Non-local energy transport in tunneling and plasmonic structures

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Abstract: Various definitions of the velocity of propagation of the electromagnetic field have been adopted in experimental and theoretical studies of tunneling and plasmonic systems. Tunneling problems are often analyzed by invoking the group delay (or dwell time) velocities. On the other hand, slow light and plasmonic systems are considered by using the wave packet group velocity. This paper discusses various definitions for the velocity of the electromagnetic wave propagation and compares them in applications to the problems of slow light and superluminality in resonant and tunneling structures. Energy propagation is, in general, a nonlocal quantity and depends on the global properties of the system, rather than being simply a local quantity. The energy propagation velocity takes into account the non-local characteristics of the wave propagation and offers a natural generalization for those situations when the group velocity is ill defined or gives unphysical results. It is shown that the group delay velocity, which may be superluminal away from the resonance, becomes equal to the energy velocity at the resonant point.

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1. Introduction

There has been renewed interest in recent years to the question of how fast electromagnetic radiation (light) propagates in material media. One of the motivations for these studies originates in the problem of apparent superluminal propagation in quantum-mechanical tunneling [1, 2]. Since the stationary Schrodinger equation is equivalent to the wave equation for electromagnetic waves, similar phenomena exist for electromagnetic waves [1, 3–6]. The problem has received new impetus again due to the discovery of the phenomenon of slow light in quantum mechanical systems and photonic structures [7–11]. Tunneling and slow light phenomena have also been investigated in metamaterials and Photonic Band Gap (PBG) systems [12–14]. These exotic properties of slow and stopped light and wave propagation in metamaterials and PBG media offer a wide range of applications in various fields.

Over the course of numerous studies, several different measures of the speed of electromagnetic wave propagation were used, such as the group delay velocity v_d, the group velocity $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$, and the energy velocity $\mathbf{v}_E$ [15]. The group delay velocity v_d is introduced by using the group delay [16], which is defined as the time at which the transmitted packet reaches its peak at the exit of the medium: $v_d = \partial\omega/\partial\kappa$, where $\kappa = L\varphi_t$ and φ_t is the phase of the transmission coefficient. The energy velocity $\mathbf{v}_E = \mathbf{S}/U$ is introduced as the ratio of the Poynting vector to the electromagnetic energy density inside the medium [15]. One can easily show that for a finite wave packet in an unbounded non-dissipative medium, the energy velocity is equal to the group velocity [17]. This work aims to study the differences and mutual relations between various measures of the propagation speed of electromagnetic radiation. One important point to note is that the various definitions of the velocity do not necessarily apply to the velocity of energy (or information) propagation.

It is generally accepted that for a transparent medium, which supports propagating waves, the concept of group velocity is a valid measure of the energy flow velocity, and therefore, the information flow velocity, given that the information flow has to be associated with a finite energy flow. The group velocity is widely used, in particular, to characterize various slow light systems. Generally, slow light is realized in systems with high dispersion, when the group velocity is orders of magnitude smaller than the speed of light. There are however a number of physical situations when the concept of group velocity has limited utility, becomes misleading, or does not exist. One such situation occurs when the group velocity is mentioned as an upper bound for the information velocity. This could be easily incorrect for wave propagation in a strongly dissipative medium where the group velocity can exceed the speed of light. In other situations, the group velocity is not well defined, such as in tunneling, in which the wave vector becomes purely imaginary and the solution is a combination of evanescent modes. Such solutions are perfectly capable of energy (and hence information) transport [18, 19], yet the group velocity cannot be defined. There is another reason why the group velocity is not necessarily the measure of the energy velocity even for the case of non-dissipative transparent media, when the group velocity is well defined. A simple illustration of this fact can be seen from the example of a standing wave in a transparent medium with a well defined $\varepsilon(\omega) > 0$ and $\partial\omega/\partial\mathbf{k} \neq 0$, which nevertheless does not transport any energy. The group velocity, $\partial\omega/\partial\mathbf{k}$, is a local quantity and is defined for wavepackets in an infinite medium. It does not take into account wave reflections, and, as such, it does not depend on the geometry of the medium, e.g. on the boundary conditions. As a matter of fact, the energy flux through the medium, and, consequently the energy flow velocity, is in general a non-local quantity. It is not defined solely by the local properties of the dielectric medium via the local function $\omega = \omega(\mathbf{k})$, but also depends on the overall geometry of the system, in particular, on boundary conditions.

In this paper we illustrate that in general the group velocity does not fully characterize the energy transport even for a simplest case of a transparent (non-dissipative) media. It suggests that the energy velocity $\mathbf{v}_E = \mathbf{S}/U$ is a better measure of the energy propagation velocity. It fully takes into account non-local effects (finite system dimensions and presence of boundaries). It is also remains well defined in those cases, when the group velocity becomes ill-determined (as in tunneling problems) and/or produces unphysical results. There exists noticeable disconnect between the approaches used to study slow light in various optical and electromagnetic structures and the methods used to study pulse propagation in tunneling problems. The slow light phenomenon is mainly interpreted as an effect of very small group velocity $v_g \equiv \partial\omega/\partial k \ll c$, resulting from large dispersion near the resonances (the nature of resonance varies for different systems). The concept of group velocity does not exist for evanescent waves in tunneling.

In the latter, the concept of the group delay and dwell time are often used as a measure of the propagation time. The interpretation of the group delay as the tunneling (traversal) time leads to apparent paradoxes such as superluminality [20–22]. Series of experiments by various groups [6, 23–26] have measured a group delay velocity which is higher than the speed of light. Superluminal velocities have also been measured in gain media [27]. It is important to note that superluminal propagation does not mean casuality violation, so that no information is transferred with velocities larger than the speed of light [23, 26].

It was noted [28, 29] that the delay time (group delay) is actually related to the storage time. Therefore saturation of the delay time for a sufficiently thick barrier becomes independent of the barrier width as the field penetrates only into a narrow part ("skin"depth) of the barrier (Hartman effect). The dwell time is a quantity related to the group delay time and has similar properties [28, 29].

The goal of this work is to analyze the relations among the energy velocity $\mathbf{v}_E$, the group velocity $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$, and the group delay velocity v_d in several configurations involving tunneling and evanescent wave energy transport. Our particular interest is in resonant regimes, in particular, those involving surface mode and standing wave resonances [30–32].

The very nature of the metamaterials is based on the resonances that are used to achieve simultaneous negative permittivity and negative permeability. It is thus of interest to study the energy transport in these resonant structures, in particular, those involving tunneling. We concentrate on two simple structures that involve tunneling barriers exhibiting two types of resonances: the surface mode resonance that occurs at the interface of the materials with opposite signs of the dielectric permittivity (permeability) and the standing wave resonance that occurs due to partial wave trapping between two barriers [18].

This article is organized as follows. In section 2, the concepts of group delay and energy velocity are explored are reviewed. In section 3, these concepts are applied to the propagation of electromagnetic waves through a slab to illustrate the main ideas and fix the notations. In section 4, energy transport is analyzed for the cases of resonant tunneling of evanescent waves through multilayered structures and the corresponding velocities are obtained. Finally, in section 5, some final remarks and the summary of this work are given.

2. Group delay time and energy velocity

To fix the notations, in this section we define the group delay or Wigner delay, which is used to determine the group delay velocity (not be confused with the group velocity). We also define the energy transport velocity, which will be compared with the group delay velocity and the standard group velocity for propagating modes.

The Wigner delay [16, 29] is defined by considering the propagation of a localized wavepacket. The localized wave packet, peaked around a frequency ω_0, can be presented in the form

$$\psi(x,t) = \int_\omega f(\omega-\omega_0)\exp[ikx - i\omega t]\,d\omega. \tag{1}$$

This packet propagates in vacuum with a group velocity $\partial\omega/\partial k$. Upon collision with a generic barrier, the reflected and transmitted wavepackets are generated

$$\psi_R(x,t) = \int_\omega f(\omega-\omega_0)|R|\exp[i\phi_r(\omega) + ikx - i\omega t]\,d\omega, \tag{2}$$

$$\psi_T(x,t) = \int_\omega f(\omega-\omega_0)|T|\exp[i\phi_t(\omega) + ikx - i\omega t]\,d\omega. \tag{3}$$

Assuming that the functions $f(\omega-\omega_0)|R|$ and $f(\omega-\omega_0)|T|$ are slowly varying functions of ω and that the phase changes rapidly with ω, using the stationary phase method [29, 33], these

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integrals can be estimated by setting the rate of change of the phase to zero

$$\frac{d}{d\omega}\left(\phi_r - \omega t\right) = 0, \tag{4}$$

$$\frac{d}{d\omega}\left(\phi_t + kd - \omega t\right) = 0. \tag{5}$$

These last equations suggest that the peaks of the reflected and transmitted packets appear at the positions $x = d$ (for the transmitted packet) and $x = 0$ (for the reflected packet) with delays given by [29, 33]

$$\tau_{gt} = \frac{d}{d\omega}\left(\phi_t + kd\right), \tag{6}$$

$$\tau_{gr} = \frac{d\phi_r}{d\omega}. \tag{7}$$

It can be shown that for symmetric barriers, the delay in transmission and in reflection are identical [34], and that will be the case considered here. In the most general case of an asymmetric barrier, the bi-directional delay is defined as [34]

$$\tau_g = |R|^2\,\tau_{gr} + |T|^2\,\tau_{gt}. \tag{8}$$

Since the group delay is defined as the time at which the transmitted packet peaks at the exit, the group delay velocity may be defined as the length of the barrier divided by the group delay

$$v_d = \frac{d}{\tau_{gt}}. \tag{9}$$

This velocity often becomes superluminal. Various interpretations of the group delay as a transit time, and the related consequences of superluminal propagation have been a topic of intense debate [29, 35, 36]. The group delay has been measured in a number of experiments [23–26], including experiments with gain media with negative group-velocity index [27]. The superluminal velocities reported in these experiments are not in disagreement with casuality since the group delay velocity is not a signal velocity (nor the energy/information transmission velocity) [23, 24, 27], which should always be less than c.

The energy velocity can be defined in general form as done by Brillouin and Sommerfeld based on the equation of energy conservation for the electromagnetic field [15]

$$\frac{\partial U}{\partial t} + \nabla\cdot\mathbf{S} = Q, \tag{10}$$

where U is the electromagnetic energy density, $\mathbf{S}$ is the Poynting vector (electromagnetic energy flux) and Q represents the Joule losses. The flux of any quantity can be represented as the density of this quantity multiplied by its flux velocity, that is, $\mathbf{S} = U\mathbf{v}_E$. This last representation of the Poynting vector is known as the Umov form [37]. For a wave packet in an infinite homogeneous medium, the energy flux and the energy density are constant with $\mathbf{v}_E = \partial\omega/\partial\mathbf{k}$, that is, the energy velocity equals the group velocity. In general, U and $\mathbf{S}$ can be position dependent and the local energy velocity can be non-uniform in space. In general, this energy velocity is always less than c even for gain media [38].

In the next section, we consider a plane electromagnetic wave incident on a one-dimensional structure in vacuum. Then, the energy flux can be conveniently written in terms of the amplitude of the incident electromagnetic field (E_0, H_0) and the transmission coefficient

$$S|_{z=0} = S|_{z=d} = \frac{1}{2}Z_0|H_0|^2|T|^2 = \frac{1}{2Z_0}|E_0|^2|T|^2, \tag{11}$$

where $Z_0 = E_0/H_0$ is the impedance of free space.

It is instructive to consider how the energy is transmitted through a tunneling barrier with only evanescent waves present. Consider an evanescence region such that the magnetic field $H_x(z)$ is given by

$$H_x(z) = C\exp(-\kappa z) + D\exp(\kappa z), \tag{12}$$

with κ a real number but C and D possibly complex, being the amplitudes of two waves in the region. By Maxwell's equations, the electric field associated with these waves is given by

$$E_y(z) = -i\frac{1}{\omega\varepsilon}\frac{\partial H_x}{\partial z}, \tag{13}$$

or

$$E_y(z) = i\frac{\kappa}{\omega\varepsilon}\left(C\exp(-\kappa z) - D\exp(\kappa z)\right). \tag{14}$$

For the Poynting flux we have

$$S_z = \frac{1}{2}\Re(E_y H_x^*) \sim (CD^* - C^*D) \sim \Im(CD^*). \tag{15}$$

As can be seen from Eq. (15) the z-component of the Poynting flux becomes finite when the product CD^* has an imaginary part other than zero; in other words, there has to be a finite phase shift between the amplitudes C and D. Therefore, a finite energy flux occurs as a result of the superposition of two evanescent modes with a finite phase shift [18]; this is called interference of evanescent waves [18]. It is important to note that the reflected and transmitted waves (separated by the evanescent region) can be viewed as the result of the energy "leaking" from the system through two different channels (reflection and transmission respectively) [18]. Energy transport by the evanescent waves inside the barrier is responsible for the tunneling. The continuity (energy conservation) Eq. (10) introduces the velocity for this energy transport, this way naturally introducing the measure for the tunneling velocity. It is important to note that this energy transport is purely electromagnetic and does not include other forms of energy present inside the medium, nor any "localized energy" associated with the degrees of freedom in the medium [38].

3. Velocities in dispersive slab

Consider y-polarized plane waves propagating in the z-direction, normally incident on a slab of dispersive material. The general electromagnetic field can be represented then in the form

$$\mathbf{E} = [0, E_y, 0]e^{i(kz-\omega t)}, \tag{16}$$

$$\mathbf{H} = [H_x, 0, 0]e^{i(kz-\omega t)}. \tag{17}$$

Since E_y and H_x are related, the full solution is uniquely defined by the magnetic field H_x. In general, Maxwell equations define the full solution as a sum of two waves propagating in opposite directions (as in the plasma region $0 < z < d$). The ratio of the amplitude of the backward wave to the amplitude of the incident wave in vacuum region on the left $(z < 0)$ defines the reflection coefficient R. There is no reflected wave in the vacuum region on the right of the plasma slab $(z > d)$:

$$H_0\left(e^{ik_v z} + Re^{-ik_v z}\right) \qquad z < 0, \tag{18}$$

$$H_0\left(Ae^{ikz} + Be^{-ikz}\right) \qquad 0 < z < d, \tag{19}$$

$$H_0 Te^{ik_v z} \qquad z > d. \tag{20}$$

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The wave vector k is a function of ω

$$k^2 = \frac{\omega^2}{c^2}\,\varepsilon(\omega), \tag{21}$$

where the dielectric constant $\varepsilon(\omega)$ is taken to be in the form

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}. \tag{22}$$

For the case where $\varepsilon(\omega) < 0$, the wavevector becomes imaginary and the fields become evanescent.

The parameters A, B, R and T are obtained by using the boundary conditions for the fields (e.g continuity of E_y and H_x at the boundaries $z = 0$ and $z = d$). The resulting reflection and transmission coefficients for this structure are given by

$$T = \frac{e^{-ik_v d}}{g}, \tag{23}$$

$$R = i\frac{(k\varepsilon_0/k_v\varepsilon - k_v\varepsilon/k\varepsilon_0)\sin kd}{2g}, \tag{24}$$

where

$$g = \cos kd - i\,(k\varepsilon_0/k_v\varepsilon + k_v\varepsilon_0/k\varepsilon)\sin kd/2, \tag{25}$$

$$k_v = \frac{\omega}{c}, \tag{26}$$

and

$$k = \frac{\omega}{c}\sqrt{\varepsilon(\omega)}. \tag{27}$$

The above formulas also remain valid in the evanescent case, where k is replaced with $i\kappa$.

The phase of the transmission coefficient is given by

$$\phi_0 = \phi_t + k_v d = \arctan\left(\frac{k\varepsilon_0/k_v\varepsilon + k_v\varepsilon_0/k\varepsilon}{2}\tan kd\right). \tag{28}$$

The reflection and transmission responses for the slab are plotted in Fig. 1 as a function of $\varepsilon(\omega)$.

The group delay velocity is given by

$$v_d = \frac{d}{\tau_{gt}} = d\left(\frac{\partial\phi_0}{\partial\omega}\right)^{-1}. \tag{29}$$

The averaged energy velocity in the direction of propagation (z-direction) is given by

$$v_E = \frac{\frac{1}{2}Z_0|H_0|^2|T|^2}{\int_0^d U dz}, \tag{30}$$

where U is given by

$$U = \frac{1}{4}\varepsilon_0\frac{\partial(\omega\varepsilon)}{\partial\omega}|E_y|^2 + \frac{1}{4}\mu_0|H_x|^2. \tag{31}$$

For the case of a dispersive slab, Eq. (30) reduces to

$$\frac{v_E}{c} = \frac{4}{(1+\varepsilon)\left(1+\frac{1}{\varepsilon}\frac{\partial(\omega\varepsilon)}{\partial\omega}\right) + (\varepsilon-1)\left(1-\frac{1}{\varepsilon}\frac{\partial(\omega\varepsilon)}{\partial\omega}\right)\frac{\sin 2kd}{2kd}}. \tag{32}$$

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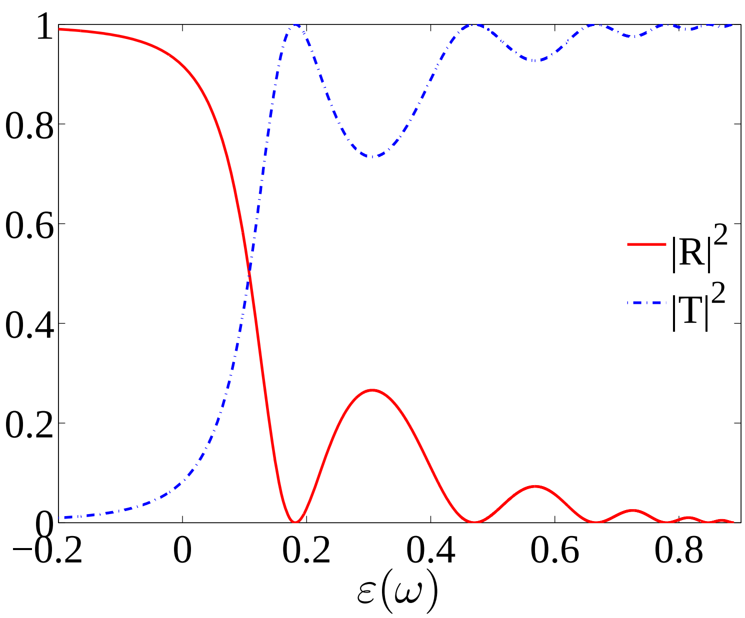

Fig. 1. Reflection and transmission responses of the slab layer: ω_p=10 GHz, and d=0.02 m.

For a medium with dielectric permittivity given by Eq. (22), such that

$$\frac{\partial(\omega\varepsilon)}{\partial\omega} = 1 + \frac{\omega_p^2}{\omega^2} = 2 - \varepsilon, \tag{33}$$

the energy velocity divided by c (normalized to the speed of light), reduces to

$$\frac{v_E}{c} = \frac{2\varepsilon}{1 + \varepsilon + \frac{\sin 2kd}{2kd}(\varepsilon - 1)^2}, \tag{34}$$

which, as can be seen clearly, remains always subluminal and will oscillate due to the term $\frac{\sin 2kd}{2kd}$.

The usual group velocity is given by the usual formula $v_g = \partial\omega/\partial k$, which is valid only for regions where $\varepsilon > 0$. The energy, group and group delay velocities are plotted in Fig. 2(a). A close-up look of the region where $\varepsilon \to 0$ is given in Fig. 2(b).

Figure 2(a) illustrates the difference between group delay velocity, group velocity and energy velocity. The group delay velocity is oscillating as a function of the dielectric constant. These oscillations correspond to the oscillations in the transmission coefficient as in Fig. 1. The energy velocity is different from the group velocity because of the finite thickness of the layer. As can be seen from Eq. (34), due to dispersion, the energy velocity also oscillates. The energy velocity becomes equivalent to the group velocity in the limit of $\varepsilon \to 1$. It can be seen that the group delay velocity becomes superluminal in the tunneling regime ($\varepsilon < 0$). It is worth noting here that while the group velocity approaches zero at $\varepsilon = 0$, the energy flow velocity is never zero. In tunneling, for the non resonant case it can be very small. It is interesting to notice that the energy velocity remains small for tunneling even in the resonant case, contrary to the resonant cases with surface wave resonances as in Section 4.

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In the case of a medium whose dielectric permittivity is given by the Drude-Lorentz model

$$\varepsilon = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}, \tag{35}$$

where ω_0 is the resonant frequency of the electron motion [17], the situation is similar. The energy velocity will oscillate and it will be equal to the group delay velocity when the transmission is total. The group velocity is not equal to the energy velocity. Similarly to the previous model, all the velocities tend to the same value as $\varepsilon \to 1$ as can be seen from Fig. 2(c).

This difference among the different velocities is valid even for propagating cases as can be seen from Figs. 3(a) and 3(b) where the different velocities are plotted as functions of frequency and barrier width for a nondispersive case with dielectric constant larger than 1. In these examples the difference between the group velocity and the energy velocity is clearly seen, with the energy velocity being smaller than the group velocity. This can be seen from the expressions for the energy velocity in a non-dispersive slab. The energy velocity as defined in Eq. (30), reduces, for a nondispersive slab, to

$$\frac{v_E}{c} = \frac{2}{\varepsilon + 1}. \tag{36}$$

As is clear from Eq. (36), the energy velocity is dependent on the dielectric constant of the slab and, contrary to the energy velocity for the dispersive case and to the group delay velocity, does not oscillate but has a constant value. This same situation happens also with the group velocity, that does not oscillate for the non-dispersive case, whereas it does for the case with dispersion. It is worth noticing that its value is always lower than that of the group velocity, which for non dispersive slabs is given by $v_g = c/\sqrt{\varepsilon}$, the only exception being of course in vacuum. The energy velocity remains always subluminal due to the fact that $\varepsilon < 1$ implies dispersion and therefore Eq. (36) is no longer valid. For $0 < \varepsilon < 1$, the group velocity will be superluminal.

The group delay velocity is oscillating along with the transmission coefficient of the slab. The group delay velocity becomes equal to the energy velocity in the regions where the transmission is resonant as can be seen from Figs. 3(a) and 3(b), in accordance to the result obtained by D'Aguanno *et al* [39].

4. Velocities for a double layer and a double barrier

Consider a two layer structure as shown in Fig. 4(a). The layers have a dielectric constants $\varepsilon_1 = \varepsilon_1(\omega)$ and $\varepsilon_2 = \varepsilon_2(\omega)$ and widths L_1 and L_2, for a total width of $L = L_1 + L_2$. The electromagnetic wave that will be considered is TM-polarized (p-polarized), with the electric field in the incidence plane (y,z) and the magnetic field in the x-direction. Denoting:

$$k_v^2 = k_0^2 - k_y^2, \qquad k_0 = \omega/c \quad k_y = k_0 \sin\theta_i, \tag{37}$$

$$\kappa_1^2 = k_y^2 - \varepsilon_1(\omega) k_0^2, \tag{38}$$

$$\kappa_2^2 = k_y^2 - \varepsilon_2(\omega) k_0^2, \tag{39}$$

$$\eta_i = \varepsilon_i / \kappa_i, \tag{40}$$

the fields in the structure are given by

$$H_0 \left(e^{ik_v z} + R e^{-ik_v z} \right) \qquad z < 0, \tag{41}$$

$$H_0 \left(A e^{-\kappa_1 z} + B e^{\kappa_1 z} \right) \qquad 0 < z < L_1, \tag{42}$$

$$H_0 \left(C e^{-\kappa_2 (z - L_1)} + D e^{\kappa_2 (z - L_1)} \right) \qquad L_1 < z < L, \tag{43}$$

$$H_0 \, T \, e^{ik_v z} \qquad z > L. \tag{44}$$

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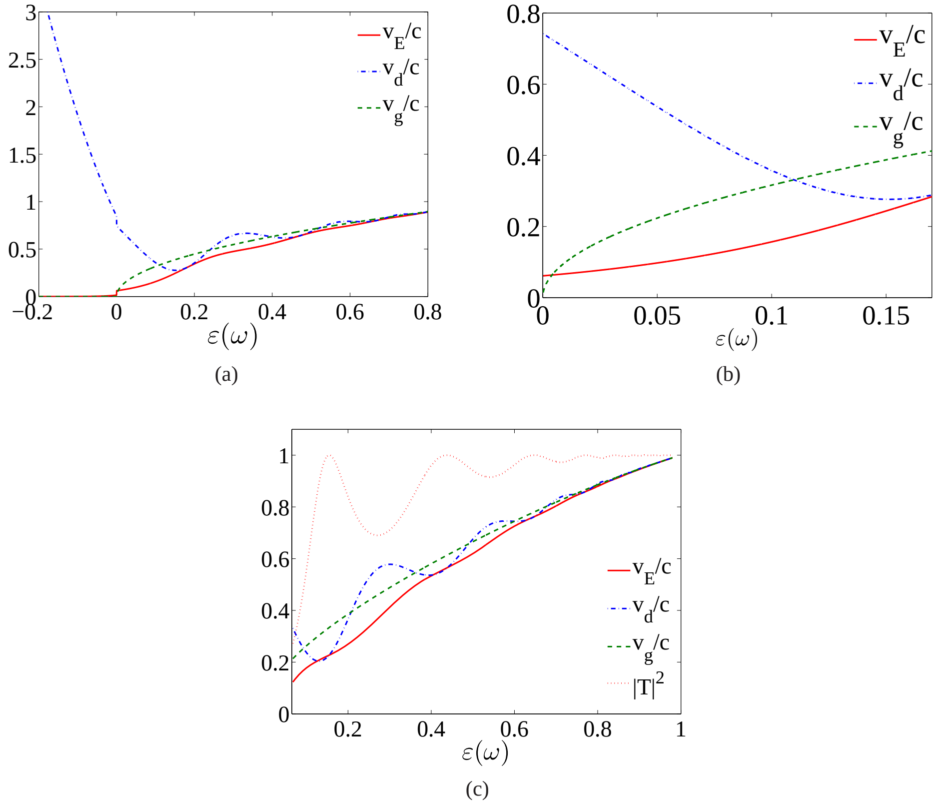

Fig. 2. (a) Normalized energy, group delay and group velocities of the slab layer, ω_p=10 GHz and d=0.02 m; (b) Close-up of the region where $\varepsilon \to 0$; (c) the case with dispersion, ε is given by Eq. (35), $\omega_0 = 0.5\,\omega_p$, d and ω_p as in (a).

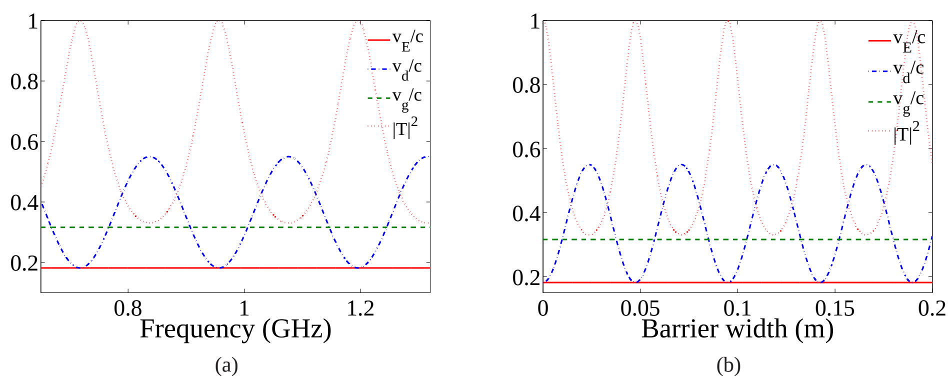

Fig. 3. Normalized energy, group delay and group velocities for $\varepsilon = 10$. (a) As a function of frequency; a=0.2 m. (b) As function of the slab width; f=1 GHz.

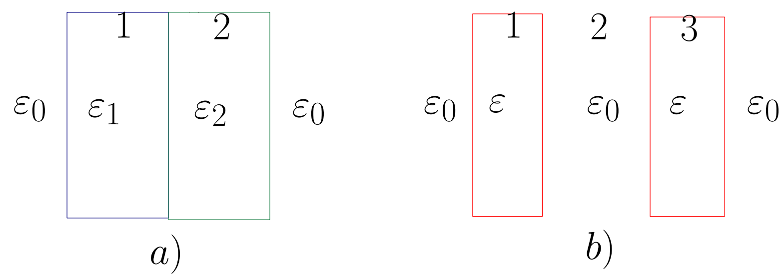

Fig. 4. (a) The two layer structure, region 1- $0 < \varepsilon_1 < 1$; region 2 - $\varepsilon_2 < 0$. Incidence angles larger than the critical are considered, so waves are evanescent in both regions. (b) The double barrier structure; regions 1,3 - $\varepsilon < 0$; region 2 - $\varepsilon = 1$.

Similar to the procedure used to obtain the reflection and transmission responses of the slab, by applying the boundary conditions for the fields at the interfaces and solving the corresponding equations, the transmission coefficient for the double layer is

$$T = \frac{e^{-ik_v L}}{g}, \tag{45}$$

$$g = \cosh \kappa_2 L_2 \cosh \kappa_1 L_1 + \Delta_1 \sinh \kappa_1 L_1 \sinh \kappa_2 L_2 + i \left[\Delta_2 \sinh \kappa_1 L_1 \cosh \kappa_2 L_2 \right]. \tag{46}$$

The phase of the transmission coefficient is given by

$$\phi_0 = -\arctan \left[\frac{\Delta_2 \sinh \kappa_1 L_1 \cosh \kappa_2 L_2 + \Delta_3 \cosh \kappa_1 L_1 \sinh \kappa_2 L_2}{\cosh \kappa_2 L_2 \cosh \kappa_1 L_1 + \Delta_1 \sinh \kappa_1 L_1 \sinh \kappa_2 L_2} \right]. \tag{47}$$

Here Δ_1, Δ_2 and Δ_3 are defined as

$$\Delta_1 = \frac{1}{2} \left(\frac{\eta_1}{\eta_2} + \frac{\eta_2}{\eta_1} \right), \tag{48}$$

$$\Delta_2 = \frac{1}{2} \left(\frac{\eta_1}{\eta_0} - \frac{\eta_0}{\eta_1} \right), \tag{49}$$

$$\Delta_3 = \frac{1}{2} \left(\frac{\eta_2}{\eta_0} - \frac{\eta_0}{\eta_2} \right). \tag{50}$$

The resonant conditions of the structure are given by the conditions [18]

$$\kappa_1 L_1 = \kappa_2 L_2, \tag{51}$$

$$\eta_1 + \eta_2 = 0. \tag{52}$$

This resonance corresponds to the excitation of surface modes in the structure. In Fig. 5 the resonance at the interface of the slabs can be seen.

The plots for the transmittance, the group delay velocity and the energy velocity are given in Figs. 6(a)–7. Note that the energy velocity stays subluminal and reaches a maximum value of 0.05 c.At the resonance, where the transmission coefficient is 1, the group delay velocity is

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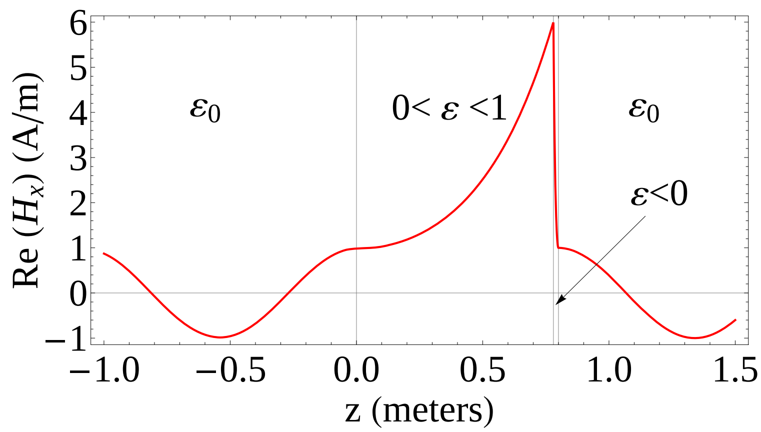

Fig. 5. Magnetic field distribution for the double layer structure. The width of the first layer $L_1 = 0.78m$, the width of the second layer $L_2 = 0.02m$, $\varepsilon_1 = 0.9$, $\varepsilon_2 = -35$. The resonance occurs at $\theta \simeq 74°$.

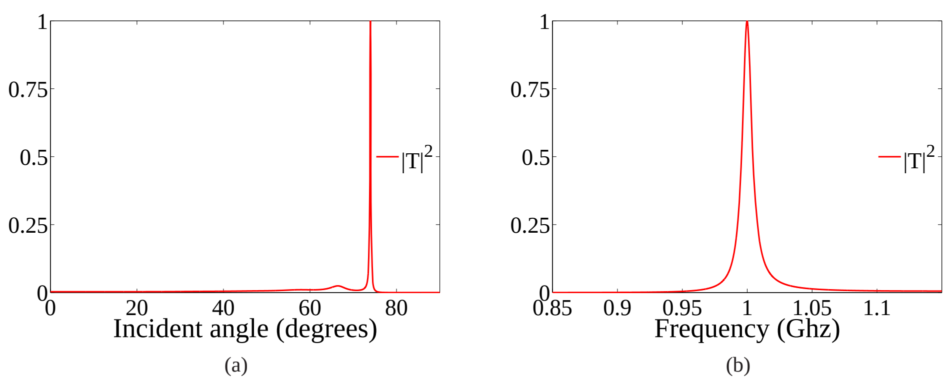

Fig. 6. Transmissivity of the double layer for $\omega_{p1} = 3.77 \times 10^{10}$ rad s^{-1} (6 GHz), $\omega_{p2} = 2 \times 10^9$ rad s^{-1} (0.32 GHz), $L_1 = 0.78m$, $L_2 = 0.02m$. (a) As a function of the incident angle with f=1 GHz. (b) As a function of frequency at resonant incidence.

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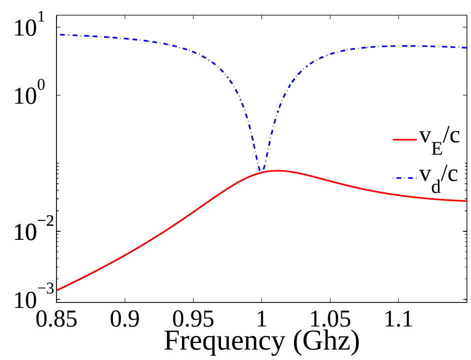

Fig. 7. Normalized energy velocity and group delay velocity of the double layer structure at resonant incidence.

equal to the energy velocity. The group delay velocity becomes superluminal away from the resonance.

For the double barrier, two symmetrical barriers of width a are bounded by vacuum and separated by another vacuum region of width L as shown in Fig. 4(b). As in the double layer case, a TM-polarized electromagnetic wave will be considered and the same notations will be used.

The fields in this configuration are given by

$$H_0\left(e^{ik_v z}+R\,e^{-ik_v z}\right) \qquad z<0, \tag{53}$$

$$H_0\left(A_0\,e^{-\kappa z}+B_0\,e^{\kappa z}\right) \qquad 0<z<a, \tag{54}$$

$$H_0\left(A_1\,e^{ik_v z}+B_1\,e^{-ik_v z}\right) \qquad a<z<L+a, \tag{55}$$

$$H_0\left(A_2\,e^{-\kappa(z-L-a)}+B_2\,e^{\kappa(z-L-a)}\right) \qquad L+a<z<L+2a, \tag{56}$$

$$H_0\,T\,e^{ik_v z} \qquad z>L+2a. \tag{57}$$

As it was done for the slab and the double layer, by using Maxwell equations to obtain the electric field in the double barrier and using the boundary conditions at the interfaces, the transmission coefficient can be calculated to be

$$T=\frac{e^{-2ik_v a}}{g}, \tag{58}$$

$$\begin{aligned} g &= \cosh^2(\kappa a)+\frac{1}{4}\sinh^2(\kappa a)\left[\sigma^2\cos(k_v L)-\delta^2\right] \\ &+ i\sinh(\kappa a)\left[\delta\cosh(\kappa a)+\frac{1}{4}\sigma^2\sinh(\kappa a)\sin(2k_v L)\right]. \end{aligned} \tag{59}$$

Here

$$\sigma=\frac{\eta_0}{\eta_1}+\frac{\eta_1}{\eta_0}, \tag{60}$$

$$\delta=\frac{\eta_1}{\eta_0}-\frac{\eta_0}{\eta_1}, \tag{61}$$

$$\sigma^2=\delta^2+4. \tag{62}$$

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The resonant conditions for the double barrier structure are given by [36, 40]

$$\cot(k_v L) = -\frac{1}{2}\delta \tanh(\kappa a), \tag{63}$$

or by [18]

$$\tan(k_v L) = \frac{2\xi}{1-\xi^2}, \qquad \xi = \frac{\eta_0}{\eta} = \frac{\kappa \varepsilon}{k_v \varepsilon_0}. \tag{64}$$

The phase of the transmission coefficient is given by

$$\phi_0 = k_v L - \arctan\left[\frac{\sinh(\kappa a)\left[\delta \cosh(\kappa a) + (1/4)\sigma^2 \sinh^2(\kappa a)\sin(2k_v L)\right]}{\cosh^2(\kappa a) + (1/4)\sinh^2(\kappa a)\left[\sigma^2 \cos(2k_v L) - \delta^2\right]}\right]. \tag{65}$$

The resonant condition are the leaky eigenmode solutions to the finite depth well problem [18]. In Fig. 8 the resonances at the interface between the slabs and the inner vacuum region can be seen.

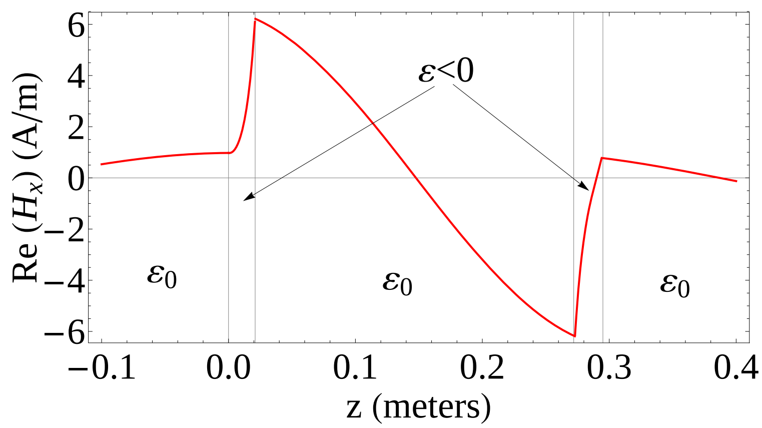

Fig. 8. Magnetic field distribution for the double barrier structure. The width of the barriers is $a = 0.021m$, the width of the middle layer is $L = 0.25m$, $\varepsilon = -35$. The resonance occurs at $\theta = 62.3°$.

The plots for the transmittance, the group delay velocity and the energy velocity are given in Figs. 9(a)–10. As before, the energy velocity is subluminal, with a maximum value of the order of 0.01 c. The group delay velocity is equal to the energy velocity at the resonance and becomes superluminal away from the resonance.

5. Discussion and summary

Various definitions of the velocities are often invoked in the discussions of slow light and superluminality problems. The conventional group velocity $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$ can be used as a measure of the energy velocity only for transparent (non-dissipative) weakly dispersive and infinite medium when the wave packet width is shorter than the characteristic width of the propagation region and no multiple reflections occur. In the latter case, the conventional group velocity is equal to the energy flow velocity given by the ratio of the Poynting flux to the energy density, $\mathbf{v}_g = \mathbf{v}_E \equiv \mathbf{S}/U$. This equality is broken however in many instances. Even in the simplest case of a transparent (non-dissipative) dispersive medium the energy transport velocity in a finite width

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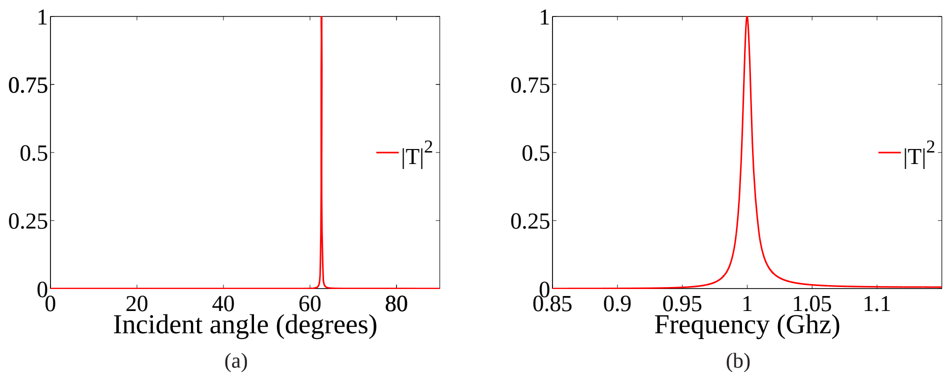

Fig. 9. Transmissivity of the double barrier structure for $\omega_p = 3.77 \times 10^{10}$ rad s^{-1} (6 GHz), $L = 0.25m$, $a = 0.021m$. (a) As a function of the incident angle with f=1 GHz. (b) As a function of frequency at resonant incidence.

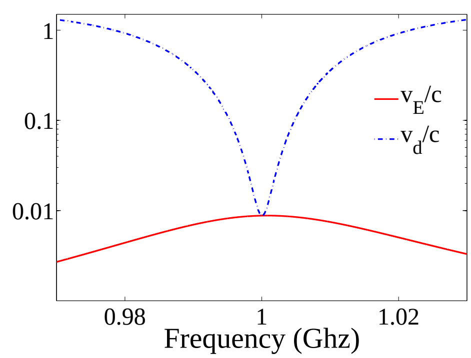

Fig. 10. Normalized energy and group delay velocities of the double barrier structure as function of frequency for resonant incidence.

region differs from the group velocity due to nonlocal (boundary) effects as it was discussed in Section 3.

The energy velocity $\mathbf{v}_E$ offers a natural generalization of the conventional group velocity $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$. It is worth noting here that the actual energy transport (and the energy velocity $\mathbf{v}_E$) remains finite even when the conventional group velocity becomes zero [41]. The energy velocity $\mathbf{v}_E$ describes the energy transport in other situations, such as in dissipative media, where the group velocity may become superluminal (and thus has not the physical meaning of the energy/information propagation velocity) and when the group velocity is not well defined (as in tunneling problems).

The other often referred measure of the propagation speed is the so called group delay velocity Eq. (9), based on the concept of the group delay which is calculated as the derivative of the phase of the transmission coefficient. The group delay saturates with the width of the tunneling barrier, leading to apparent superluminal propagation. Superluminal propagation of the group delay was measured in many experiments. It is not considered to result in any casuality violation since no information is transferred by a smooth wave packet. Signal/information transfer is related to the non-analytical points of the pulse and thus the velocity at which it is transmitted is that of the front (or precursor) and is limited by the speed of light [15]. It is worth noting that information transfer requires a finite energy transport which always remains subluminal.

In this paper, we have compared the different velocities associated with tunneling for different tunneling configurations that involve standing wave and surface mode resonances. The standing wave resonances result in bandgaps of multi-layer dielectric structures. The delay time in such structures were experimentally studied in [25], where it was shown that the delay time is very sensitive to the number of $\lambda/4$ layers. It is shown here that strong variations of the group delay velocity are related to system resonances due to standing waves and/or surface waves. These resonances result in oscillations of the propagation velocities as functions of the system width and frequency (Figs. 2 and 3). At the resonance, the group delay velocity v_d becomes subliminal and equal to the energy velocity v_E (Figs. 7 and 10). In the deeply tunneling regime, away from the resonance, the group delay velocity v_d may become superluminal while the energy velocity always remains subluminal. The evanescent resonant regimes studied in this paper are characterized by large amplitudes of the electromagnetic field and result in low energy velocities, reaching resonance values of 0.05 c for the double layer and of 0.01 c for the double barrier. These values suggest the possibility of using these resonant tunneling structures as slow light systems.

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