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On nonlinear effects in inductively coupled plasmas

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Nonlinear current and potential oscillations in low pressure inductively coupled plasmas are analyzed within the framework of electron magnetohydrodynamics. It is shown that both current and potential oscillations can be attributed, respectively, to the solenoidal and potential components of nonlinear Lorentz and inertial forces. Scaling of the nonlinear force with the phase shift between the electric current and the electric field is analyzed. It is demonstrated that the solenoidal part of the force that provides a source of nonlinear current vanishes in neglect of collisions and collisionless absorption, while oscillations of the electrostatic potential remain finite. It is shown that these oscillations are the result of plasma polarization due to Hall drifts. © *2000 American Institute of Physics.* [S1070-664X(00)01311-2]

I. INTRODUCTION

Low pressure inductively coupled discharges have recently emerged as promising plasma sources for material processing and lighting technologies.[1,2] The operation of such plasma sources in low frequency and low pressure regimes reveals a number of interesting properties of such discharges. The effects of anomalous electromagnetic field penetration, collisionless heating, and negative energy absorption have recently attracted a great deal of experimental and theoretical investigations.[3–22] One of the main reasons for such rich underlying physics is the nonlocal nature of low pressure inductively coupled plasma (ICP) discharges. In particular, it turns out that such plasmas are almost collisionless, with the electron mean free path exceeding the length of a system. In such situations, electron thermal motion becomes important and leads to such phenomena as the anomalous skin effect and collisionless absorption. Recent experiments have also shown that nonlinear effects are important in such a plasma and give rise to nonlinear harmonics in current,[23] plasma potential[6,24] and enhanced penetration of the electromagnetic field.[5,8,9] In this paper we analyze possible sources of nonlinearity of low pressure ICP. Note that the effects considered in this paper are due to electron dynamics and thus are complementary to nonlinear effects of ions dynamics, such as those in Ref. 25.

A modern low pressure ICP is characterized by a unique set of operating parameters. In addition to the above noted collisionless (nonlocal) nature of the electron component [when inequalities $\omega \gtrsim \nu$, $kv_{th} \gtrsim (\nu,\omega)$ are satisfied], the influence of the induced rf (radio frequency) magnetic field B becomes important. The effective electron cyclotron frequency, eB/mc, in the induced magnetic field B may exceed the electron-neutral collisional frequency ν and characteristic frequency of the oscillations $\omega \simeq B^{-1}\partial B/\partial t$ by an order of magnitude. At low frequencies, the nonlinear Lorentz force acting on electrons may become much larger than the force from the inductive electric field.[23] Under these conditions plasmas are in the regime of electron (Hall) magnetohydrodynamics (EMHD),[26] which has been traditionally applied to high-density high-frequency phenomena in plasmas typical for high power pulsed systems. The low pressure ICP turns out to be even more complex because of the effects of thermal particle motion, which are important for ICP but were typically neglected in EMHD.[26]

In this paper we concentrate on nonlinear effects due to the modification of Ohm's law by the Lorentz force and nonlinear inertia. Nonlinear electron inertia is important when the characteristic length scale k^{-1} becomes comparable to the collisionless electron skin depth, $k^2c^2/\omega_{pe}^2 \simeq 1$ (this condition is typically satisfied in ICP). Both types of nonlinearity (Lorentz force and nonlinear inertia) contribute to the nonlinear force acting on the plasma due to the fluctuating rf field. It has been shown recently[19,20] that ponderomotive and second harmonic plasma potentials are nonlinearly excited due to the action of nonlinear forces associated with the rf magnetic field. We show in this paper, that both, the nonlinear potential and the nonlinear current are produced as a result of nonlinear forces. We also investigate the structure of these forces and analyze their scaling with respect to the collision frequency.

We describe the electron component by the following momentum equation:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{e}{m}\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right] - \nu\mathbf{v}, \tag{1}$$

where ν is the collisional frequencies due to electron-neutral interaction, $e>0$.

We assume that ions are immobile so that the electric current is due to the electron motion only:

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}=-\frac{4\pi}{c}en\mathbf{v}. \tag{2}$$

These two coupled equations (1) and (2) form a nonlinear electron magnetohydrodynamic system. It is often convenient to write the electron equation of motion (1) in the form of the conservation of the generalized momentum $\mathbf{p}=m\mathbf{v}-e\mathbf{A}/c$, where $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B}=\nabla\times\mathbf{A}$. Taking the curl of (1), one obtains identically[26]

$$\frac{\partial}{\partial t}\nabla\times\mathbf{p}-\nabla\times(\mathbf{v}\times\nabla\times\mathbf{p})=-m\nabla\times(\nu\mathbf{v}). \tag{3}$$

The left-hand side of this equation has a structure similar to the equation for vorticity conservation in an ideal fluid. In our case, the vorticity $\nabla\times\mathbf{p}$ of the generalized momentum is conserved in the absence of dissipative effects ($\nu=0$). In other words, for $\nu=0$, the vorticity $\nabla\times\mathbf{p}$ is simply convected by electron flow so that the flux of the vorticity through any closed contour that moves with the electron fluid remains constant, i.e., the vorticity is ''frozen-in'' the electron fluid. In the limit $k^2c^2/\omega_{pe}^2\ll 1$, when the electron inertia is not important, $m\mathbf{v}\ll e\mathbf{A}/c$, the generalized vorticity is proportional to the magnetic field, $\nabla\times\mathbf{p}\simeq -e\mathbf{B}/\mathbf{c}$. Then, Eq. (3) describes the convection of the magnetic field by electron flow (second term on the left-hand side) and the diffusion of the magnetic field through the plasma due to finite ν [the right-hand side of Eq. (3)].

II. NONLINEAR EFFECTS IN CYLINDRICAL ICP

A. Magnetic field and nonlinear polarization electric field

The excitation of oscillatory and dc components of the electric field due to the nonlinear effects of an rf magnetic field was analyzed in Refs. 19 and 20 for a semi-infinite slab model of ICP. In this section we demonstrate that the dynamics of an axial rf magnetic field are not affected by nonlinearity in one-dimensional cylindrical ICP, and the generation of the radial polarization field which has time average (dc) and second harmonic components is the only nonlinear effect.

We consider an infinitely long cylindrical plasma with an applied external rf magnetic field in the axial direction, $\mathbf{B}=B_z(r)\hat{\mathbf{z}}$. The inductive electric field $\mathbf{E}$ and the electron velocity are both in the azimuthal direction, $\mathbf{E}=E_\phi\hat{\phi}$, $\mathbf{v}=\mathbf{v}_\phi\hat{\phi}$, where $\hat{\mathbf{z}}$ and $\hat{\phi}$ are respective unit vectors. We also assume axial $\partial/\partial z=0$, and azimuthal $\partial/\partial\phi=0$ symmetries for all parameters. The plasma density and magnetic field, may vary in the radial direction. Under such conditions, the equation for B_z remains linear and no other components of the magnetic field are excited in a one-dimensional cylindrical ICP. This is easily shown by noting that in this geometry, both nonlinear terms in (1), $\mathbf{v}\cdot\nabla\mathbf{v}$ and $\mathbf{v}\times\mathbf{B}$, are curl-free vectors, so that no nonlinear terms occur in Eq. (3) for the magnetic field B_z after applying $\nabla\times$ to Eq. (1). From Eq. (3) one finds a linear equation for B_z

$$\frac{\partial}{\partial t}\left[B_z-\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{c^2}{\omega_{pe}^2}r\frac{\partial B_z}{\partial r}\right)\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\nu c^2}{\omega_{pe}^2}\frac{\partial B_z}{\partial r}\right), \tag{4}$$

where $\omega_{pe}^2(r)=4\pi e^2n(r)/m$, is the local value of the electron plasma frequency, $n(r)$ is the plasma density, and m is the electron mass.

In the one-dimensional ICP discharge, the only nonlinear effect is plasma potential oscillations that occur due to plasma polarization from the action of a nonlinear force.[19,20] Note that the nonlinear polarization field will be accompanied by a modification of the plasma density profile.[27,28] Plasma charge (and electric field) arise to compensate the nonlinear force applied in the radial direction. The equation for the nonlinear polarization field $\mathbf{E}^p=-\nabla\Phi^p$ is obtained by taking the divergence of (1):

$$\begin{aligned}\nabla^2\Phi^p&=\nabla\cdot\left(\frac{m}{e}\mathbf{v}\cdot\nabla\mathbf{v}+\frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\\&=\frac{e}{m\omega_{pe}^2}\frac{1}{r}\frac{\partial}{\partial r}r\left(\frac{\partial}{\partial r}\frac{B_z^2}{2}-\frac{m}{e}\frac{v_\phi^2}{r}\right)\\&=\frac{e}{m\omega_{pe}^2}\frac{1}{r}\frac{\partial}{\partial r}r\left(\frac{\partial}{\partial r}\frac{B_z^2}{2}-\frac{c^2}{\omega_{pe}^2}\frac{1}{r}\left(\frac{\partial B_z}{\partial r}\right)^2\right).\end{aligned} \tag{5}$$

The first term on the right-hand side is a Lorentz or magnetic pressure force, and the second is an inertial (centrifugal) force. We note that the centrifugal force was included in the calculation of ponderomotive forces in Ref. 27. It is easy to see that the second term is operative only when there is curvature of the electric field which yields the curvature of the electron flow; this term vanishes in the slab approximation when the electric field lines are straight. The first term, due to the magnetic pressure, is present even in the slab approximation when the wave amplitude varies in the radial (r) direction. The curvature term always reduces the magnitude of the magnetic pressure gradient.[27] Assuming harmonic time dependence of B_z, $B_z\sim\exp(i\omega t)$, one can see from Eq. (5) that the polarization field has time average (ponderomotive) and second harmonic components.

B. Nature of the polarization electric field in cylindrical ICP

The nature of nonlinear polarization fields in inductively coupled plasma is closely related to the Hall drift of electrons in the rf magnetic field. It also bears upon the interesting fact that the penetration of the external electromagnetic wave into the one-dimensional ICP is not affected by a dc magnetic field. To analyze this in greater detail, it is instructive to consider the penetration of electromagnetic fields into a one-dimensional cylindrical ICP with an external stationary (dc) axial magnetic field B_0 in the z direction. The rf magnetic field B_z is also applied in the z direction, so the total magnetic field is $\mathbf{B}=(B_0+B_z)\hat{\mathbf{z}}$, and the inductive electric field (generated by the external rf magnetic field) is in the $\hat{\phi}$

direction, $\mathbf{E}=E_\phi\hat{\boldsymbol{\phi}}$. This field configuration corresponds to the polarization in the extraordinary electromagnetic wave.

The polarization effect already occurs in the linear case. In this section, we consider electron motion in the linear approximation and neglect effects of the oscillating magnetic field:

$$i\omega m v_\phi=-\frac{e}{m}E_\phi+\frac{e}{mc}v_r B_0-\nu v_\phi, \tag{6}$$

$$i\omega m v_r=-\frac{e}{m}E_r-\frac{e}{mc}v_\phi B_0-\nu v_r. \tag{7}$$

Solving (7) and (6) one obtains

$$v_r=-\frac{e}{m}\frac{E_r(i\omega+\nu)-E_\phi\Omega_c}{\Omega_c^2+(i\omega+\nu)^2}, \tag{8}$$

$$v_\phi=-\frac{e}{m}\frac{E_\phi(i\omega+\nu)+E_r\Omega_c}{\Omega_c^2+(i\omega+\nu)^2}, \tag{9}$$

where $\Omega_c=eB_0/mc$. The first terms in Eqs. (8) and (9) are due to the Pedersen conductivity representing the conductivity along the electric field, the second terms are due to the Hall conductivity describing the electric current in the direction perpendicular to both the electric field and the magnetic field. The electric current can be written in a vector form[29]

$$\mathbf{J}=\sigma_P\mathbf{E}+\sigma_H\mathbf{E}\times\mathbf{b}, \tag{10}$$

where $\mathbf{b}=\mathbf{B}/B$ is the unit vector along the magnetic field; it is assumed in (10) that $\mathbf{E}\perp\mathbf{b}$. In the low frequency case, $\omega \ll(\nu,\Omega_c)$, the standard expressions for the Pedersen and Hall conductivities are $\sigma_P=\sigma_0/(1+\Omega_c^2/\nu^2)$, and $\sigma_H=\sigma_0(\Omega_c/\nu)/(1+\Omega_c^2/\nu^2)$, where $\sigma_0=e^2n/m\nu$ is the collisional plasma conductivity. One might expect that the modification of plasma conductivity by the dc magnetic field may modify the skin-effect. In what follows we show that the external dc magnetic field does not modify the penetration of the rf field into the plasma because of the mutual compensation of Pedersen and Hall effects in the azimuthal direction.

In our calculations we neglect the displacement current. This assumption, which is critical for our conclusions, is well justified for typical ICP parameters. Then, from Maxwell equations one obtains

$$\nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\mathbf{J}. \tag{11}$$

Taking the radial component of this equation one finds that the left-hand side vanishes identically,

$$\nabla(\nabla\cdot\mathbf{E})-\nabla^2\mathbf{E}=\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rE_r)\right]-\left[\nabla^2E_r-\frac{E_r}{r^2}\right]=0, \tag{12}$$

where $\nabla^2(\cdots)=r^{-1}\partial(r(\cdots)/\partial r)$. Equations (11) and (12) imply that the radial current on the right-hand side of (11) must be equal to zero, $J_r=0$. The absence of radial current is quite an obvious result for the one-dimensional (with azimuthal and axial symmetry) configuration when there is no displacement current. To compensate the radial Hall current due to the E_ϕ field and thus make $J_r=0$, a finite radial electric field E_r must be generated. This is the polarization field. We can find it from (8) by taking into account that $J_r=0$:

$$E_r(i\omega+\nu)-E_\phi\Omega_c=0. \tag{13}$$

Using this expression for the radial electric field in (9), one obtains that the net current in the $\hat{\boldsymbol{\phi}}$ direction is not changed in the presence of a dc magnetic field

$$v_\phi=-\frac{e}{m}\frac{E_\phi(i\omega+\nu)+E_r\Omega_c}{\Omega_c^2+(i\omega+\nu)^2}=-\frac{e}{m}\frac{E_\phi}{(i\omega+\nu)}. \tag{14}$$

As a result, the penetration of the rf magnetic field into the plasma (skin-effect) is not affected by the dc magnetic field, because the Pedersen modification of the azimuthal current due to the dc magnetic field is cancelled by the Hall current from the induced radial polarization field E_r.

In an ICP, where the electric field E_ϕ and magnetic field B_z are coupled, the induced rf magnetic field can play a role of the dc field in the above arguments. Then, the radial polarization electric field becomes nonlinear. Quantitatively, one can estimate the magnitude of the polarization field from (14) and (13) by replacing Ω_c with its counterpart due to the rf magnetic field, $\Omega_c\to eB_z/mc$:

$$E_r=\frac{1}{i\omega+\nu}E_\phi\frac{eB_z}{mc}=-\frac{1}{c}B_z v_\phi=-\frac{1}{4\pi en}\frac{\partial}{\partial r}\frac{B_z^2}{2}, \tag{15}$$

where $v_\phi=c/(4\pi en)\partial B_z/\partial r$ was used in the last step.

In fact, Eq. (13) corresponds to the radial component of the equation of motion (1) which reads

$$m\frac{\partial v_r}{\partial t}+m(\mathbf{v}\cdot\nabla\mathbf{v})_r=-eE_r-\frac{e}{c}v_\phi B_z-\nu m v_r. \tag{16}$$

Using the condition of zero radial current, $v_r=0$, one obtains for the radial polarization field [compare to (5)]

$$E_r=-\frac{e}{m\omega_{pe}^2}\left(\frac{\partial}{\partial r}\frac{B_z^2}{2}-\frac{c^2}{\omega_{pe}^2}\frac{1}{r}\left(\frac{\partial B_z}{\partial r}\right)^2\right). \tag{17}$$

The last term here is due to nonlinear inertia which has been neglected in (13). The radial polarization field has time average (dc) and second harmonic components. In Sec. IV we show, that more generally, the induced polarization field can be viewed as the potential part of the Lorentz and inertial forces.

III. NONLINEAR MAGNETIC FIELD AND POLARIZATION ELECTRIC FIELD IN A PLANAR ICP

In a planar two-dimensional ICP discharge (pancake geometry) we have two magnetic field components created by the external coil, $\mathbf{B}=B_r(r,z)\hat{\mathbf{r}}+B_z(r,z)\hat{\mathbf{z}}$. The primary inductive electric field is in the azimuthal direction, $\mathbf{E}=E\hat{\boldsymbol{\phi}}$. We show in this section that nonlinear effects result in the azimuthal magnetic field $B_\phi(r,z)$ and polarization potential $\Phi^p(r,z)$ that are generated at the second harmonic of a driving frequency.

The equation for the magnetic field (3) can be written as

$$\frac{\partial}{\partial t}\mathbf{B}+\frac{\partial}{\partial t}\nabla\times\left(\frac{c^2}{\omega_{pe}^2}\nabla\times\mathbf{B}\right)+\nabla\times\left(\frac{c^2}{\omega_{pe}^2}\nabla\times\mathbf{B}\right)$$
$$=\nabla\times(\mathbf{v}\times\mathbf{B})+\frac{m_e c}{e}\nabla\times((\mathbf{v}\cdot\nabla)\mathbf{v}). \quad (18)$$

As one can observe from this equation, there are two possible sources of nonlinear effects here: the Lorentz force, $\mathbf{v}\times\mathbf{B}$, and the nonlinear inertial force associated with electron flow along curved trajectories (centrifugal force), $m\mathbf{v}\cdot\nabla\mathbf{v}$. The last term on the left-hand side describes the resistive diffusion of the magnetic field, and the second term on the left-hand side is associated with linear electron inertia effects.

Taking the ϕ component of (19) one obtains

$$\frac{\partial}{\partial t}B_\phi-\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial}{\partial t}+\nu\right)\left(\nabla^2 B_\phi-\frac{B_\phi}{r^2}\right)$$
$$+\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial}{\partial t}+\nu\right)\left(\kappa_z\frac{\partial B_\phi}{\partial z}+\kappa_r\frac{\partial(rB_\phi)}{r\partial r}\right)$$
$$=\frac{c}{8\pi e n_0}\hat{\phi}\cdot\boldsymbol{\kappa}\times(\nabla\mathbf{B}^2/2-\mathbf{K})+\frac{c}{4\pi e n_0}$$
$$\times\left(\frac{\partial}{\partial z}K_r-\frac{\partial}{\partial r}K_z-\frac{c^2}{\omega_{pe}^2}\frac{\partial}{\partial z}\frac{1}{r}\left[\frac{\partial}{\partial z}B_r-\frac{\partial}{\partial r}B_z\right]^2\right). \quad (19)$$

Here $\boldsymbol{\kappa}\equiv\nabla n/n=\kappa_r\hat{\mathbf{r}}+\kappa_z\hat{\mathbf{z}}$ is a vector characterizing the plasma density gradient, $\kappa\sim L^{-1}$, where L is the characteristic length scale of plasma density variation, and $\mathbf{K}\equiv(\mathbf{B}\cdot\nabla)\mathbf{B}$ is a vector characterizing the curvature of the magnetic field:

$$\mathbf{K}=K_r\hat{\mathbf{r}}+K_z\hat{\mathbf{z}}$$
$$=\left(\frac{1}{2}\frac{\partial}{\partial r}B_r^2+B_z\frac{\partial}{\partial z}B_r\right)\hat{\mathbf{r}}+\left(\frac{1}{2}\frac{\partial}{\partial z}B_z^2+B_r\frac{\partial}{\partial r}B_z\right)\hat{\mathbf{z}}. \quad (20)$$

Note that $\mathbf{K}=0$ in the one-dimensional cylindrical ICP.

The right-hand side of Eq. (20) is given by nonlinear terms due to the Lorentz force (represented here by the components of the magnetic stress tensor $\nabla\mathbf{B}^2$ and curvature $\mathbf{K}$) and the inertial (centrifugal) term v_ϕ^2/r written here as the electric current in terms of the magnetic field gradient. All of these terms are quadratic in the amplitude of the primary rf magnetic field (B_r and B_z components) at the fundamental driving frequency. Thus, these terms provide direct sources of the dc and second harmonic components of the azimuthal magnetic field B_ϕ. Note that plasma inhomogeneity provides additional nonlinear terms to the equation for B_ϕ field. The linear terms in (19) (on the left-hand side) define the time dependence and spatial structure of the B_ϕ field.

The B_r and B_z components of the magnetic field are not generated in the second order, i.e., the equations for these components remain linear in neglect of the nonlinearly generated B_ϕ field. In this approximation, equations for B_r and B_z can be obtained by taking appropriate components of (18) and neglecting B_ϕ. These equations have the form

$$\frac{\partial}{\partial t}B_z+\frac{\partial}{\partial t}\frac{\partial}{\partial r}\left(\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)+\frac{\partial}{\partial r}\left(\frac{c^2\nu}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)$$
$$=0, \quad (21)$$

$$\frac{\partial}{\partial t}B_r-\frac{\partial}{\partial t}\frac{\partial}{\partial z}\left(\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)-\frac{\partial}{\partial z}\left(\frac{c^2\nu}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)$$
$$=0. \quad (22)$$

When solved with appropriate boundary conditions, these equations will give the distribution of the linear magnetic field in the planar ICP. In fact, these equations are equivalent to a single equation for the azimuthal electric field E_ϕ [see Eq. (B1)].

When the nonlinear second order azimuthal field $B_\phi\sim\mathcal{O}(B_r^2,B_z^2)$, is present in the ICP, the B_r and B_z components will be modified in the third order. Effectively, B_ϕ produces the terms of the order of $\mathcal{O}(B_r^3,B_z^3)$ in the equations for B_r and B_z. Retaining in (18) terms with B_ϕ, one obtains the following nonlinear equations describing the interaction of B_r and B_z with the nonlinear azimuthal field B_ϕ:

$$\frac{\partial}{\partial t}B_z+\frac{\partial}{\partial t}\frac{\partial}{\partial r}\left(\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)+\frac{\partial}{\partial r}\left(\frac{c^2\nu}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)$$
$$=-\frac{1}{r}\frac{\partial}{\partial r}r\frac{ce^2}{m\omega_{pe}^2}\left[\frac{B_r}{r}\frac{\partial}{\partial r}rB_\phi+B_z\frac{\partial}{\partial z}B_\phi\right], \quad (23)$$

$$\frac{\partial}{\partial t}B_r-\frac{\partial}{\partial t}\frac{\partial}{\partial z}\left(\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)-\frac{\partial}{\partial z}\left(\frac{c^2\nu}{\omega_{pe}^2}\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}\right)\right)$$
$$=\frac{\partial}{\partial z}\frac{ce^2}{m\omega_{pe}^2}\left[\frac{B_r}{r}\frac{\partial}{\partial r}rB_\phi+B_z\frac{\partial}{\partial z}B_\phi\right]. \quad (24)$$

The azimuthal magnetic field B_ϕ enters Eqs. (23) and (24) in combination with the first spatial derivatives of B_r and B_z. It means that B_ϕ gives rise to convective (wavelike) effects in the dynamics of B_r and B_z, contrary to the diffusive behavior described in Eqs. (23) and (24) by the second order spatial derivatives of B_r and B_z. Note that the gradient of plasma density may also modify the magnetic field penetration compared to a simple diffusive regime with a constant diffusion coefficient. Such effects were observed in experiments reported in Ref. 22.

Thus, as follows from (19), in the leading order, dc and second harmonic components of B_ϕ are generated. Coupling to the second harmonic of B_ϕ in Eqs. (23) and (24) leads to chain generation of odd (3rd, 5th, ...) B_r and B_z higher order harmonics. Backward coupling of higher order harmonics of B_r and B_z in (19) will generate higher order even harmonics of B_ϕ (2nd, 4th, ...).

The nonlinear azimuthal magnetic field $B_\phi(r,z)$ corresponds to two components of the nonlinear current J_r and J_z. These components can be found from Ampere equations

$$J_r=\frac{c}{4\pi}\frac{\partial B_\phi}{\partial z}, \tag{25}$$

$$J_z=-\frac{c}{4\pi}\frac{1}{r}\frac{\partial rB_\phi}{\partial r}. \tag{26}$$

For bounded inductive plasma surrounded by a low conducting wall sheath, nonlinear currents are closed within the plasma volume.[23]

To obtain the equation for the polarization field, we take the divergence of the equation of electron motion (1). Taking into account that $\nabla\cdot\mathbf{v}=0$ and neglecting the effects of plasma inhomogeneities, we obtain for the polarization potential

$$\nabla^2\Phi^p=\frac{1}{4\pi en}\left(\nabla^2\frac{B^2}{2}-\nabla\cdot\mathbf{K}-\frac{c^2}{\omega_{pe}^2}\frac{1}{r}\frac{\partial}{\partial r}r\left[\frac{\partial B_z}{\partial r}-\frac{\partial B_r}{\partial z}\right]^2\right). \tag{27}$$

This expression reduces to (5) in the case of a one-dimensional cylindrical ICP.

IV. NONLINEAR POLARIZATION FIELD AND NONLINEAR CURRENT AS A MANIFESTATION OF NONLINEAR FORCES

In this section we show that the nonlinear current and nonlinear polarization field represent two different parts (solenoidal and potential) of the nonlinear magnetic (Lorentz) and inertial forces and consider some general characteristics of these components.

We rewrite the momentum equation (1) in the form

$$m\frac{\partial\mathbf{v}}{\partial t}-\nu\mathbf{v}+e\mathbf{E}=\mathbf{F}, \tag{28}$$

where $\mathbf{F}$ represents a total nonlinear force,

$$\mathbf{F}\equiv-\frac{e}{c}\mathbf{v}\times\mathbf{B}-m(\mathbf{v}\cdot\nabla)\mathbf{v}$$
$$=-\frac{e}{c}\mathbf{v}\times\mathbf{B}-m\left(\nabla\frac{\mathbf{v}^2}{2}-\mathbf{v}\times\nabla\times\mathbf{v}\right). \tag{29}$$

In general, this force has potential and solenoidal parts (any vector can be represented as a sum of a solenoidal and potential part):

$$\mathbf{F}=e\boldsymbol{\nabla}\Phi+\nabla\times\mathbf{G}. \tag{30}$$

It will be shown below that, with this normalization, Φ becomes simply a polarization potential; $\mathbf{G}$ is some vector function playing the role of the vector potential for $\mathbf{F}$. One can also show that only the azimuthal component of $\mathbf{G}$ is required to describe the nonlinear B_ϕ field, however, it is more convenient to work directly with $\nabla\times\mathbf{F}=\nabla\times\nabla\times\mathbf{G}$ rather than with $\mathbf{G}$.

By taking $\nabla\times\mathbf{F}$ and $\nabla\cdot\mathbf{F}$ one can decouple solenoidal and potential parts. The potential part is responsible for the generation of the polarization field while the solenoidal part gives rise to the nonlinear current. By taking the curl of (28) we obtain equations for the magnetic field that are equivalent to (19), (23), and (24). The most interesting component is the azimuthal component,

$$\frac{\partial}{\partial t}B_\phi-\frac{c^2}{\omega_{pe}^2}\left(\frac{\partial}{\partial t}+\nu\right)\left(\nabla^2B_\phi-\frac{B_\phi}{r^2}\right)=-\frac{c}{e}\hat{\boldsymbol{\phi}}\cdot\nabla\times\mathbf{F}. \tag{31}$$

Thus, $c/e\,\hat{\boldsymbol{\phi}}\cdot\nabla\times\mathbf{F}$ represents the nonlinear source of the azimuthal magnetic field.

The equation for nonlinear polarization potential Φ^p is obtained by taking the divergence of (28):

$$e\nabla^2\Phi^p=\nabla\cdot\mathbf{F}. \tag{32}$$

Thus, $\nabla\cdot\mathbf{F}/e$ gives a source of nonlinear polarization field, $\mathbf{E}^p=-\nabla\Phi^p$. The nonlinear force associated with the polariation field enters the momentum balance equation. It can be measured[30] as an imbalance force between the ambipolar potential and the pressure gradient force

$$\mathbf{F}^p\equiv-e\mathbf{E}^p=e\mathbf{E}^a+T\nabla\ln n, \tag{33}$$

where $\mathbf{E}^a$ is the ambipolar electric field.

The potential and solenoidal parts of $\mathbf{F}$ have different scaling with collision frequency ν. In fact, the solenoidal part vanishes for $\nu=0$. Indeed, in the absence of collisions, we have

$$\frac{\partial\mathbf{v}}{\partial t}=-\frac{e}{m_e}\mathbf{E} \tag{34}$$

or

$$\nabla\times\mathbf{v}=\frac{e}{mc}\mathbf{B}. \tag{35}$$

Using it in Eq. (30), we obtain

$$\mathbf{F}\equiv-\frac{e}{c}\mathbf{v}\times\mathbf{B}-m(\mathbf{v}\cdot\nabla)\mathbf{v}=-\nabla\left(m\frac{\mathbf{v}^2}{2}\right). \tag{36}$$

This is a purely potential force which does not produce any nonlinear current. The polarization potential in this case is $\Phi^p=-m\mathbf{v}^2/2$, and the nonlinear force is the gradient of electron oscillatory energy. Thus any contribution to the nonlinear current solely occurs due to collisions, or more generally, due to a finite phase shift between the time derivative of the electron velocity and the electric field. Such a phase shift is produced either due to collisions or due to collisionless absorption mechanisms. As noted in Appendix B, collisionless absorption can be described as a viscosity effect in the momentum balance,[15,31] so one can use a representation in the form

$$i\omega\mathbf{v}=-\frac{e}{m}\mathbf{E}-\nu_{\text{eff}}\mathbf{v}, \tag{37}$$

where $\nu_{\text{eff}}=\nu+k^2\eta_{z\phi}$ includes the contribution of collisionless absorption processes. One has to remember that ν_{eff} is in fact a nonlocal operator and, in general, also has an imaginary part.

As a way of approximation we will use (37) to investigate the scaling of the nonlinear sources $\nabla\times\mathbf{F}$ and $\nabla\cdot\mathbf{F}$ with effective collision frequency ν_{eff} assuming that it is local.

Using (37) we obtain for $\mathbf{F}=\overline{\mathbf{F}}+\tilde{\mathbf{F}}$:

$$\overline{\mathbf{F}}=\frac{1}{4}\frac{e^2}{m}\frac{1}{\omega^2+|\nu_{\mathrm{eff}}|^2}\left[\nabla(\mathbf{E}\cdot\mathbf{E}^*)+\frac{1}{c}(\nu_{\mathrm{eff}}\mathbf{E}\times\mathbf{B}^*+\nu_{\mathrm{eff}}^*\mathbf{E}^*\times\mathbf{B})\right],\tag{38}$$

$$\tilde{\mathbf{F}}=\frac{1}{4}\frac{e^2}{m}\left[\frac{i\nu_{\mathrm{eff}}}{\omega(i\omega+\nu_{\mathrm{eff}})^2}\mathbf{E}\cdot\nabla\mathbf{E}-\frac{i}{\omega(i\omega+\nu_{\mathrm{eff}})}\nabla\frac{\mathbf{E}^2}{2}-\frac{i\nu_{\mathrm{eff}}^*}{\omega(-i\omega+\nu_{\mathrm{eff}}^*)^2}\mathbf{E}^*\cdot\nabla\mathbf{E}^*+\frac{i}{\omega(-i\omega+\nu_{\mathrm{eff}}^*)}\nabla\frac{\mathbf{E}^{*2}}{2}\right],\tag{39}$$

where X^* means complex conjugate of X; $\overline{\mathbf{F}}$ represent the time average (dc) value of the force, and $\tilde{\mathbf{F}}$ is oscillating part at the second harmonic. The second term in (38) is due to the damping of the wave momentum and corresponds to the variation of the phase of the electromagnetic field.[32]

Formally, nonlinear sources $\nabla\times\mathbf{F}$ and $\nabla\cdot\mathbf{F}$, with $\mathbf{F}$ written in terms of the electric field via (38) and (39), are equivalent to the nonlinear sources in Eqs. (19) and (27) which are written in terms of the magnetic field B_r and B_z. It may be advantageous to use expressions in terms of the magnetic field if the experimental profiles of B_r and B_z (including the relative phase information) are available. One can also use directly the definition of $\mathbf{F}$ in (29) if the information on $\mathbf{v}=-\mathbf{J}/en_0$ and $\mathbf{B}$ and their relative phase is available. In this way, the effects of nonlocal absorption will be effectively taken into account through profiles and phase of the magnetic field and the electric current. Thus, the expressions for nonlinear sources in (19) and (27) are preferred in nonlocal regimes. Expressions (38) and (39) derived in the local approximation could be used with ν_{eff} as an approximation to a nonlocal operator.

V. SUMMARY AND CONCLUSIONS

We have investigated mechanisms for the generation of nonlinear currents and nonlinear polarization fields in inductively coupled plasmas. The curvature of the magnetic field in planar ICP and nonlinear electron inertia are identified as direct sources of nonlinear poloidal currents (J_r and J_z) and the azimuthal magnetic field B_ϕ. Both the time average and the second harmonics of B_ϕ may be generated by these currents. The amplitude of nonlinear current may become comparable with the amplitude of the primary current at the fundamental driving frequency.[30] Finite B_ϕ may lead to enhanced penetration of the magnetic field due to secondary nonlinear effects (due to the dc part of B_ϕ) and generation of odd harmonics of B_r and B_z (due to coupling to the even harmonics of the B_ϕ). Expressions for B_ϕ are obtained in terms of B_z and B_r. The generation of B_ϕ is only possible in two-dimensional configurations, so that the enhanced penetration of the external magnetic field in the one-dimensional approximation[5] cannot be explained by this mechanism. Note that the approximation of infinite length, $\partial/\partial z=0$, may not be applicable to the ICP discharge in Ref. 5.

We have also shown that the models of enhanced resistivity[5,12] such as those due to Pedersen conductivity [e.g., given by Eq. (A4)] are incorrect in applications to one-dimensional cylindrical ICP. Any modifications in the dynamics of B_r and B_z fields should come only through generation of the nonlinear azimuthal field B_ϕ which occurs only in two-dimensional geometry. It is important to note that effects of finite B_ϕ on the evolution of the B_r and B_z magnetic field in planar ICP appear as nonlinear convection (wavelike) rather than diffusion phenomena. The present theoretical analysis of nonlinear dynamics in ICP confirms the experimental observations in Refs. 6 and 30.

We have investigated the structure of nonlinear forces which consist of Lorentz and inertial force contributions. It was shown that the generation of the B_ϕ field is due to the solenoidal part of the nonlinear force, while the potential part is responsible for the generation of the nonlinear polarization field. Potential and solenoidal parts have different scalings with the collision frequency; moreover, the solenoidal part vanishes in the limit $\nu\to 0$. The nonlinear polarization field occurs with predominantly dc and second harmonic components. In the absence of a phase shift, the dc and second harmonic components have equal amplitudes. Thus, the difference in amplitude of these components could be used as an effective measure of the phase shift which could be associated with collisional and collisionless absorption mechanisms.

It should be noted that our results are obtained neglecting the nonlocal effects of electron thermal motion such as the anomalous skin effect. The anomalous skin effect is known to significantly modify the penetration and structure of the electric and magnetic field inside the plasma.[6] Our equations for nonlinear magnetic field B_ϕ and polarization potential Φ are based on a hydrodynamic approach. The structure of these nonlinear terms is not affected by the presence of collisionless nonlocal effects. Thus, the expressions in Eqs. (19) and (27) for nonlinear sources of magnetic field and polarization potential can also be used in collisionless regimes as long as the linear fields B_z, B_r, and electron velocity v_ϕ, are determined from more complicated equations which include nonlocal effects of thermal motion (or are found experimentally). As an approximation, general expressions for the nonlinear forces in terms of the electric field E_ϕ can be used in nonlocal regimes with a modified value of the effective collision frequency as described in Appendix B. Alternatively, using Eq. (37), one can estimate the effective collision frequency ν_{eff} (37) from experimental values of the phase shift between the electric field and electric current. Then nonlinear expressions (38) and (39) can be used to evaluate the nonlinear forces in (31) and (32) in nonlocal regimes.

Nonlinear effects due to the rf field may also modify the electron stress tensor[33] that give nonlinear terms similar to those considered in this paper. We have neglected here such effects of the stress tensor modification that will be considered separately.

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APPENDIX A: A NOTE ON MODELS OF THE MAGNETIC FIELD ENHANCED RESISTIVITY

It has been recently suggested that the enhanced penetration of magnetic fields observed experimentally[5] can be explained by the reduction of the plasma conductivity due to the induced magnetic field using the model of enhanced resistivity.[5,8,9,12] For $\omega \ll \omega_c$, this model can be written in the following form:

$$\sigma_P = \frac{\sigma_0}{1+\omega_c^2/\nu^2}, \tag{A1}$$

where $\omega_c = eB/mc$ is the electron cyclotron frequency due to the induced magnetic field. For $\omega_c^2/\nu^2 > 1$ the resistivity is enhanced and one may think that this leads to the enhanced penetration of the magnetic field.[5,8,9,12] We have already noted that this conclusion is incorrect for the one-dimensional cylindrical ICP considered in Sec. II A: Eq. (4) derived for such a configuration has no nonlinear terms and the dynamics of B_z are strictly linear. As was explained in Sec. II B, the reduction of electric current due to the Pedersen conductivity is exactly compensated by the Hall current due to the polarization field. In what follows we show that in neglect of electron inertia, this result can be obtained for arbitrary two-dimensional situation, $\partial/\partial z = 0$, $\partial/\partial x \neq 0$, $\partial/\partial y \neq 0$; $B = B_z(x,y)\mathbf{z}$; for the cylindrical discharge, $x \to r$, $y \to \phi$.

It is instructive to first review the derivation of Eq. (A1). We use the electron equation of motion neglecting effects of electron inertia

$$J_x + \omega_c/\nu J_y = \sigma_0 E_x, \tag{A2}$$

$$-\omega_c/\nu J_x + J_y = \sigma_0 E_y. \tag{A3}$$

Here $\omega_c = eB_z/mc$.

The usual reasoning is as follows:[34,35] from symmetry one assumes $E_x = 0$. Then from (A2) one finds $J_x = -\omega_c/\nu J_y$. Using it in (A3) one obtains

$$J_y = \frac{\sigma_0}{1+\omega_c^2/\nu^2} E_y = \sigma_P E_y. \tag{A4}$$

Then from Ampere's law $\partial E_y/\partial x = -(1/c)\partial B_z/\partial t$ and (A4) we derive a nonlinear equation[34,35]

$$\frac{\partial}{\partial t} B_z = \frac{\partial}{\partial x}\left(\left(\frac{c^2}{4\pi\sigma_0}(1+\omega_c^2/\nu^2)\right)\frac{\partial}{\partial x} B_z\right), \tag{A5}$$

which seemingly suggests an enhanced nonlinear diffusion due to the enhanced diffusion coefficient $D_P = (c^2/4\pi\sigma_0) \times (1+\omega_c^2/\nu^2)$. A similar model was also used in Refs. 5, 8, and 9 with additional time averaging of σ_P in (A5).

As was noted above, Eq. (A5) contradicts Eq. (4) which shows no nonlinear effects and was derived under the same assumptions and in the same geometry. To understand the discrepancy, let us remove the symmetry assumption, $E_x = 0$, and write the more general equation

$$-\frac{1}{c}\frac{\partial}{\partial t} B_z = \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial x} E_x. \tag{A6}$$

The E_x and E_y components of the electric field can be found from (A2) and (A3). Substituting them in (A6) we have

$$-\frac{1}{c}\frac{\partial}{\partial t} B_z = \frac{1}{\sigma_0}\frac{\partial}{\partial x}(J_y - \omega_c/\nu J_x) - \frac{1}{\sigma_0}\frac{\partial}{\partial y}(J_x + \omega_c/\nu J_y). \tag{A7}$$

The enhancement of the diffusion coefficient originates from the second term in first brackets on the right-hand side of (A7). It can be readily be seen, however, that this term is exactly cancelled by the terms in the second brackets on the right-hand side. Indeed, opening brackets in (A7) we have

$$\begin{aligned} -\frac{1}{c}\frac{\partial}{\partial t} B_z &= \frac{1}{\sigma_0}\left(\frac{\partial}{\partial x} J_y - \frac{\partial}{\partial y} J_x\right) - \frac{1}{\sigma_0}\frac{\partial}{\partial x}(\omega_c/\nu) J_x \\ &\quad - \frac{\partial}{\partial y}(\omega_e/\nu) J_y - \omega_c/\nu \frac{1}{\sigma_0}\frac{\partial}{\partial x} J_x \\ &\quad - \omega_c/\nu \frac{1}{\sigma_0}\frac{\partial}{\partial y} J_y. \end{aligned} \tag{A8}$$

Two last term cancel each other because of the quasistationarity condition $\nabla \cdot \mathbf{J} = 0$. The second term and third terms in (A8) also cancel each other if one takes into account that

$$J_y = -\frac{c}{4\pi}\frac{\partial}{\partial x} B_z, \tag{A9}$$

$$J_x = \frac{c}{4\pi}\frac{\partial}{\partial y} B_z. \tag{A10}$$

Thus, from (A8) we obtain an exact consequence of Eqs. (A2) and (A3) which is

$$\begin{aligned} -\frac{1}{c}\frac{\partial}{\partial t} B_z &= \frac{1}{\sigma_0}\left(\frac{\partial}{\partial x} J_y - \frac{\partial}{\partial y} J_x\right) \\ &= \frac{c^2}{4\pi\sigma_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) B_z, \end{aligned} \tag{A11}$$

and no effects of the enhanced penetration are present here. The incorrect Eq. (A5) is obtained because of the use of an incomplete electric current (A4) where the Hall current is neglected. Here, we have neglected electron inertia that introduces nonlinear convection of $\nabla^2 B_z$ with the electron flow velocity. In the general case of $\partial/\partial x \neq 0$, $\partial/\partial y \neq 0$ such nonlinear effects are finite; for a symmetrical case with $\partial/\partial\phi = 0$, the contribution of this convection to equation for B_z is identically zero as was shown in Eq. (4).

APPENDIX B: LINEAR FIELDS IN THE CYLINDRICAL ICP

Linear fields in ICP are described by Eqs. (21) and (22). It is convenient to use instead a single equation for the azimuthal electric field E_ϕ[22]

$$-\frac{\partial^2}{\partial z^2}E_\phi-\frac{\partial^2}{\partial r^2}E_\phi+\frac{E_\phi}{r^2}-\frac{1}{r}\frac{\partial}{\partial r}E_\phi=-\frac{4\pi}{c^2}i\omega\hat{\sigma}E_\phi, \tag{B1}$$

where $\hat{\sigma}$ is the plasma conductivity introduced via $J_\phi=\hat{\sigma}E_\phi$, $\hat{\sigma}=e^2n_0/(i\omega+\nu)m$. Equation (B1) has a solution

$$E_\phi=E_0\exp(ikz)J_1(\mu_1 r/a), \tag{B2}$$

where

$$-k^2=\frac{\mu_1^2}{a^2}+\frac{\omega_{pe}^2}{c^2}\frac{1}{1-i\nu/\omega}, \tag{B3}$$

and $\mu_1=3.8$ is the first root of the Bessel function $J_1(\mu_1)=0$. The magnetic field is determined via relations

$$B_r=\frac{ic}{\omega}\frac{\partial}{\partial z}E_\phi, \tag{B4}$$

$$B_z=-\frac{ic}{\omega}\frac{1}{r}\frac{\partial}{\partial r}(rE_\phi). \tag{B5}$$

Equations (B1), (B4), and (B5) are equivalent to the linear part of Eqs. (23) and (24).

When the second term in (B3) can be neglected, the electromagnetic field becomes close to the vacuum field in a subcritical waveguide with the exponential decay factor $k\simeq\mu_1/a$. The second term in (B3) is small in low density and/or low frequency regimes and finite collisions, $\nu/\omega\gg 1$. It is interesting to note that a similar situation may occur in the strongly nonlocal case when $kv_{th}\gg(\omega,\nu)$.[30] In this case, effects of the thermal motion can be described in terms of plasma viscosity,[15,31,36] so that the generalized conductivity is written as[36]

$$\hat{\sigma}=\frac{e^2n_0}{m}\frac{1}{i\omega+\nu+k^2\eta_{z\phi}}, \tag{B6}$$

where $\eta_{z\phi}$ is the viscosity coefficient introduced as $\pi_{z\phi}=-\eta_{z\phi}mn_0(ikV_\phi)$. In the nonlocal regime, $kv_{th}\gg(\omega,\nu)$, and the viscosity coefficient is $\eta_{z\phi}=v_{th}/(\sqrt{\pi}k)$,[36] and

$$-k^2=\frac{\mu_1^2}{a^2}+\frac{\omega_{pe}^2}{c^2}\frac{1}{1-i\nu/\omega-ikv_{th}/\sqrt{\pi}\omega}. \tag{B7}$$

In the strongly nonlocal regime, $kv_{th}/\omega\gg\nu/\omega$, one obtains from (B7) a standard expression for the anomalous skin-depth in the slab approximation,

$$\delta=\frac{1}{k}=\left(\frac{c^2}{\omega_{pe}^2}\frac{v_{th}}{\sqrt{\pi}\omega}\right)^{1/3}, \tag{B8}$$

where $v_{th}=\sqrt{2T/m}$. This is exactly the same expression as obtained from kinetic theory.[37]

In nonlocal regimes the second term can be neglected in Eq. (B7) when

$$\frac{\mu_1^2}{a^2}\gg\frac{\omega_{pe}^2}{c^2}\frac{1}{kv_{th}/\sqrt{\pi}\omega}, \tag{B9}$$

which leads to the vacuum electromagnetic field. It is important to remember though, that a small but finite phase shift due to dissipation (either collisional, related to ν, or collisionless, related to kv_{th}) is important for calculations of the source terms for the nonlinear current and polarization field as noted in Sec. V. This finite phase shift yields an imaginary part of the k parameter found from (B7) that is important for energy absorption even when the electromagnetic field is close to the vacuum field.

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