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Stabilization of magnetic islands due to the sheared plasma flow and viscosity

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Abstract
It is shown that the asymmetric deformation of the magnetic island caused by the finite plasma viscosity gives a stabilizing effect on the magnetic islands in the Rutherford regime. Such stabilization typically overcomes the destabilizing effect of the ion polarization current.

1. Introduction

Magnetic islands associated with low m (poloidal mode number) helical modes often limits the performance of a tokamak plasma and may lead to plasma disruptions. When a tearing mode stability parameter $\triangle'$ is negative, $\triangle' < 0$, so that the standard magnetohydrodynamic (MHD) tearing modes are stable, magnetic islands modes can be excited by the perturbed bootstrap current or by the external resonant magnetic perturbation due to the residual error fields. Both types of modes are strongly detrimental for tokamak plasmas so that their avoidance is a crucial issue for the reactor scale devices [1]. Experimental data indicate that only sufficiently large initial magnetic perturbations grow to a larger amplitude so that there exists a threshold level below which the magnetic islands do not grow. It was found [2–4] that the ion polarization current may affect stability of a magnetic island in the Rutherford regime [5]. In particular, it has been concluded that the ion polarization current may provide a stabilizing effect that leads to a threshold for the magnetic islands excitation. The stabilizing effect of the ion polarization current was used in early evaluations of the critical level of the error magnetic field [6–12]. It has also been suggested [13] that the ion polarization current provides a threshold for the neoclassical tearing modes (NTM). Later, it has been realized [14] (see also [15, 16]) however that the ion polarization current is destabilizing for a typical situation of a magnetic island driven by the external magnetic perturbation (and in neglect of the ion finite Larmor radius (FLR) effects). A simple analysis shows [14] that with the destabilizing effect of the ion polarization current there exists no threshold for the magnetic islands driven by the external field. On the contrary, it appears that due to the destabilizing effect of the ion polarization current, any small initial perturbation in the Rutherford regime will be growing. Such a result is in apparent contradiction with a number of experimental observations. In this paper we re-examine the problem of the magnetic island interacting with a static external perturbation such as an error field. We show that in a viscous plasma, there is an additional stabilizing factor due to the shear plasma flow near the magnetic islands. Such a stabilizing effect occurs due to the deformation of the magnetic island by the plasma flow. We note that such a deformation has recently been studied theoretically [17, 18]. It appears that experimental data [17, 18] also indicate asymmetric deformations of the magnetic island.

2. Basic equations

The helically symmetric perturbation is described by an auxiliary flux function ψ that contains both the unperturbed sheared magnetic field and the perturbation

$$\psi = -\frac{x^2}{2L_s}B_0 + \tilde{\psi}\cos\xi, \tag{1}$$

where $\xi = m\hat{\theta}$, $x = r - r_s$ is the distance from the corresponding rational surface, $L_s = qR/S$ is the shear length, $S = r_s q'/q$, q_s is the safety factor at the rational surface. The helical coordinate $\hat{\theta}$ is $\hat{\theta} = \theta - \zeta/q_s$, where $q_s \equiv q(r_s) = m/n$ is the safety factor on the rational surface, and θ, ζ are the poloidal and toroidal angles, respectively. Such a perturbation creates a magnetic island with a half width w, $w^2 = 4\tilde{\psi}L_s/B_0$.

We consider the magnetic island dynamics within the one-fluid magnetohydrodynamic model. The vorticity equation is

$$-\frac{c^2}{4\pi v_A^2}\frac{d_0}{dt}\nabla_\perp^2\phi + \nu\frac{c^2}{4\pi v_A^2}\nabla_\perp^4\phi + \nabla_\parallel J_\parallel = 0. \tag{2}$$

Here $v_A^2 = B_0^2/4\pi n_0 m_i$ is the Alfvén velocity, $\nabla_\parallel$ is the gradient along the total magnetic field, J is the longitudinal current, and ν is the viscosity coefficient.

In the leading order, the relation between the electrostatic potential ϕ and the magnetic flux function ψ can be found from the parallel component of Ohm's law which gives

$$\phi = F(\psi). \tag{3}$$

The electric current found from (2) is used in the Ampere law giving

$$\int_{-\infty}^{\infty} dx \int_{-\pi}^{\pi} d\xi\, J(x,\xi)\cos\xi = \frac{c}{4}\Delta_c'\tilde{\psi}, \tag{4}$$

$$\int_{-\infty}^{\infty} dx \int_{-\pi}^{\pi} d\xi\, J(x,\xi)\sin\xi = \frac{c}{4}\Delta_s'\tilde{\psi}. \tag{5}$$

The $\triangle_c'$ and $\triangle_s'$ are respectively the $\cos\xi$ and $\sin\xi$ components of the general tearing stability parameter. Examination of (4) and (5) shows that, in fact, equation (1) represents only a zero-order approximation. More accurately, expression (1) has to be modified to account for derivatives of $\tilde{\psi}$:

$$\psi = -\frac{x^2}{2L_s}B_0 + \tilde{\psi}\left(1 + \frac{\triangle_c'}{2}|x|\right)\cos\left(\xi - \frac{\triangle_s'}{2}|x|\right). \tag{6}$$

The term $\triangle_c'|x|/2$ gives quasilinear corrections to the island growth [19]. As follows from the momentum balance equation, in a viscous plasma the $\triangle_s'$ parameter is defined by the value of the velocity shear outside the magnetic islands. The term $\triangle_s' |x|/2$ gives a finite phase shift investigated in [17, 18].

Usually, a symmetric island perturbation is considered (as given by (1)), so that $\tilde{\psi}$ is even in ξ, and $\tilde{\phi}$ is also even in ξ. As a result the second term in (2) does not contribute to the cos component of the parallel current, and the viscous term in (2) does not affect directly the island stability in equation (4). When the magnetic island is deformed due to Δ_s' as in (6), there is a component of $\tilde{\phi}$ which is odd in ξ, then there is a finite contribution of the viscous term to the cos component of $J(x, \xi)$; thus there is an additional contribution to (4). Investigation of this contribution is the subject of the present paper.

3. Plasma flow around the magnetic island

In the lowest order, the viscous term in (2) does not enter the equation for the longitudinal current because of symmetry in ξ. However, the solubility condition for (2) allows one to find a one-parametric family of the function $F(\psi)$. Averaging equation (2) over the magnetic surface, we obtain the following solubility condition [14]:

$$\left\langle \nabla_\perp^4 F(\psi) \right\rangle = 0. \tag{7}$$

This equation, together with boundary conditions, may be used to completely determine the velocity profile outside the magnetic island. For thin islands we can write

$$\nabla_\perp^4 F(\psi) \simeq \frac{\partial^4}{\partial x^4} F(\psi) = \psi_x \frac{\partial^2}{\partial \psi^2} \left(\psi_x^3 \frac{\partial^2 F(\psi)}{\partial \psi^2} \right). \tag{8}$$

Then equation (7) takes the form

$$\frac{\partial^2}{\partial \psi^2} \left(\oint \psi_x^3 \, \mathrm{d}\xi \frac{\partial^2 F(\psi)}{\partial \psi^2} \right) = 0. \tag{9}$$

Outside the magnetic separatrix we have two separate regions: (+) for $x > 0$ and (−) for $x < 0$. The solubility condition can be integrated separately for both, giving

$$\frac{\partial^2 F}{\partial \psi^2} = -\frac{C_1^{(\pm)}}{4\tilde{\psi}} \frac{1}{\lambda_3(k)}. \tag{10}$$

Here, the function $\lambda_3(k)$ is determined by the relation

$$\oint \psi_x^3 \, \mathrm{d}\xi = Ak \left(\tfrac{2}{3}(2k^2 - 1)E(1/k) - \tfrac{1}{3}(k^2 - 1)K(1/k) \right) \equiv A\lambda_3(k), \tag{11}$$

where A is an unimportant constant, E and K are the elliptic integrals, and k is the magnetic surface label introduced by the relation $2k^2 - 1 = -\psi/\tilde{\psi}$.

Integrating (10) one more time, we obtain

$$\frac{\partial F}{\partial \psi} = C_1 g(k), \tag{12}$$

where

$$g(k) = \int_1^k \frac{k \, \mathrm{d}k}{\lambda_3(k)}. \tag{13}$$

Constant $C_1^{(\pm)}$ is determined from external boundary conditions.

We consider the simplest case of a static magnetic island created by a stationary error field magnetic perturbation in a moving plasma. The plasma flow is zero at the magnetic separatrix and approaches the equilibrium value $V_0(x)$ away from the magnetic island. In the limit of large x, we find from (12)

$$\frac{\partial F}{\partial \psi} = \mathrm{sgn}(x) C_1 \left(g_\infty - \frac{2}{\pi k} \right) + \mathcal{O}(\frac{1}{k^2}), \qquad k \gg 1, \tag{14}$$

where

$$g_\infty = \int_1^\infty \frac{k\,\mathrm{d}k}{\lambda_3(k)} \simeq 0.86.$$

By using the asymptotic form $V \to V_0^{'} x$ at $x \gg w$, we obtain from (14)

$$C_1 = -\frac{V_0^{'} L_\mathrm{s}}{c g_\infty}. \tag{15}$$

The velocity profile defined by equations (14) and (15) was also used in [14].

4. Effect of plasma inertia

Plasma inertia modifies the stability of magnetic islands via the ion polarization current. From (2) we find

$$J_\mathrm{c} = \frac{c^3}{4\pi v_\mathrm{A}^2} \frac{\partial F}{\partial \psi} \left(\nabla_\perp^2 \tilde{\phi} - \langle \nabla_\perp^2 \tilde{\phi} \rangle \right), \tag{16}$$

or after using $\tilde{\phi} \simeq F(\psi)$ and $\nabla_\perp^2 \simeq \partial^2/\partial x^2$, we obtain [4]

$$J_\mathrm{c} = \frac{c^3}{4\pi v_\mathrm{A}^2} \frac{2 B_0 \tilde{\psi}}{L_\mathrm{s}} \frac{\partial F}{\partial \psi} \frac{\partial^2 F}{\partial \psi^2} \left(\cos\xi - \langle \cos\xi \rangle \right). \tag{17}$$

For the velocity profile defined by (12) we obtain

$$\triangle_\mathrm{c}' + g_2 \frac{V_0^{'2} L_\mathrm{s}^2}{v_\mathrm{A}^2} \frac{1}{w} = 0, \tag{18}$$

where the numerical coefficient

$$g_2 = \frac{16 \lambda_2}{\pi g_\infty^2},$$

and λ_2 is defined by

$$\lambda_2 = \int_1^\infty \frac{K(1/k)}{\lambda_3(k)} g(k) \left(\left\langle \cos^2 \xi \right\rangle - \langle \cos\xi \rangle^2 \right) \mathrm{d}k \simeq 0.1.$$

Equation (18) was obtained in [14]. The last term in (18) indicates that the ion polarization current is destabilizing for a static magnetic island in a moving plasma. Note that in this paper we consider a nonrotating (locked) magnetic island in a surrounding plasma that is moving with velocity V_0. For magnetic islands rotating with frequency ω, equation (18) is modified with the transformation $V_0^{'} w \to (\omega - \omega_\mathrm{E}) / k_\theta$ [11, 20], where ω_E is the Doppler shift frequency due to the equilibrium electric field, and $k_\theta = m/r_\mathrm{s}$. In the next section we consider how (18) is modified taking into account the deformation of the magnetic island in a viscous plasma.

5. Effects of the viscous shear flow

5.1. Momentum balance equation

The $\sin\xi$ component of Ampere's law corresponds to the poloidal component of the momentum balance equation so that the $\triangle_s'$ parameter describes the momentum absorption at the rational surface [21–23]. The island equilibrium equation (5) can also be derived by integrating the current closure equation over $d\psi\, d\xi$.

By integration of (5) by parts, one obtains

$$\int \nabla_\parallel J\, d\psi d\xi = \frac{k_\theta c\tilde{\psi}^2}{4L_s}\triangle_s'. \tag{19}$$

Integration of the viscous term gives

$$\int \nabla_\perp^4\phi\, d\psi\, d\xi = -4\pi\frac{B_0^2}{L_s^2}\left.\frac{\partial F}{\partial\psi}\right|_{x\gg w} = -4\pi\frac{B_0^2}{L_s^2}C_1 g_\infty. \tag{20}$$

Then, the final momentum balance equation takes the form

$$\triangle_s^{'} = -64\frac{\nu V_0' L_s^2}{k_\theta v_A^2 w^4}, \tag{21}$$

which is similar to the equation in [14]. This equation illustrates the fact that the viscous shear flow forces a finite value of $\triangle_s'$, thus making the magnetic surfaces asymmetric due to the contribution of a $\sin\xi$ component to ψ.

5.2. Viscous shear layer

Near the separatrix the electrostatic potential is no longer a magnetic flux function resulting in a finite value of the electric current along the magnetic field lines. The plasma inertia can be neglected within such a viscous-resistive layer that is described by the equation

$$-\nu\frac{c^2}{4\pi v_A^2}\nabla_\perp^4\phi - \sigma\nabla_\parallel^2\phi = 0. \tag{22}$$

Near the separtrix we approximate $x = w\cos(\xi/2)$ so that

$$\frac{\partial}{\partial x} = \psi_x\frac{\partial}{\partial\psi} \simeq -\frac{B_0}{L_s}w\cos\frac{\xi}{2}\frac{\partial}{\partial\psi}. \tag{23}$$

Then equation (22) takes the form

$$\epsilon\cos^4\frac{\xi}{2}\tilde{\psi}^4\frac{\partial^4\phi}{\partial\psi^4} - \cos\frac{\xi}{2}\frac{\partial}{\partial\xi}\left(\cos\frac{\xi}{2}\frac{\partial\phi}{\partial\xi}\right) = 0, \tag{24}$$

where the small parameter ϵ is given by

$$\epsilon = \left(\frac{w_c}{w}\right)^6. \tag{25}$$

The width of the viscous-resistive layer w_c is defined by the expression

$$w_c^6 = 256\nu D_m\frac{L_s^2}{k_\theta^2 v_A^2}, \tag{26}$$

where $D_m = c^2/4\pi\sigma$.

We look for the solution of (24) in the form

$$\hat{\phi} = F(\psi)Y(\xi), \tag{27}$$

where the radial and angular functions satisfy the equations

$$\epsilon\,\tilde{\psi}^4 \frac{\partial^4 F}{\partial \psi^4} + \lambda_0 F = 0, \tag{28}$$

$$\frac{\partial}{\partial \xi}\left(\cos\frac{\xi}{2}\frac{\partial Y}{\partial \xi}\right) + \lambda_0 \cos^3\frac{\xi}{2} Y = 0. \tag{29}$$

The positive eigenvalue λ_0 can be estimated from the variational form

$$\lambda_0 = \frac{\int_{-\pi}^{\pi} \cos(\xi/2)(\partial Y/\partial \xi)^2\,\mathrm{d}\xi}{\int_{-\pi}^{\pi} \cos^3(\xi/2) Y^2\,\mathrm{d}\xi}. \tag{30}$$

The general solution of equation (28) has the form

$$F = a_1 \cos(\chi/\sqrt{2})\exp(-\chi/\sqrt{2}) + a_2 \sin(\chi/\sqrt{2})\exp(-\chi/\sqrt{2}), \tag{31}$$

where a new variable χ was introduced as

$$\chi = \frac{\psi + \tilde{\psi}}{\tilde{\psi}}\left(\frac{\lambda_0}{\epsilon}\right)^{1/4}. \tag{32}$$

Expression (27) together with (28) and (29) describe the transition layer where the odd part of the electrostatic potential ϕ_{s} decays to zero inside the magnetic island, so that $F \to 0$ for $\chi \gg 1$. The outer boundary condition can be obtained from the outer solution given by equation (12). In the outer region

$$\phi = F(\psi). \tag{33}$$

We are interested only in the $\sin\xi$ component, so we use

$$\psi = -\frac{x^2}{2L_{\mathrm{s}}} B_0 + \tilde{\psi}\cos\xi + \tilde{\psi}\frac{\triangle_{\mathrm{s}}'}{2}|x|\sin\xi. \tag{34}$$

Assuming that $\triangle_{\mathrm{s}}' w < 1$, we obtain

$$\frac{\partial \phi_{\mathrm{s}}}{\partial x} = \frac{\partial F(\psi)}{\partial \psi}\tilde{\psi}\frac{\triangle_{\mathrm{c}}'}{2}\sin\xi + \frac{\partial^2 F(\psi)}{\partial \psi^2}\psi_x \frac{\tilde{\psi}\triangle_{\mathrm{c}}' x}{2}\sin\xi \tag{35}$$

for the $\sin\xi$ component of the electrostatic potential. Then the outer boundary conditions at the boundary of the transition layer are

$$\phi_{\mathrm{s}}|_{\psi=\psi_{\mathrm{s}}} = 0, \tag{36}$$

$$\left.\frac{\partial \phi_{\mathrm{s}}}{\partial x}\right|_{\psi=\psi_{\mathrm{s}}} = -\left.\frac{\partial^2 F_{\mathrm{out}}(\psi)}{\partial \psi^2}\frac{\tilde{\psi}\triangle_{\mathrm{c}}'\psi_x x}{2}\right|_{\psi=\psi_{\mathrm{s}}}\sin\xi. \tag{37}$$

Note that the exact location of the outer boundary of the transition layer is not important; we chose it to be $\psi = \psi_{\mathrm{s}} \equiv -\tilde{\psi}$ or $\chi = 0$. Then, the following are the boundary conditions for $F(\chi)$:

$$F|_{\chi=0} = 0, \tag{38}$$

$$\left.\frac{\partial F}{\partial \chi}\right|_{\chi=0} = -\left.\frac{\partial^2 F_{\mathrm{out}}(\psi)}{\partial \psi^2}\left(\frac{\epsilon}{\lambda_0}\right)^{1/4}\frac{\tilde{\psi}^2\triangle_{\mathrm{s}}'}{2}\right|_{\psi=\psi_{\mathrm{s}}}, \tag{39}$$

$$F|_{\chi\gg 1} = 0. \tag{40}$$

To match the outer solution with the transition layer, we take $Y(\xi)$ in the form

$$Y(\xi) = \cos\frac{\xi}{2}\sin\xi. \tag{41}$$

We use this expression also as a trial function to evaluate λ_0 in (30).

Applying boundary conditions (38)–(40), we find

$$a_1 = 0, \tag{42}$$

$$\frac{a_2}{\sqrt{2}} = -\left.\frac{\partial^2 F_{\rm out}(\psi)}{\partial\psi^2}\left(\frac{\epsilon}{\lambda_0}\right)^{1/4}\frac{\tilde{\psi}^2\triangle_{\rm s}'w}{2}\right|_{\psi=\psi_{\rm s}}. \tag{43}$$

The electric current within the viscous-resistive layer is found from Ohm's law

$$J = -\sigma\nabla_\parallel\phi = -\sigma\frac{k_\theta}{B_0}\psi_x\frac{\partial\hat{\phi}}{\partial\xi}. \tag{44}$$

Using (44) and (27) in (4), we have

$$\frac{c\triangle_{\rm c}'\tilde{\psi}}{4} = \int J_\parallel\cos\xi\,{\rm d}x\,{\rm d}\xi = -4\sigma\frac{k_\theta}{B_0}\left(\frac{\epsilon}{\lambda_0}\right)^{1/4}R_0G_0, \tag{45}$$

where

$$R_0 = \int_0^\pi \cos\xi\frac{\partial}{\partial\xi}Y(\xi)\,{\rm d}\xi = \frac{16}{15},$$

$$G_0 = \int_0^\infty F(\chi)\,{\rm d}\chi = \frac{a_2}{\sqrt{2}}.$$

Finally we obtain

$$\frac{R_0}{l_3}C_1\sigma\frac{k_\theta}{8L_{\rm s}}\left(\frac{\epsilon}{\lambda_0}\right)^{1/2}\frac{\triangle_{\rm s}'w^3}{2} = \frac{c\triangle_{\rm c}'}{4}, \tag{46}$$

where $l_3 = \lambda_3(1) = 2/3$.

By using the momentum balance (21) for $\triangle_{\rm s}'$ in (46), we obtain the island width equation

$$g_3\left(\frac{\epsilon}{\lambda_0}\right)^{1/2}\frac{\nu}{D_{\rm m}}\frac{V_0'^2L_{\rm s}^2}{v_{\rm A}^2}\frac{1}{w} = \triangle_{\rm c}', \tag{47}$$

where

$$g_3 = \frac{8R_0}{\pi l_3 g_\infty} \simeq 4.74.$$

The left-hand side of this equation is positive, which means that the sheared plasma flow in a viscous plasma provides a stabilizing effect on the magnetic islands in the Rutherford regime.

6. Summary

In a stationary state, the external electromagnetic torque associated with the resonant magnetic perturbation is balanced by the viscous force [21, 22]. Thus, transverse plasma viscosity maintains a finite velocity gradient around the magnetic island. It is shown in this paper that plasma viscosity leads to an asymmetric deformation of the magnetic island such that a $\sin\xi$ component in the perturbed electrostatic potential is excited. As a result, the viscous force directly affects the island evolution equation modifying its stability. Such modification is a novel effect which was not considered earlier. In this paper we show that this leads to the stabilization of the magnetic islands.

It follows from (18) and (47) that the overall stability of a magnetic island is determined by the competition of the destabilizing effect of the polarization current in (18) and stabilizing effects of the viscosity and the sheared flow in (47). For smaller magnetic islands, the stabilizing effect is dominant. To evaluate the critical island width, we use the following parameters (typical for the JET tokamak): $B_0 = 2.5$ Tl, minor radius $a = 100$ cm, major radius $R = 300$ cm, $T = 1.25$ keV, $m/n = 3/2$, $n_e = 1.9 \times 10^{13}$ cm^{-3}. We use a classical expression for plasma diffusivity $D_m = c^2/(4\pi)\eta_\perp/2$, with the electron resistivity $\eta_\perp = 1.15 \times 10^{-14}$ $Z_{eff} \ln \Lambda T_e^{-3/2}$ s, $Z_{eff} = 2$ and Coulomb logarithm $\ln \Lambda = 17$. The transverse plasma viscosity is assumed to be anomalous with an empirical viscosity coefficient with a gyro-Bohm type scaling $\nu = A\rho^2 v_{ti}/a^2$,where we take $A = 1$.

For the above parameters, the width of the viscous-resistive layer around the magnetic island is $w_c = 0.5$ cm. From (18) and (47) we conclude that for magnetic islands with a width less than $w < w_{th} \equiv w_c(g_3\nu/D_m g_2 \lambda_0^{1/2})^{1/3} \simeq 1.7$ cm, the stabilizing effect due to the shear flow and viscosity overcomes destabilization due to the polarization current. The magnetic island of a width w_{th} corresponds to the magnetic perturbation (at the rational surface) of the order of $B_r/B_0 = w_{th}^2/(r_s L_s)$. For the above parameters of JET, this gives $B_r/B_0 \simeq 9.4 \times 10^{-5}$. The above expression for the threshold magnetic island w_{th} leads to the following scaling of the critical magnetic field $B_r/B_0 \sim T n^{1/3} B^{-8/3}$. This is not too far from JET experimental scaling $B_r/B_0 \sim n^{0.55} B^{-1.25} \omega_0$ [25] assuming that $\omega_0 \sim T/B$.

In general, finite ion Larmor radius effects [16, 24] can also be important within the viscous-resistive layer. Note that for the above parameters the viscous-resistive layer width is of the same order of magnitude as the ion Larmor radius. In our estimates, we have used classical plasma conductivity, while the perpendicular viscosity was taken to be anomalous. In fact, electrical conductivity can also be anomalous due to the stochastization of electron trajectories (such effects could be especially important near the separatrix of a magnetic island). Then one may need to employ a mean field Ohm's law [26, 27] for a turbulent collisionless plasma. Such a hierarchical MHD and turbulence model in the context of the magnetic island evolution was proposed in [26].

Influence of the equilibrium plasma flow on tearing modes has long been a subject of intense interest [30–38]. It was shown that the stability of the linear tearing modes is changed by a finite plasma flow. Earlier studies have shown that the perpendicular plasma flow caused by plasma diffusion is generally stabilizing [30, 31], though later a new region of the instability was also found [37]. Large shear flow may induce new instability due to coupling to Kelvin–Helmholtz modes [33, 35]. Nonlinear coupling of modes with different helicity may also destabilize magnetic islands and lead to oscillatory behaviour [38]. In the nonlinear stage, the shear flow and viscosity was shown to reduce the growth rate and decrease the saturated island width [33, 34]. Deformation of the magnetic flux surfaces due to the plasma flow is clearly noticeable in numerical simulations [34]; however, no detailed studies of these effects have been made so far.

In summary, we have investigated an effect of the asymmetric deformation of the magnetic island caused by the finite plasma viscosity and sheared plasma flow. Such an effect is stabilizing for sufficiently narrow magnetic islands and may be responsible for the threshold typically observed in experiments with externally applied resonant magnetic perturbations [29, 39–41].

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