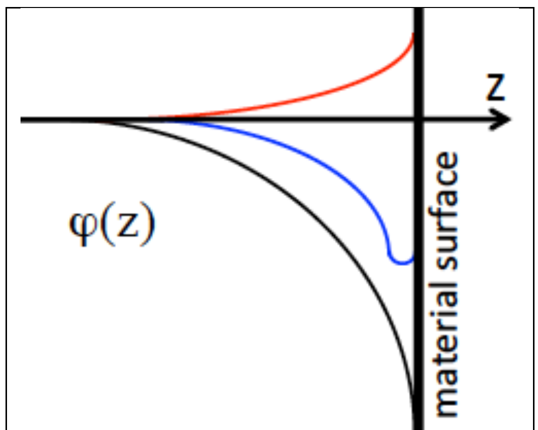

Fig. IV.6. Electrostatic potential profiles for: a standard sheath, $\gamma_{see} << 1$, (black), the SCL sheath for $1 > \gamma_{see} \geq \gamma_{see}^{(crit)}$ (blue), and "inverse sheath", $\gamma_{see} >> 1$, (red).

As it is indicated in Eq. (IV.18), the secondary electron emission can significantly alter the magnitude of φ_{sh}. Moreover, closer consideration shows that for $1 > \gamma_{see} \geq \gamma_{see}^{(crit)}$, the structure of the sheath becomes non-monotonic. This is the so-called space-charge-limited (SCL) sheath (see Fig. IV.6) where to maintain the ambipolarity of plasma flow, a part of the emitted electrons are reflected back to the target by the hump of the electrostatic potential [20]. In Ref. [21] it was suggested that the secondary electron emission could be used for cooling the edge plasma in magnetic confinement devices. In [22] it was shown that the secondary electron emission can result in "focusing" the heat flux and the formation of the "hot spots" on the plasma-facing components. However, in [23] it was argued that for the case of a strongly emitting surface, the so-called "inverse sheath" (IS) (see Fig. IV.6) could be formed. In this case, the positive (with respect to the plasma) potential at the target prevents ions from reaching the target and ambipolarity is maintained by equilibrating the fluxes of the plasma and emitted electrons. Moreover, further studies show that in the presence of cold neutrals, both the ionization and charge exchange processes within the hump of the electrostatic potential of the SCL sheath make the SCL sheath "unstable" and it evolves into IS [24]. The authors of [25] speculated that the IS can promote divertor detachment.

However, a more thorough investigation of the effect of the IS on divertor detachment with a 2D code UEDGE has shown that the IS *per se* does virtually not alter the SOL plasma parameters and does not advance divertor detachment [26]. However, since the IS conditions inhibit the ion flux to the target, the IS regime could be beneficial from the point of view of the strong reduction of the first wall erosion.

The formation of the IS could also explain some, otherwise puzzling experimental data on positive plasma floating potential with respect to strongly electron-emitting objects observed in [27] and [28], e.g. see Fig. IV.7.

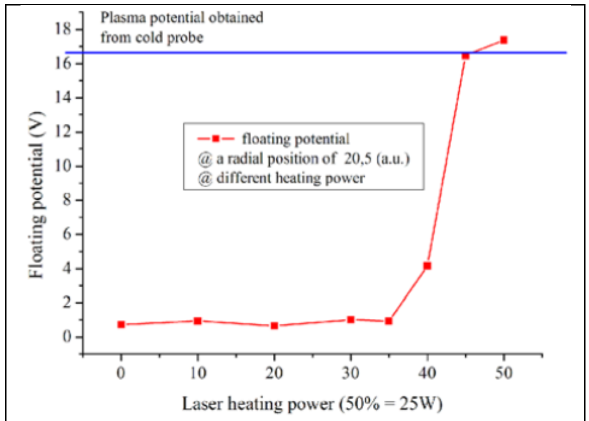

Fig. IV.7. Floating potential of the probe versus the laser heating power that is used to facilitate thermionic emission from the probe. Reproduced with permission from [27], © John Wiley and Sons 2011.

Conclusions for Chapter IV

In conclusion of this chapter, we note that the sheath plays quite a unique role in the edge plasma physics. Even though it occupies a tiny region close to the plasma-facing components, the sheath can make a large impact on the erosion of plasma-facing components. It imposes some constraints on the plasma flow to the material surfaces and sets the boundary conditions for both kinetic and fluid-based plasma codes, which are used to study different phenomena in the edge plasma, ranging from plasma transport and going to edge plasma turbulence. Finally, as we will see in Chapter VII, the effective boundary conditions at the sheath can result in specific sheath driven instabilities of the edge plasma, which might alter cross-field plasma transport and, therefore, the heat and particle fluxes on the plasma-facing components.

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