BRIEF COMMUNICATIONS

The purpose of this Brief Communications section is to present important research results of more limited scope than regular articles appearing in Physics of Plasmas. Submission of material of a peripheral or cursory nature is strongly discouraged. Brief Communications cannot exceed four printed pages in length, including space allowed for title, figures, tables, references, and an abstract limited to about 100 words.

Role of the shear flow profile on the stability of magnetic islands

A. I. Smolyakov
Department of Physics and Engineering Physics University of Saskatchewan, Saskatoon S7N 5E2, Canada and Japan Atomic Energy Research Institute, Naka 311-0193, Japan

E. Lazzaro and R. Coelho
Istituto di Fisica del Plasma, CNR-EURATOM, Milano, Italy

T. Ozeki
Japan Atomic Energy Research Institute, Naka 311-0193, Japan

(Received 11 June 2001; accepted 28 September 2001)

Plasma flow affects the stability of a magnetic island via modification of the ion inertial current. It is shown here that certain profiles of plasma velocity with shear may provide a stabilizing influence on the magnetic island. Such profiles of the plasma flow are characterized by inite plasma circulation inside a magnetic island.
[DOI: 10.1063/1.1420741]

In this Brief Communication we analyze how effects of plasma shear flow may affect the stability of magnetic islands in the Rutherford regime.[1] The plasma flow modifies the stability of the magnetic island via the inertial (polarization) current due to the ion drift in the electric field associated with the relative plasma-island motion.[2] This current results in an additional term[2–7] in the Rutherford evolution equation. If stabilizing, this term may provide a threshold for the modes driven by external sources such as neoclassical tearing modes (NTM)[6] or the modes driven by the external resonant perturbations.[5,8] It has been realized recently[9–11] that in absence of ion drift effects the ion polarization current is destabilizing for a certain class of the velocity profiles near the magnetic island. In this note we analyze how do more general plasma velocity profiles with a flow inside a magnetic island change the effect of the ion polarization current. We show that an internal circulation maintained by the external shear flow may stabilize magnetic islands.

The magnetic island dynamics is analyzed within the one-fluid magnetohydrodynamic approximation so that the current closure relation is

$$-\frac{1}{4\pi v_A^2}\frac{d_0}{dt}\nabla_\perp^2\phi-\nu\frac{c^2}{4\pi v_A^2}\nabla_\perp^4\phi+\nabla_\parallel J_\parallel=0. \tag{1}$$

Here $v_A^2=B_0^2/4\pi n_0 m_i$ is the Alfvén velocity, $\nabla_\parallel$ is the gradient along the total magnetic field, J is the longitudinal current, ν is the viscosity coefficient, and the total time derivative is given by

$$\frac{d_0}{dt}=\frac{\partial}{\partial t}+\frac{c}{B_0}\mathbf{b}\times\nabla\phi\cdot\nabla.$$

We consider magnetic islands whose width is larger than the ion Larmor radius ρ, $w>\rho$, but much smaller than the minor plasma radius a, $w\ll a$, so that a slab approximation can be adopted in the vicinity of the rational surface. The rotating helically symmetric perturbation is described by an auxiliary flux function ψ

$$\psi=-\frac{x^2}{2L_s}B_0+\tilde{\psi}\cos\xi, \tag{2}$$

where $\xi=(m\hat{\theta}-\omega t)$, $x=r-r_s$ is the distance from the corresponding rational surface, $L_s=qR/S$ is the shear length, $S=r_s q'/q$, q_s is the safety factor at the rational surface. The helical coordinate $\hat{\theta}$ is $\hat{\theta}=\theta-\zeta/q_s$, where $q_s\equiv q(r_s)=m/n$ is the safety factor on the rational surface, and θ, ζ are the poloidal and toroidal angles, respectively.

The perturbation in the form Eq. (2) corresponds to a quasi steady-state ($\tilde{\psi}=$const) magnetic island with a half-width $w=(4L_s\tilde{\psi}/B_s)^{1/2}$ rotating with a frequency ω. For such perturbations the lowest order equation for the parallel electric field $E_\parallel=0$ gives $\tilde{\phi}=F(\psi)$ where the $\tilde{\phi}$ is the potential function

$$\tilde{\phi}\equiv\phi-\frac{\omega B_0}{ck_\theta}x. \tag{3}$$

Using Eqs. (2) and (3), Eq. (1) can be integrated[2] giving

$$J_\parallel=\frac{c^3}{4\pi v_A^2}\frac{\partial F}{\partial\psi}(\nabla_\perp^2\tilde{\phi}-\langle\nabla_\perp^2\tilde{\phi}\rangle). \tag{4}$$

In the approximation $\nabla_\perp^2\simeq\partial^2/\partial x^2$, we obtain

$$J_{\parallel}=\frac{c^3}{4\pi v_A^2}\frac{2B_0\tilde{\psi}}{L_S}\frac{\partial F}{\partial\psi}\frac{\partial^2 F}{\partial\psi^2}(\cos\xi-\langle\cos\xi\rangle), \tag{5}$$

where the average over the magnetic surface is

$$\langle\cdots\rangle=\frac{\oint d\xi(\cdots)/\psi_x}{\oint d\xi/\psi_x}.$$

The Rutherford equation is derived by integrating the Ampère law

$$\int_{-\infty}^{\infty}dx\int_{-\pi}^{\pi}d\xi J_{\parallel}(x,\xi)\cos\xi=\frac{c}{4}\Delta_c'\tilde{\psi}. \tag{6}$$

Equations (6) and (5) should be closed by the relation between the potential function $\tilde{\phi}$ and magnetic flux ψ.

The function $\tilde{\phi}=F(\psi)$ can be found from Eq. (1). Averaging Eq. (1) over the magnetic surface we obtain[9]

$$\langle\nabla_{\perp}^4 F(\psi)\rangle=0, \tag{7}$$

or

$$\frac{\partial^2}{\partial\psi^2}\left(\oint\psi_x^3\,d\xi\frac{\partial^2 F(\psi)}{\partial\psi^2}\right)=0. \tag{8}$$

Outside the magnetic separatrix we have two separate regions: $(+)$ for $x>0$ and $(-)$ for $x<0$. The solubility condition can be integrated separately for both sides giving

$$\frac{\partial F}{\partial\psi}=C_0+C_1^{(\pm)}g(k), \tag{9}$$

where

$$g(k)=\int_1^k\frac{k\,dk}{\lambda_3(k)}. \tag{10}$$

Here, the function $\lambda_3(k)$ is determined by the relation

$$\oint\psi_x^3\,d\xi=Ak\left(\frac{2}{3}(2k^2-1)E(1/k)-\frac{1}{3}(k^2-1)K(1/k)\right)$$
$$\equiv A\lambda_3(k), \tag{11}$$

where A is an unimportant constant, E and K are the elliptic integrals, and k is the magnetic surface label introduced by the relation $2k^2-1=-\psi/\tilde{\psi}$.

Constants C_0 and $C_1^{(\pm)}$ are determined from external boundary conditions. The constant C_0 characterizes plasma flow inside the magnetic island. Such internal circulation occurs as a result of the shearing action of the external flow.

Substituting expressions (9) into (5) and using the latter in Eq. (6) we obtain equation describing the steady state island

$$\Delta_c'+\frac{c^2}{v_A^2}\frac{\Gamma}{w}=0. \tag{12}$$

Here Γ is given by

$$\Gamma=\sum\Gamma^{(\pm)}, \tag{13}$$

where

$$\Gamma^{(\pm)}=\frac{8}{\pi}[C_0C_1^{(\pm)}\lambda_1+C_1^{(\pm)2}\lambda_2]. \tag{14}$$

The $(\pm)$ sign refers to the right $(x>0)$ and left $(x<0)$ side of the magnetic island. The numerical coefficients λ_1 and λ_2 are defined by

$$\lambda_1=\int_1^{\infty}\frac{K(1/k)}{\lambda_3(k)}(\langle\cos^2\xi\rangle-\langle\cos\xi\rangle^2)dk\simeq 0.51,$$

$$\lambda_2=\int_1^{\infty}\frac{K(1/k)}{\lambda_3(k)}g(k)(\langle\cos^2\xi\rangle-\langle\cos\xi\rangle^2)dk\simeq 0.14.$$

One can see that the second term in Eq. (14) is definitely destabilizing, while the first term can be stabilizing for $C_0C_1^{(\pm)}<0$. Since C_0 corresponds to the plasma circulation inside the magnetic islands, one can conclude that the internal circulation may lead to the stabilization effect.

The plasma flow corresponding to the potential Eq. (9) has to be matched to the plasma flow in the external region. Asymptotic behavior of the poloidal velocity away from the magnetic island is obtained from Eq. (9) in the limit of large x

$$\frac{\partial F}{\partial\psi}=C_0+C_1^{(\pm)}\left(g_{\infty}-\frac{2}{\pi k}\right)+\mathcal{O}\left(\frac{1}{k^2}\right),\quad k\gg 1, \tag{15}$$

where

$$g_{\infty}=\int_1^{\infty}\frac{k\,dk}{\lambda_3(k)}\simeq 0.87.$$

It is obvious from Eq. (15) that both C_0 and $C_1^{(\pm)}$ contribute to the asymptotic part of plasma flow that is increasing linearly with x at large $x\gg w$

$$C_0+C_1^+g_{\infty}=-\frac{L_s}{c}\frac{dV_0}{dx}\bigg|^+, \tag{16}$$

$$C_1^+=\frac{\pi L_s}{2cw}V_0|^+, \tag{17}$$

$$C_0+C_1^-g_{\infty}=-\frac{L_s}{c}\frac{dV_0}{dx}\bigg|^-, \tag{18}$$

$$C_1^-=-\frac{\pi L_s}{2cw}V_0|^-. \tag{19}$$

These matching conditions are the most general for the three-parametric $(C_0,\,C_1^+$ and $C_1^-)$ family of solutions to Eq. (7). Parameters C_0, C_1^+ and C_1^- are determined in terms of asymptotic values of plasma flow V_0 and its derivatives on both sides of a magnetic island.

For a given w, a generic velocity profile $V=V_0+x\,dV_0/dx$ requires C_0 to be finite. In particular, localized profile of plasma flow (that is, velocity profile that does not have a linearly growing component, $\sim V_0'x$) also requires a finite C_0,

$$C_0+C_1^{(\pm)}g_{\infty}=0. \tag{20}$$

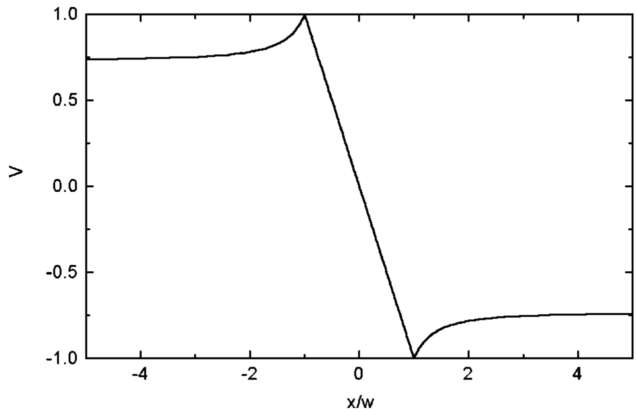

FIG. 1. Qualitative behavior of plasma flow velocity (in arbitrary units) for a localized profile, $C_0+C_1^{(\pm)}g_\infty=0$.

There is only one possible type of the velocity profile that is localized on both sides (schematically shown in Fig. 1 as a function of x for $\xi=0$). For such velocity profile the ion polarization current is stabilizing.

In a general case one obtains from Eqs. (16), (17) (for each side)

$$\Gamma=\frac{8}{\pi}\frac{4L_s^2}{\pi^2c^2w^2}\left[(\lambda_2-\lambda_1g_\infty)V_0^2-\frac{\pi}{2}\lambda_1V_0V_0'w\right]. \qquad (21)$$

Since $\lambda_2-\lambda_1g_\infty<0$ one can see that the polarization current is stabilizing when the second term in Eq. (21) is small compared to the first one, $V_0'w<V_0$. The first term in Eq. (21) corresponds to the localized part of the velocity profile, while the second term $\sim V_0'$, is due to the non-localized part of the plasma velocity. For an arbitrary velocity profile, such as in Fig. 2, the net effect will depend on relative contributions of the stabilizing and destabilizing factors. The net effect of the polarization current can be stabilizing when $V_0V_0'>0$, or when $V_0V_0'<0$ and V_0' is not too large:

$$(\lambda_2-\lambda_1g_\infty)V_0^2-\frac{\pi}{2}\lambda_1V_0V_0'w<0. \qquad (22)$$

In all cases, the stabilizing effect occurs due to finite plasma velocity inside the magnetic separatrix. Note that we have four equations (16)–(19) for three variables, C_0, C_1^+ and

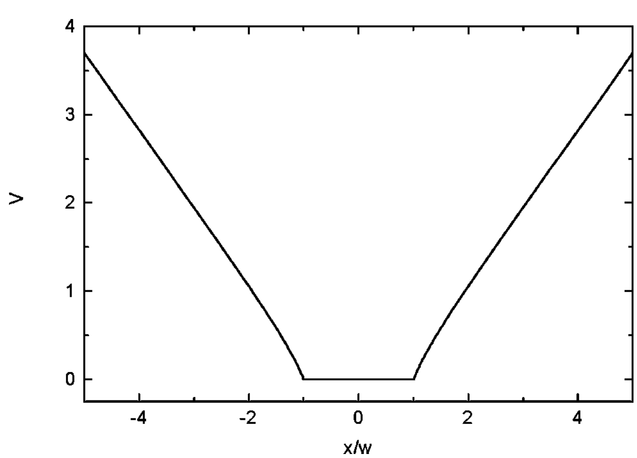

FIG. 2. Qualitative behavior of plasma flow velocity (in arbitrary units) for a general case: weakly localized for $x<0$, $C_0+C_1^-g_\infty=0$, not localized for $x>0$, $C_0+C_1^+g_\infty\neq0$.

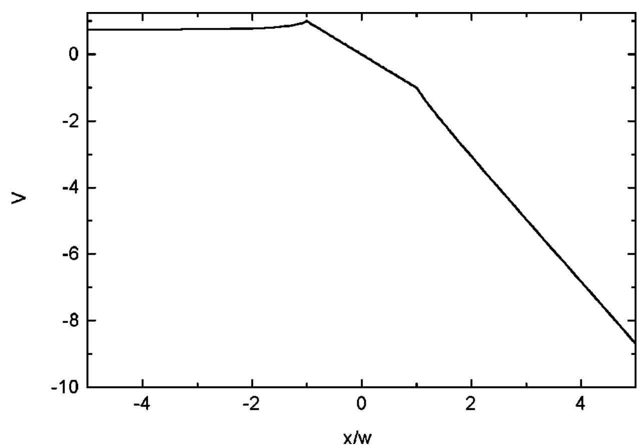

FIG. 3. Qualitative behavior of plasma flow velocity (in arbitrary units) for a nonlocalized velocity profile, no internal circulation, $C_0=0$.

C_1^-, so that values of plasma flow and its derivatives are not completely independent on both sides of a magnetic island. Therefore, conditions (22) on both sides are not independent either; they are related via Eqs. (16) and (17).

In absence of the internal circulation $C_0=0$ the polarization current is destabilizing. In this case

$$\Gamma^{(\pm)}=\frac{8}{\pi}C_1^{(\pm)2}\lambda_2^2. \qquad (23)$$

The velocity profile for this case is schematically shown in Fig. 3. The result of Ref. 9 can be obtained from Eq. (23) by using Eqs. (16) and (18) with $C_0=0$.

In Eqs. (16)–(19) we have neglected the effects of the island rotation, $\omega=0$. A finite value of the island rotation frequency ω changes definitions of C_0, $C_1^{(\pm)}$ coefficients, but a qualitative picture of the shear flow effects remains the same.

The plasma velocity profile with internal circulation inside the magnetic separatrix are similar to the "cat's-eyes" configurations that often occur in shear fluid flows. We are not aware of any direct measurements of plasma velocity profiles with internal circulations as suggested in this work. However, asymmetrical deformation of the magnetic island have been observed recently.[12,13] Such deformations were explained as a result of a combined effects of plasma viscosity and shear flow across the magnetic island region.[12,13] One can readily see that such an explanation is equivalent to the occurrence of plasma flow inside the magnetic islands. The excitation of strongly sheared plasma flow and, as a result, the excitation of the internal circulation can also be expected in the experiments on active control of magnetic islands in a tokamak[8] where the frequency of the external control field was strongly modulated. Such a frequency modulation appears to be crucial for a successful suppression of the otherwise unstable magnetic islands. As we suggest in our work, the mechanism of suppression may be related to the excitation of plasma flow inside the magnetic island.

We would like also to note the relation between different types of the velocity profiles and the net deposition of the momentum in the resistive layer. The latter is characterized by the parameter Δ_s'

$$\int_{-\infty}^{\infty} dx \int_{-\pi}^{\pi} d\xi J_{\parallel}(x,\xi \sin)\xi = \frac{c}{4}\Delta_s' \tilde{\psi}. \tag{24}$$

From the momentum balance equation, or equivalently, from the current closure equation (1),[4,14] it can be readily seen that the Δ_s' parameter is proportional to the difference in the nonlocalized parts of the velocity perturbation on both sides of a magnetic island

$$\Delta_s' \sim \left(\left.\frac{dV_0}{dx}\right|_{+\infty} - \left.\frac{dV_0}{dx}\right|_{-\infty} \right). \tag{25}$$

For the velocity profile shown in Fig. 2, the net force applied across the magnetic island is maximal. Alternatively, for the localized velocity profiles (such as in Fig. 1) we obtain $\Delta_s' = 0$ [contrary to the statement in Ref. 10, where the net momentum deposition remains finite even for the localized profiles]. Since ultimately Δ_s' is determined from the outer (ideal) region, the Δ_s' parameter becomes finite due to either wall interaction or to the interaction with error field (or with other magnetic islands mediated by toroidal coupling). In situations when such interaction is expected to be small (such as NTM modes localized deeply inside the plasma) Δ_s' should be negligible. In this case one should expect that velocity profile to be localized so the net momentum applied across the magnetic island is small. This suggest that for magnetic islands that do not experience a strong interaction with external fields, localized velocity profiles are more likely to occur.

In summary, in this work we have suggested that the plasma circulation inside the magnetic island may provide a stabilizing influence on the magnetic island. Such a circulation occurs when the external sheared flow is imposed on plasma.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Y. Kishimoto (JAERI) and Dr. M. Azumi (JAERI) for encouragement of this work and helpful discussions.

This work supported in part by Natural Science and Engineering Research Council of Canada.

[1] H. Rutherford, Phys. Fluids **16**, 1903 (1973).
[2] A. I. Smolyakov, Sov. J. Plasma Phys. **15**, 667 (1989).
[3] X. Garbet, F. Morgoues, and A. Samain, Plasma Phys. Controlled Fusion **30**, 343 (1988); **32**, 131 (1990).
[4] A. I. Smolyakov, A. Hirose, E. Lazzaro, G. Re, and J. D. Callen, Phys. Plasmas **2**, 1581 (1995).
[5] R. Fitzpatrick and T. C. Hender, Phys. Fluids B **3**, 644 (1991).
[6] H. R. Wilson, J. W. Connor, R. J. Hastie, and C. C. Hegna, Phys. Plasmas **3**, 248 (1996).
[7] B. Mikhailovskii, V. D. Pustovitov, V. S. Tsypin, and A. I. Smolyakov, Phys. Plasmas **7**, 1204 (2000).
[8] G. A. Navratil, C. Gates, M. E. Mauel, D. Maurer, N. Nadle, E. Taylor, Q. Xiao, W. A. Reass, and G. A. Wurden, Phys. Plasmas **5**, 1855 (1998).
[9] F. Waelbroeck and R. Fitzpatrick, Phys. Rev. Lett. **78**, 1703 (1997).
[10] R. Fitzpatrick and F. Waelbrock, Phys. Plasmas **7**, 4983 (2000).
[11] J. W. Connor, F. L. Waelbrock, and H. R. Wilson, Phys. Plasmas **8**, 2835 (2001).
[12] M. S. Chu, L. Chen, L.-J. Zheng, C. Ren, and A. Bondeson, Nucl. Fusion **39**, 2107 (1999).
[13] C. Ren, M. S. Chu, and J. D. Callen, Phys. Plasmas **6**, 1203 (1999).
[14] A. B. Mikhailovskii, S. V. Konovalov, V. D. Pustovitov, and V. S. Tsypin, Phys. Plasmas **7**, 2530 (2000).