

## On the electron drift velocity in plasma devices with $E \times B$ drift

O. Chapurin, and A. Smolyakov

Citation: *Journal of Applied Physics* **119**, 243306 (2016); doi: 10.1063/1.4954994

View online: <https://doi.org/10.1063/1.4954994>

View Table of Contents: <http://aip.scitation.org/toc/jap/119/24>

Published by the [American Institute of Physics](#)

---

### Articles you may be interested in

[Tutorial: Physics and modeling of Hall thrusters](#)

*Journal of Applied Physics* **121**, 011101 (2017); 10.1063/1.4972269

[How does a probe inserted into the discharge influence the plasma structure?](#)

*Journal of Applied Physics* **119**, 183302 (2016); 10.1063/1.4948981

[Ion properties in a Hall current thruster operating at high voltage](#)

*Journal of Applied Physics* **119**, 163305 (2016); 10.1063/1.4947523

[Appropriate use of the particle-in-cell method in low temperature plasmas: Application to the simulation of negative ion extraction](#)

*Journal of Applied Physics* **120**, 213303 (2016); 10.1063/1.4971265

[Tutorial: Reactive high power impulse magnetron sputtering \(R-HiPIMS\)](#)

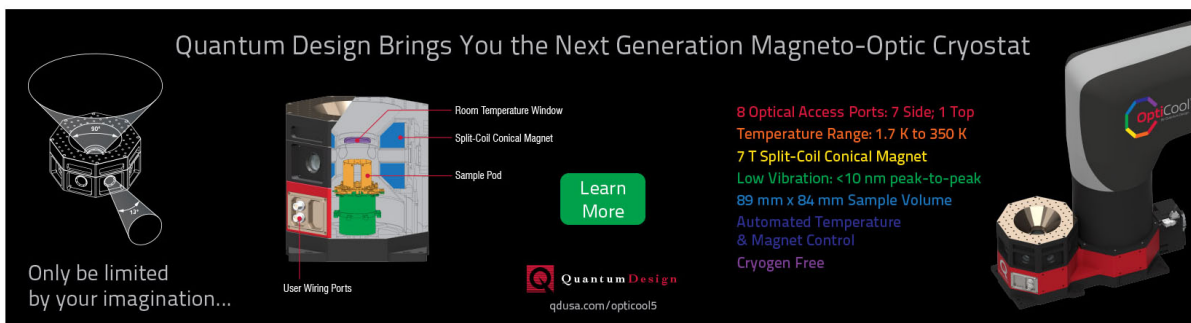
*Journal of Applied Physics* **121**, 171101 (2017); 10.1063/1.4978350

[Plasma filamentation and shock wave enhancement in microwave rockets by combining low-frequency microwaves with external magnetic field](#)

*Journal of Applied Physics* **120**, 063303 (2016); 10.1063/1.4960805

---

Quantum Design Brings You the Next Generation Magneto-Optic Cryostat



Only be limited by your imagination...

Room Temperature Window  
Split-Coil Conical Magnet  
Sample Pod  
User Wiring Ports

Learn More

Quantum Design  
qdusa.com/opticool5

8 Optical Access Ports: 7 Side; 1 Top  
Temperature Range: 1.7 K to 350 K  
7 T Split-Coil Conical Magnet  
Low Vibration: <10 nm peak-to-peak  
89 mm x 84 mm Sample Volume  
Automated Temperature & Magnet Control  
Cryogen Free

# On the electron drift velocity in plasma devices with $\mathbf{E} \times \mathbf{B}$ drift

O. Chapurin and A. Smolyakov

Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Saskatchewan S7N 5E2, Canada

(Received 8 May 2016; accepted 17 June 2016; published online 30 June 2016)

The structure and various components of the electron drift velocity are discussed in application to plasma discharges with the  $\mathbf{E} \times \mathbf{B}$  drift. In high density plasmas, the contribution of the diamagnetic drift can be of the same order magnitude as the  $\mathbf{E} \times \mathbf{B}$  drift. It is pointed out that curvature and gradient drifts associated with magnetic field inhomogeneities manifest themselves via the electron pressure anisotropy. Estimates show that the components of the diamagnetic drift related to the electron pressure anisotropy and magnetic field gradients can be important for the parameters of modern magnetrons and Hall thrusters. Similar additional terms appear in the momentum balance as mirror forces which may affect the distribution of the electrostatic potential in Hall devices. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4954994>]

## I. INTRODUCTION

Plasma discharges with the electron  $\mathbf{E} \times \mathbf{B}$  drift are widely used as ion sources in various applications such as magnetron sputtering<sup>1-3</sup> and Hall thrusters for electric propulsion.<sup>4,5</sup> The magnitude of the electron current is an important parameter characterizing the energetics of such discharges.<sup>6</sup> It has been noted that for the parameters of the modern magnetrons,<sup>1,7</sup> the currents associated with magnetic curvature and magnetic gradient drifts may be comparable or larger than that of the  $\mathbf{E} \times \mathbf{B}$  drift current. Geometry of the magnetic field and the associated magnetic field gradients also were shown to affect the distribution of the electrostatic potential<sup>5,8-10</sup> and the electron cross field transport.<sup>11,12</sup> The currents associated with magnetic gradients are ultimately related to particle kinetic energy (or thermal pressure), and thus, depend on the temperature profiles.<sup>13</sup> In this paper, we clarify the structure and the role of the diamagnetic (related to thermal pressure) contributions to the electron current and investigate the effect of the magnetic field inhomogeneities and pressure anisotropy. The role of pressure anisotropy and magnetic field gradients in the force balance is elucidated. It is pointed out that magnetic field gradients appear in the diamagnetic current and force balance in combination with pressure anisotropy.

## II. FLUID AND PARTICLE DRIFT VELOCITIES

In a strong magnetic field,  $\omega \ll \omega_{ce}$ ,  $\nu \ll \omega_{ce}$ , the velocity of the electron fluid can be found from the momentum balance equation in the form

$$m_e n \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -en(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \nabla p. \quad (1)$$

Neglecting the electron inertia on the left hand side, one gets a fluid drift velocity

$$\mathbf{V} = \mathbf{V}_E + \mathbf{V}_P + V_{\parallel} \mathbf{b} = \frac{\mathbf{E} \times \mathbf{b}}{B} - \frac{1}{enB} \mathbf{b} \times \nabla p + v_{\parallel} \mathbf{b}, \quad (2)$$

where  $\mathbf{b}$  is the unit vector along magnetic field,  $\mathbf{b} = \mathbf{B}/B$ . The first term in Eq. (2) represents the  $\mathbf{E} \times \mathbf{B}$  drift

$$\mathbf{V}_E = \frac{\mathbf{E} \times \mathbf{b}}{B}, \quad (3)$$

the second is the diamagnetic drift due to electron thermal pressure

$$\mathbf{V}_P = -\frac{1}{enB} \mathbf{b} \times \nabla p. \quad (4)$$

These are the expressions for the fluid velocity that is for the mean velocity obtained by averaging over ensemble of many particles. Note that although these expressions do not involve explicitly the magnetic field gradients, they are valid for arbitrary geometry of the magnetic field.

The guiding center drift theory<sup>14,15</sup> offers an alternative perspective on particle motion. In this theory, the equations of motion of individual particles are solved by averaging method, using the small parameter due to the strong magnetic field  $\omega \ll \omega_{ce}$ . The averaged drift (guiding center) velocity of an individual particle is given by the expression<sup>14,15</sup>

$$\begin{aligned} \mathbf{v}_D = \mathbf{v}_E - \frac{mv_{\perp}^2}{2eB} \mathbf{b} \times \frac{\nabla \mathbf{B}}{B} - \frac{mv_{\parallel}^2}{eB} \mathbf{b} \times (\mathbf{b} \cdot \nabla) \mathbf{b} \\ - \frac{mv_{\perp}^2}{2eB} \mathbf{b} (\mathbf{b} \cdot \nabla \times \mathbf{b}) + v_{\parallel} \mathbf{b}, \end{aligned} \quad (5)$$

$e > 0$ .

In this form, the guiding center drifts explicitly include the components of the velocity due to the magnetic field gradients: the gradient drift (the second term) and the curvature drift (the third term). The key difference between (2) and (5) is that the velocity in Eq. (5) is for a single particle, while Eq. (2) is the fluid drift, averaged over many particles. The relation between (2) and (5) can be found by averaging Eq. (5) over the particle distribution function

$$n \langle \mathbf{v}_D \rangle = \int \mathbf{v}_D f(\mathbf{r}, \mathbf{v}, t) d^3 v. \quad (6)$$

Averaging Equation (5) over the Maxwellian distribution and using standard vector identities, one can show that the fluid velocity in (2) and guiding center velocity (5) are related<sup>16</sup> via the magnetization current as follows:

$$n\mathbf{V} = n\langle \mathbf{v}_D \rangle - \nabla \times \mathbf{M}, \quad (7)$$

where the vector  $\mathbf{M}$  (magnetization vector) is a magnetic moment per unit of volume

$$\mathbf{M} = -\frac{cp}{eB}\mathbf{b}. \quad (8)$$

We have used also

$$m\langle v_{\parallel}^2 \rangle = T; \quad m\langle v_{\perp}^2 \rangle = 2T; \quad \langle v_{\parallel} \rangle = V_{\parallel}.$$

Therefore, the curvature and gradient drift (guiding center drifts) and diamagnetic drift are different representations of the same phenomena. The curvature and gradient drifts cannot be added to the diamagnetic drift contrary to the assertion made in Ref. 1. For the purposes of calculation of the electron current (fluid drift) velocity, the expression given in Eq. (2) is complete and is valid for arbitrary profile of the magnetic field.

### III. ANISOTROPIC PLASMA PRESSURE EFFECTS

Plasma confined by strong magnetic field may develop pressure anisotropy in the directions parallel and perpendicular to the magnetic field,  $p_{\parallel} \neq p_{\perp}$ . Such anisotropy may also be related to the particles' losses along the magnetic field into the plasma boundary sheath as well as secondary electron emission effects.<sup>17-19</sup> Anisotropic pressure adds additional terms to the fluid drifts, and some of these additional contribution explicitly depend on the magnetic field gradients. The particle drifts in case of anisotropic pressure can be derived either by direct averaging of the guiding center equations or using the definitions of parallel and perpendicular plasma pressure

$$p_{\parallel} = m \int v_{\parallel}^2 f d^3v, \quad (9)$$

$$p_{\perp} = \frac{m}{2} \int v_{\perp}^2 f d^3v, \quad (10)$$

$$p = \frac{2p_{\perp} + p_{\parallel}}{3} = \frac{m}{3} \int (v_{\perp}^2 + v_{\parallel}^2) f d^3v. \quad (11)$$

Alternatively, one can use the momentum balance equation which includes the electron stress tensor

$$0 = -en(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla p - \nabla \cdot \mathbf{\Pi}, \quad (12)$$

where  $\mathbf{\Pi}$  is the stress (viscosity) tensor in the CGL form<sup>20</sup>

$$\mathbf{\Pi} = (p_{\parallel} - p_{\perp}) \left( \mathbf{b}\mathbf{b} - \frac{1}{3}\mathbf{I} \right). \quad (13)$$

The contribution of the stress tensor to the electron drift velocity has the same nature as the diamagnetic drift and has to be added to the fluid drift. From Eq. (12), one has

$$\mathbf{V} = \mathbf{V}_E + \mathbf{V}_p + \mathbf{V}_{\pi}, \quad (14)$$

where the stress tensor contribution is

$$\mathbf{V}_{\pi} = -\frac{1}{enB^2} \mathbf{B} \times \nabla \cdot \mathbf{\Pi}. \quad (15)$$

Using (13) one finds the divergence of the stress tensor in the form

$$\begin{aligned} \nabla \cdot \mathbf{\Pi} &= (p_{\parallel} - p_{\perp})(\mathbf{b}\nabla \cdot \mathbf{b} + \mathbf{b} \cdot \nabla \mathbf{b}) \\ &+ \mathbf{b}(\mathbf{b} \cdot \nabla(p_{\parallel} - p_{\perp})) - \frac{1}{3}\nabla(p_{\parallel} - p_{\perp}), \end{aligned} \quad (16)$$

and

$$\begin{aligned} \mathbf{B} \times \nabla \cdot \mathbf{\Pi} &= (p_{\parallel} - p_{\perp})\mathbf{B} \times (\mathbf{b} \cdot \nabla)\mathbf{b} - \frac{1}{2}\mathbf{B} \times \pi_{\parallel} \\ &= (p_{\parallel} - p_{\perp})\mathbf{B} \times \nabla_{\perp} \ln B - \frac{1}{3}\mathbf{B} \times \nabla(p_{\parallel} - p_{\perp}). \end{aligned} \quad (17)$$

Here, we have used  $\mathbf{b} \cdot \nabla \mathbf{b} \simeq \nabla_{\perp} \ln B$  valid for low plasma pressure case when the magnetic field can be assumed to be the vacuum field,  $\nabla \times \mathbf{B} \simeq 0$ .

Combining all terms, we obtain for the drift velocity

$$\begin{aligned} \mathbf{V}_{\perp} &= \frac{\mathbf{E} \times \mathbf{b}}{B} - \frac{1}{enB}\mathbf{b} \times \nabla p - \frac{(p_{\parallel} - p_{\perp})}{enB^2}\mathbf{B} \times \nabla_{\perp} \ln B \\ &+ \frac{1}{3enB^2}\mathbf{B} \times \nabla(p_{\parallel} - p_{\perp}). \end{aligned} \quad (18)$$

This expression also can be written in the form

$$\mathbf{V}_{\perp} = \mathbf{V}_E - \frac{1}{enB^2}\mathbf{B} \times \nabla p_{\perp} - \frac{1}{enB^2}(p_{\parallel} - p_{\perp})\mathbf{B} \times \nabla_{\perp} \ln B. \quad (19)$$

The additional terms in Eq. (18) (the third and fourth terms) due to pressure anisotropy are of the same nature as the standard diamagnetic drift (the second term) and thus are components of the total diamagnetic drift.

### IV. ANISOTROPIC PRESSURE AND THE FORCE BALANCE

The effects of the anisotropic pressure also manifest themselves in the force (momentum) balance and in energy equations. The full system of continuity, momentum, and energy balance equations that take into account full diamagnetic terms, anisotropic pressure, and non-uniform magnetic field can be derived from the lowest order drift-kinetic equation in the form

$$\frac{\partial f}{\partial t} + \nabla \cdot \left( \frac{d\mathbf{R}}{dt} f \right) + \frac{\partial}{\partial v_{\parallel}} \left( \frac{dv_{\parallel}}{dt} f \right) + \frac{\partial}{\partial v_{\perp}^2} \left( \frac{dv_{\perp}^2}{dt} f \right) = 0, \quad (20)$$

where  $\mathbf{R}$  is the guiding center coordinate. The drift equations written for the case of low pressure plasma have the form<sup>14</sup>

$$\frac{d\mathbf{R}}{dt} = v_{\parallel}\mathbf{b} + \mathbf{v}_E + \mathbf{v}_d, \quad (21)$$

$$\frac{dv_{\parallel}}{dt} = \frac{q}{m} \mathbf{E} \cdot \mathbf{b} + \frac{1}{2} v_{\perp}^2 \nabla \cdot \mathbf{b} + v_{\parallel} \mathbf{v}_E \cdot \nabla \ln B, \quad (22)$$

$$\frac{dv_{\perp}^2}{dt} = -v_{\perp}^2 v_{\parallel} \nabla \cdot \mathbf{b} + v_{\perp}^2 \mathbf{v}_E \cdot \nabla \ln B. \quad (23)$$

These equations conserve the phase space volume as follows from Hamiltonian nature of this system:

$$\nabla \cdot \left( \frac{d\mathbf{R}}{dt} \right) + \frac{\partial}{\partial v_{\parallel}} \left( \frac{dv_{\parallel}}{dt} \right) + \frac{\partial}{\partial v_{\perp}^2} \left( \frac{dv_{\perp}^2}{dt} \right) = 0. \quad (24)$$

Taking moments of Eq. (20) with the  $(1, \mathbf{v}, \mathbf{v}^2)$  weights, one obtains the reduced continuity, momentum balance, and energy equations, respectively. The full system of these equations is given in Ref. 21.

It was shown that the mirror force may affect the potential distribution in the Hall thruster.<sup>9</sup> The mirror force is related to the second term on the right hand side of Equation (22). However, the equation for the evolution of the perpendicular energy (23) has a related term to maintain the conservation of the phase space: this is the first term on the right hand side of Equation (23). Thus, the mirror force in the parallel momentum balance equation appears from both terms, in Eqs. (22) and (23), corresponding to the stress tensor<sup>21</sup>

$$m_e n \frac{dV_{\parallel}}{dt} = -enE_{\parallel} - \nabla_{\parallel} p - \mathbf{b} \cdot \nabla \cdot \mathbf{\Pi}, \quad (25)$$

where  $\nabla_{\parallel} = \mathbf{b} \cdot \nabla$  is the parallel gradient operator and  $E_{\parallel} = \mathbf{b} \cdot \mathbf{E}$ . Using the expression (13), one finds

$$\mathbf{b} \cdot \nabla \cdot \mathbf{\Pi} = -(p_{\parallel} - p_{\perp}) \mathbf{b} \cdot \nabla \ln B + \mathbf{b} \cdot \nabla (p_{\parallel} - p_{\perp}), \quad (26)$$

and

$$m_e n \frac{dV_{\parallel}}{dt} = -enE_{\parallel} - \nabla_{\parallel} p_{\parallel} + (p_{\parallel} - p_{\perp}) \mathbf{b} \cdot \nabla \ln B. \quad (27)$$

This expression shows that the mirror force in the parallel momentum balance manifests itself via the pressure anisotropy.

It is of interest to look at the total force balance in quasi-neutral plasmas. For unmagnetized cold ions, the respective momentum balance equation has the form

$$m_i n (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i = en(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}). \quad (28)$$

We have neglected here the ion pressure and ion viscous stress,  $\nabla \cdot \mathbf{\Pi}_i$  and assumed the stationary state,  $\partial/\partial t = 0$ . Equation (28) emphasizes the electrostatic mechanism of ion acceleration. Note that Equation (28) does include the Lorentz force effects on ions, though small it can be noticeable in some applications, e.g., in Hall thrusters. The only assumption made in Eq. (28) is neglecting of the thermal pressure and thermal spread of the distribution function; the latter would lead to the finite ion viscosity term. The respective electron equation is

$$m_e n (\mathbf{V}_e \cdot \nabla) \mathbf{V}_e = -en(\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) - \nabla p_e - \nabla \cdot \mathbf{\Pi}, \quad (29)$$

which now includes the pressure and viscosity of finite temperature electrons. Summing the electron and ion equations,

one has the total momentum balance for the quasineutral plasma in the following form:

$$m_e n (\mathbf{V}_e \cdot \nabla) \mathbf{V}_e + m_i n (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i = \mathbf{J} \times \mathbf{B} - \nabla p_e - \nabla \cdot \mathbf{\Pi}. \quad (30)$$

From (13), one can find the following expression for the force due to pressure anisotropy:

$$\begin{aligned} \nabla \cdot \mathbf{\Pi} &= (p_{\parallel} - p_{\perp}) (\mathbf{b} \nabla \cdot \mathbf{b} + \mathbf{b} \cdot \nabla \mathbf{b}) \\ &+ \mathbf{b} (\mathbf{b} \cdot \nabla (p_{\parallel} - p_{\perp})) - \frac{1}{3} \nabla (p_{\parallel} - p_{\perp}). \end{aligned} \quad (31)$$

The momentum balance in the form of Equation (30) emphasizes the electromagnetic nature of the ion acceleration in Hall devices.<sup>23</sup> The momentum deposited into the ions,  $m_i n (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i$  ultimately comes from the  $\mathbf{J} \times \mathbf{B}$  force due to the interaction of the plasma current and the magnetic field created by the magnetic coils. It is this force that is transmitted to the hardware of the Hall thruster. In situations when the electron diamagnetic current (including the anisotropic components) is large, the diamagnetic forces ultimately affect the discharge characteristics, in particular, the ion acceleration as in Hall thrusters. It is worth noting that the electron inertial forces,  $m_e n (\mathbf{V}_e \cdot \nabla) \mathbf{V}_e$ , in particular, the centrifugal radial force may also affect the ion acceleration and deflection for supersonic rotation of electrons.<sup>24</sup>

## V. SUMMARY

The importance of the diamagnetic, curvature, and magnetic gradient drifts in modern magnetrons was noted in Ref. 1, where it has been shown that the electron drift velocity associated with diamagnetic terms and magnetic field inhomogeneities can reach or exceed the  $\mathbf{E} \times \mathbf{B}$  velocity. In this article, we point out that the standard diamagnetic drift (due to the isotropic pressure gradient) and curvature/gradient drifts are different representation of the same phenomena, and, for Maxwellian plasma, do not explicitly involve magnetic field gradients. In a general case, with anisotropic pressure, the diamagnetic drift as given by the last three terms in Equation (18) gives the full account of the thermal drifts. This also includes the additional term due to the pressure anisotropy that explicitly involves the magnetic gradients. Such terms are additive to the standard diamagnetic drift and may be important for plasmas with the pressure anisotropy which may develop due to the boundary sheath losses along the magnetic field, secondary electron emission, and electron acceleration in the sheath due to biasing. Elucidation of the role of the pressure anisotropy in the electron drift velocity and momentum balance is one of the main results of this paper.

The pressure anisotropy in combination with magnetic gradients provides additional contribution to the momentum balance and thus will affect the potential distribution in Hall thrusters.<sup>5,9,10</sup> Such modification may violate the simple assumption of the thermalized potential.<sup>22</sup> The anisotropic pressure contribution (the last term in Eq. (27)) has to be included to describe the variation of the electric potential along the magnetic field lines.

For typical HPIMS magnetron parameters, such as in Refs. 1 and 7, the  $\mathbf{E} \times \mathbf{B}$  drift velocity was estimated to be of the order of  $1 \div 2 \times 10^5$  m/s, for  $B = 60$  mT and  $E = 10^4$  V/m. For the characteristic length scale of the magnetic field inhomogeneity  $R = 10$  mm,  $R \simeq |\nabla B/B|^{-1}$ , the magnitude of the curvature and gradient drift  $v_d \simeq m_e v^2 / (eB) \nabla \ln B = T / (eBR) \simeq v_{Te}^2 / (\omega_{ce} R)$ , will be small for the relatively cold bulk plasma with the electron temperature of few eV, e.g., for  $T_e = 1$  eV and  $B = 60$  mT,  $v_d \simeq 1.6 \times 10^3$  m/s. The standard diamagnetic velocity for these parameters  $V_p = 1 / (enB) \nabla p_e$  will be of the same order if one assumes the pressure gradient length scale to be of the order of  $R$ ,  $|\nabla p/p| \simeq R^{-1}$ . However, for the highly energetic electrons accelerated by the voltage drop in the sheath, the general diamagnetic drift can be much higher and comparable or larger than the  $\mathbf{E} \times \mathbf{B}$  drift velocity. Thus for  $V_e \simeq 500$  eV, the diamagnetic drift would be two orders of magnitude higher than the value of  $v_d \simeq 1.6 \times 10^3$  m/s for cold  $T_e = 1$  eV plasma. The pressure of the electrons accelerated in the biased sheath is likely to be highly anisotropic. As noted above, the diamagnetic contributions from such high energy electrons have to be calculated taking into account pressure anisotropy and magnetic field gradients, as shown by two last terms in Equation (19).

For the parameters of the Hall thruster in Ref. 13, at the location of the maximum of the magnetic field,  $B = 90$  G and  $E = 10^4$  V/m, and the  $\mathbf{E} \times \mathbf{B}$  drift velocity is of the order of  $1 \times 10^6$  m/s. At this location, the electron temperature  $T_e \simeq 20$  eV and the characteristic pressure gradient  $R \simeq |\nabla p_e / p_e|^{-1} \simeq 10$  mm. For these parameters, the diamagnetic drift velocity  $V_p = 1 / (enB) \nabla p_e \simeq 2 \times 10^5$  m/s is smaller than the  $\mathbf{E} \times \mathbf{B}$  drift. However, closer to the anode, the electric field drops faster than the electron temperature and the diamagnetic drift velocity quickly becomes larger than the  $\mathbf{E} \times \mathbf{B}$  drift. The important point is that the electron distribution function in Hall thruster can be strongly anisotropic due to the directional injection of electrons from the cathode<sup>13,25</sup> as well as due to secondary electron emission and sheath depletion.<sup>17,18</sup> Therefore, the total diamagnetic drift in these conditions should be evaluated taking into account the full pressure anisotropy and magnetic gradients, as given in Equation (19).

The above estimates show that pressure anisotropy and magnetic field gradients should be fully included in

calculations of the total electron drift velocity for typical experimental conditions and have to be incorporated into the fluid models simulations.<sup>4,26</sup>

## ACKNOWLEDGMENTS

The authors are grateful to Y. Raitses for useful discussions. This work was supported in part by NSERC Canada and U.S. Air Force Office for Scientific Research FA9550-15-1-0226.

<sup>1</sup>A. Rauch and A. Anders, *Vacuum* **89**, 53 (2013).

<sup>2</sup>M. Panjan, S. Loquai, J. E. Klemberg-Sapieha, and L. Martinu, *Plasma Sources Sci. Technol.* **24**, 065010 (2015).

<sup>3</sup>T. Ito, C. V. Young, and M. A. Cappelli, *Appl. Phys. Lett.* **106**, 254104 (2015).

<sup>4</sup>L. Garrigues *et al.*, *IEEE Trans. Plasma Sci.* **36**, 2034 (2008).

<sup>5</sup>M. Keidar, A. D. Gallimore, Y. Raitses, and I. D. Boyd, *Appl. Phys. Lett.* **85**, 2481 (2004).

<sup>6</sup>J. W. Bradley, S. Thompson, and Y. A. Gonzalvo, *Plasma Sources Sci. Technol.* **10**, 490 (2001).

<sup>7</sup>A. Anders, P. Ni, and A. Rauch, *J. Appl. Phys.* **111**, 053304 (2012).

<sup>8</sup>Y. J. Choi, I. D. Boyd, and M. Keidar, *J. Appl. Phys.* **105**, 013303 (2009).

<sup>9</sup>M. Keidar and I. D. Boyd, *Appl. Phys. Lett.* **87**, 121501 (2005).

<sup>10</sup>M. Keidar and I. D. Boyd, *J. Appl. Phys.* **86**, 4786 (1999).

<sup>11</sup>D. Tang *et al.*, *J. Appl. Phys.* **102**, 123305 (2007).

<sup>12</sup>Y. Raitses, A. Smirnov, and N. J. Fisch, *J. Appl. Phys.* **104**, 066102 (2008).

<sup>13</sup>D. Staack, Y. Raitses, and N. J. Fisch, *Appl. Phys. Lett.* **84**, 3028 (2004).

<sup>14</sup>D. Sivukhin, *Reviews of Plasma Physics* (Consultants Bureau, New York, 1963), Vol. 1, p. 1.

<sup>15</sup>A. Morozov and L. Solovov, *Reviews of Plasma Physics* (Consultants Bureau, New York, 1966), Vol. 2, p. 201.

<sup>16</sup>S. Braginskii, *Reviews of Plasma Physics* (Consultants Bureau, New York, 1965), Vol. 1, p. 205.

<sup>17</sup>I. D. Kaganovich, Y. Raitses, D. Sydorenko, and A. Smolyakov, *Phys. Plasmas* **14**, 057104 (2007).

<sup>18</sup>D. Sydorenko, A. Smolyakov, I. Kaganovich, and Y. Raitses, *Phys. Plasmas* **14**, 013508 (2007).

<sup>19</sup>D. Sydorenko, A. Smolyakov, I. Kaganovich, and Y. Raitses, *IEEE Trans. Plasma Sci.* **34**, 815 (2006).

<sup>20</sup>G. F. Chew, M. L. Goldberger, and F. E. Low, *Proc. R. Soc. London, Ser. A* **236**, 112 (1956).

<sup>21</sup>A. I. Smolyakov and X. Garbet, *Phys. Plasmas* **17**, 042105 (2010).

<sup>22</sup>A. Morozov and V. Savelyev, *Reviews of Plasma Physics* (Kluwer, New York, 2000), Vol. 21, p. 203.

<sup>23</sup>V. P. Kim, *Tech. Phys.* **60**, 362 (2015).

<sup>24</sup>N. J. Fisch, Y. Raitses, and A. Fruchtman, *Plasma Phys. Controlled Fusion* **53**, 124038 (2011).

<sup>25</sup>V. Y. Fedotov, A. A. Ivanov, G. Guerrini, A. N. Vesselovzorov, and M. Bacal, *Phys. Plasmas* **6**, 4360 (1999).

<sup>26</sup>G. J. M. Hagelaar, J. Bareilles, L. Garrigues, and J. P. Boeuf, *Contrib. Plasma Phys.* **44**, 529 (2004).