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Geodesic acoustic modes and zonal flows in rotating large-aspect-ratio tokamak plasmas

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Abstract

The effect of equilibrium plasma rotation (toroidal and poloidal) on low-frequency, electrostatic modes—the geodesic acoustic modes (GAMs) and the zonal flows (ZFs)—in large aspect ratio tokamaks is studied within the framework of ideal MHD. It is shown that the plasma rotation results in a frequency up-shift of the ordinary GAM. The new branch of continuum modes induced by the poloidal rotation is found. This mode originates from the opposite sign Doppler shift of frequency due to poloidal rotation for $m = \pm 1$ poloidal side-band harmonics of the perturbed mass density, pressure and parallel velocity. In the case of slow poloidal rotation ($\Omega_P \ll c_s/qR_0$) its frequency is close to the sound frequency c_s/qR_0 (Ω_P is the poloidal angular velocity, c_s is the speed of sound, q is the safety factor and R_0 is the major radius of tokamak). The mode can be called the rotation-induced acoustic mode. This mode disappears in the case of purely toroidal plasma rotation. The frequency of the new mode in the case of relatively slow poloidal rotation ($\Omega_P \leqslant c_s/qR_0$) is lower than the frequency of the ordinary GAM modified by plasma rotation. In the case of larger poloidal angular velocities Ω_P ($\Omega_P \geqslant 2c_s/qR_0$) the mode becomes unstable and is identified as the unstable ZF. With a further increase in the poloidal angular velocity at constant toroidal angular velocity the instability is suppressed, and the mode turns again into a marginally stable, oscillating mode.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Geodesic acoustic modes (GAMs) have been actively studied in recent years. These modes involve oscillations of perturbed pressure and mass density induced by plasma compressibility due to geodesic curvature of the magnetic lines of forces, inherently present in toroidal configurations. The modes are localized on the flux surfaces and characterized by poloidally uniform radial electric field and poloidally oscillating perturbations of plasma pressure and mass density. These modes were predicted a long time ago by Winsor *et al* [1] and were observed in a variety of tokamaks [2–5]. Theoretical analysis suggests an important role of such modes in turbulence and anomalous transport regulation [6, 7], in particular in plasmas with a significant fraction of high-energy particles [8]. Most of the theoretical studies of GAMs have been performed for static plasma equilibria. However, tokamak equilibria with essential mass flows may exist. Toroidal plasma velocities comparable to the sound velocity have been reached in experiments in both large-aspect-ratio and spherical tokamaks with neutral beam injection (see, e.g., [9, 10]).

The effects of toroidal rotation on GAMs and similar Alfvénic modes have been investigated recently [11–17]. For the case of plasma equilibrium with isothermal magnetic surfaces two branches of the continuum spectrum have been found [11–13]. One is a modification of an ordinary GAM. This branch smoothly matches with the standard GAM in the absence of toroidal rotation. The GAM's frequency is shifted up by the toroidal rotation. The other mode is a new, lower frequency branch induced by toroidal plasma flow. The new mode appears as a consequence of the non-uniform equilibrium plasma density and pressure created by the centrifugal force on the magnetic surfaces of the tokamak. The frequency of this mode goes to zero in the limit of zero toroidal rotation. Both branches of continuum spectrum in a toroidally rotating plasma are marginally stable within the MHD model. Also, the effect of toroidal rotation on the Alfvén continuum for the case of plasma equilibrium with isothermal magnetic surfaces was numerically studied in [14] within the framework of MHD equations.

Another type of plasma equilibrium with toroidal flow and uniform plasma density on magnetic surfaces was considered in [15] and in our recent paper [16]. An unstable continuum was numerically found in [15], in studying the toroidal rotation effect on Alfvén continuum, but no analytical expressions for such a continuum were presented. An analytical study of electrostatic perturbations of the equilibrium with uniform plasma density on magnetic surfaces was carried out in [16]. Similar to the case of the equilibrium with isothermal magnetic surfaces in [16] two branches of the continuum spectrum for the electrostatic perturbations have been found. The first branch is an ordinary GAM modified by the toroidal rotation. It was shown that the second branch (low frequency) has a real part of frequency equal to zero and can be identified as the unstable zonal flow (ZF). The latter, unstable mode is induced by the toroidal rotation and is a consequence of the non-uniform plasma pressure created by the centrifugal force on the magnetic surfaces.

All possible plasma equilibria with a mass flow are not reduced to equilibria with toroidal rotation only. The tokamak plasmas can also rotate in the poloidal direction. Such rotation can be caused by both neoclassical and turbulent effects [18, 19]. In general, tokamak equilibria have both toroidal and poloidal rotation [20]. In [17] the Alfvén modes in tokamaks with mass flow parallel to the equilibrium magnetic field have been considered within the kinetic theory. In this paper we generalize the analytical theory of GAMs for the case of plasma equilibrium with both toroidal and poloidal flows. This problem is analyzed using the standard ideal MHD equations with adiabatic equation of state. In section 2 we consider the general plasma equilibrium with poloidal and toroidal flows and calculate the non-uniform (on the magnetic surfaces) parts of the plasma mass density and pressure created by the centrifugal forces. The axisymmetric (independent of the toroidal angle), electrostatic perturbations are treated in section 3. The linearized equations for perturbations are simplified in section 3.1. Details of the derivation are given in appendices A and B. Using these equations we obtain the poloidally dependent parts of the perturbations of parallel plasma velocity and of mass density (section 3.2). The normal mode equation for the axisymmetric perturbations is derived in section 3.3. Continuum modes described by this equation are studied, which are not the eigenmodes of the equation (see, for example, [21, 22]). The dispersion relation for the continuum spectrum is derived and the properties of this continuum are analyzed analytically (in the limit of slow rotation) and numerically (in the case of a faster rotation) in section 3.4. Conclusions are presented in section 4.

2. Equilibrium

We describe tokamak plasma by ideal MHD equations. In the CGS system they have the form

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)\boldsymbol{v}=-\nabla p+\frac{1}{c}\boldsymbol{j}\times\boldsymbol{B},\qquad \boldsymbol{j}=\frac{c}{4\pi}\nabla\times\boldsymbol{B},\tag{1}$$

$$\frac{\partial\boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B}),\qquad \nabla\cdot\boldsymbol{B}=0,\tag{2}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\boldsymbol{v})=0,\tag{3}$$

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)\frac{p}{\rho^{\Gamma}}=0,\tag{4}$$

where ρ is the mass density, p is the pressure, $\boldsymbol{v}$ is the velocity, $\boldsymbol{B}$ is the magnetic field, c is the speed of light and Γ is the ratio of specific heats.

We consider the general stationary equilibrium of the axisymmetric tokamak with the flow. As follows from equations (1)–(4) such an equilibrium is defined by the equations

$$\rho_0(\boldsymbol{v}_0\cdot\nabla)\boldsymbol{v}_0=-\nabla p_0+\frac{1}{c}\boldsymbol{j}_0\times\boldsymbol{B}_0,\qquad \boldsymbol{j}_0=\frac{c}{4\pi}\nabla\times\boldsymbol{B}_0,\tag{5}$$

$$\nabla\times(\boldsymbol{v}_0\times\boldsymbol{B}_0)=0,\qquad \nabla\cdot\boldsymbol{B}_0=0,\tag{6}$$

$$\nabla\cdot(\rho_0\boldsymbol{v}_0)=0,\tag{7}$$

$$(\boldsymbol{v}_0\cdot\nabla)\frac{p_0}{\rho_0^{\Gamma}}=0.\tag{8}$$

Hereafter the subscript 0 is used to denote the equilibrium value of the corresponding function. We use cylindrical coordinates $\{R,\varphi,z\}$. Because of the symmetry, the equilibrium quantities do not depend on the toroidal angle φ ($\partial/\partial\varphi=0$). It is assumed that the magnetic field of the tokamak forms the set of nested magnetic surfaces $\psi(R,z)=\text{const}$, $\boldsymbol{B}_0\cdot\nabla\psi=0$. Here $2\pi\psi$ is the poloidal flux. Then from $\nabla\cdot\boldsymbol{B}_0=0$ we obtain the equilibrium magnetic field

$$\boldsymbol{B}_0=F(R,z)\nabla\varphi+\nabla\psi\times\nabla\varphi.\tag{9}$$

The current density and the square of magnetic field magnitude are given by the equations

$$\boldsymbol{j}_0=\frac{c}{4\pi}(\nabla F\times\nabla\varphi-\Delta^{\star}\phi\nabla\varphi),\qquad \Delta^{\star}\psi\equiv R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right),$$

$$B_0^2=\frac{F^2+|\nabla\psi|^2}{R^2}.\tag{10}$$

Integrating equation (6), we find

$$\boldsymbol{v}_0\times\boldsymbol{B}_0=c\nabla\phi,\tag{11}$$

where ϕ is the electrostatic potential. It is obvious from equation (11) that $\boldsymbol{B}_0\cdot\nabla\phi=0$, which means that the electrostatic potential is constant on the magnetic surface, $\phi=\phi(\psi)$. Taking into account equation (9), we obtain the flow velocity

$$\boldsymbol{v}_0=\lambda(R,z)\boldsymbol{B}_0+R^2\Omega(\psi)\nabla\varphi,\tag{12}$$

where

$$\Omega = c\frac{\mathrm{d}\phi}{\mathrm{d}\psi}. \tag{13}$$

Equation (12) splits the flow into parallel to magnetic field and toroidal parts. In the absence of parallel flow ($\lambda = 0$) the plasma rotation is toroidal, and Ω is the toroidal angular velocity. Substituting equation (12) in equation (7), we find that $\lambda = \kappa(\psi)/\rho_0$. As a result, the general plasma flow admitted by the equilibrium equations is described by the equation

$$\boldsymbol{v}_0 = \frac{\kappa(\psi)}{\rho_0}\boldsymbol{B}_0 + R^2\Omega(\psi)\nabla\varphi. \tag{14}$$

The magnetic field has both toroidal and poloidal components. Therefore, the first term in equation (14) includes both the poloidal plasma rotation and part of its toroidal rotation.

The angular velocity of the toroidal plasma rotation in the presence of plasma flow parallel to the magnetic field is described by

$$\Omega_\mathrm{T} = \Omega(\psi) + \frac{\kappa(\psi)}{\rho_0 R^2}F. \tag{15}$$

It is necessary to note that, in accordance with [20], no tokamak equilibrium with purely poloidal flow is possible. There is no possibility to cancel the toroidal flow because of the different poloidal modulation of the poloidal and parallel parts of the toroidal angular velocity, as follows from an analysis of equation (15).

An analysis of the poloidal rotation effect, related to the parallel plasma flow, on geodesic acoustic modes in the plasma is the main objective of this paper. For the description of MHD waves, it is convenient to convert from cylindrical coordinates $\{R, \varphi, z\}$ to straight field line coordinates $\{\psi, \theta, \varphi\}$. The Jacobian of such a coordinate transformation is

$$J = (\nabla\psi \times \nabla\varphi)\cdot\nabla\theta. \tag{16}$$

The poloidal angle θ is chosen such that the field lines are straight in the (θ, φ)-plane, i.e. the slope is the flux function

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\theta} = \frac{\boldsymbol{B}_0\cdot\nabla\varphi}{\boldsymbol{B}_0\cdot\nabla\theta} = \frac{F}{JR^2} \equiv q(\psi), \tag{17}$$

where q is the safety factor.

Then the poloidal angular velocity of the plasma is given by the equation

$$\Omega_\mathrm{P} \equiv \boldsymbol{v}_0\cdot\nabla\theta = \frac{\kappa(\psi)J}{\rho_0}. \tag{18}$$

Let us note that, taking into account equations (17) and (18), the toroidal angular velocity can be rewritten as

$$\Omega_\mathrm{T} = \Omega(\psi) + q\Omega_\mathrm{P}, \tag{19}$$

and, generally speaking, both the toroidal angular velocity and the poloidal angular velocity depend on the poloidal angle θ.

Multiplying equation (5) by $\nabla\varphi$, we obtain

$$\boldsymbol{B}_0\cdot\nabla\left\{F\left(1 - \frac{4\pi\kappa^2}{\rho_0}\right) - 4\pi\kappa\Omega R^2\right\} = 0. \tag{20}$$

Then it follows that

$$F\left(1 - \frac{4\pi\kappa^2}{\rho_0}\right) - 4\pi\kappa\Omega R^2 = I(\psi). \tag{21}$$

The force balance along the magnetic field yields

$$\rho_0 \boldsymbol{B}_0 \cdot \nabla \left(\frac{\kappa^2 B_0^2}{2\rho_0^2} - \frac{R^2 \Omega^2}{2} \right) + \boldsymbol{B}_0 \cdot \nabla p_0 = 0. \tag{22}$$

It follows from the equation of state (8) that the equilibrium entropy is a function of magnetic flux

$$\frac{p_0}{\rho_0^\Gamma} = S(\psi). \tag{23}$$

Substituting equation (23) into equation (22), we get

$$\frac{\kappa^2 B_0^2}{2\rho_0^2} - \frac{R^2 \Omega^2}{2} + \frac{\Gamma p_0}{(\Gamma - 1)\rho_0} = H(\psi). \tag{24}$$

Finally, the normal ($\nabla\psi$) component of equation (5) defines the poloidal flux function and can be written as a Grad–Shafranov-type equation (see, e.g., [23])

$$\left(1 - \frac{4\pi\kappa^2}{\rho_0}\right) R^2 \nabla \cdot \left(\frac{\nabla\psi}{R^2}\right) + 4\pi R^2 \left(\frac{\partial p_0}{\partial\psi}\right)_R + F \left(\frac{\partial F}{\partial\psi}\right)_R$$
$$-4\pi\kappa \nabla\psi \cdot \nabla \frac{\kappa}{\rho_0} + 2\pi\rho_0 \frac{\partial}{\partial\psi} \left(\frac{\kappa^2}{\rho_0^2} |\nabla\psi|^2\right)_R = 0. \tag{25}$$

Here the partial derivative over ψ is taken for $R = \text{const}$.

The set of equations (21), (23)–(25) completely defines the equilibrium of the rotating tokamak plasma. Further, we restrict ourselves to the case of large-aspect-ratio tokamaks $R_0/a \equiv 1/\epsilon \gg 1$ (R_0 and a are the major and minor radii of the torus) and of low-beta plasma $\beta \equiv 8\pi p_0/B_0^2 \sim \epsilon^2$. Furthermore, we assume that both the poloidal angular velocity and the toroidal angular velocity are sufficiently small, so that $(\Omega_P, \Omega_T) \leqslant c_s/R_0$. Here c_s is the speed of sound, $c_s^2 = \Gamma p_0/\rho_0$. Under such assumptions the effect of plasma rotation does not exceed the plasma pressure effects. The magnetic surfaces of the tokamak are considered to be circular and concentric. The Shafranov shift can be neglected to the main order in small parameter ϵ. They can be described by the following expressions:

$$\psi = \psi(r): \ R \approx R_0 + r\cos\theta, \qquad z \approx r\sin\theta, \tag{26}$$

where r is the label of magnetic surface meaning its radius. One can also expect that, as in the case of plasma equilibrium without rotation, to the main order in ϵ the functions $f = (p_0, \rho_0, F, J, \Omega_P, \Omega_T)$ are constant on the magnetic surfaces. They can be represented in the form

$$f = \bar{f}(\psi) + \tilde{f}(\psi, \theta), \tag{27}$$

where $\tilde{f} \sim \epsilon \bar{f}$. The additional applicability condition of this assumption will be given below.

Then it follows from equation (23) that the poloidal angle dependent part of plasma pressure p_0 is related to the corresponding part of the mass density by

$$\tilde{p}_0 = \frac{\Gamma \bar{p}_0}{\bar{\rho}_0} \tilde{\rho}_0 \equiv \bar{c}_s^2 \tilde{\rho}_0, \qquad \bar{c}_s^2 \equiv \frac{\Gamma \bar{p}_0}{\bar{\rho}_0}. \tag{28}$$

Taking into account equations (27) and (28), from equations (20) and (22) we obtain the set of two equations

$$\left(1 - \frac{4\pi\kappa^2}{\bar{\rho}_0}\right) \boldsymbol{B}_0 \cdot \nabla \tilde{F} + \frac{4\pi\kappa^2}{\bar{\rho}_0^2} \bar{F} \boldsymbol{B}_0 \cdot \nabla \tilde{\rho}_0 = 4\pi\kappa\Omega \boldsymbol{B}_0 \cdot \nabla R^2, \tag{29}$$

$$\frac{\kappa^2 \bar{F}}{\bar{\rho}_0^2 R_0^2} \boldsymbol{B}_0 \cdot \nabla \tilde{F} + \left(\bar{c}_s^2 - \frac{\kappa^2 \bar{F}^2}{\bar{\rho}_0^2 R_0^2}\right) \frac{1}{\bar{\rho}_0} \boldsymbol{B}_0 \cdot \nabla \tilde{\rho}_0 = \frac{1}{2} \left(\Omega^2 + \frac{\kappa^2 \bar{F}^2}{\bar{\rho}_0^2 R_0^4}\right) \boldsymbol{B}_0 \cdot \nabla R^2. \tag{30}$$

It is convenient to introduce two dimensionless coefficients λ_ρ and λ_F according to

$$\frac{1}{\bar\rho_0}\boldsymbol{B}_0\cdot\nabla\tilde\rho_0 \equiv \frac{\lambda_\rho}{R_0^2}\boldsymbol{B}_0\cdot\nabla R^2, \qquad \frac{1}{\bar F}\boldsymbol{B}_0\cdot\nabla\tilde F \equiv \frac{\lambda_F}{R_0^2}\boldsymbol{B}_0\cdot\nabla R^2. \tag{31}$$

Then equations (29) and (30) can be rewritten as

$$(\bar\omega_{\rm A}^2 - \bar\Omega_{\rm P}^2)\lambda_F + \bar\Omega_{\rm P}^2\lambda_\rho = \frac{\bar\Omega_{\rm P}\bar\Omega_{\rm T}}{q} - \bar\Omega_{\rm P}^2, \tag{32}$$

$$q^2\bar\Omega_{\rm P}^2\lambda_F + (\bar\omega_{\rm s}^2 - q^2\bar\Omega_{\rm P}^2)\lambda_\rho = \frac{\bar\Omega_{\rm T}^2}{2} - q\bar\Omega_{\rm P}\bar\Omega_{\rm T} + q^2\bar\Omega_{\rm P}^2. \tag{33}$$

Here

$$\bar\Omega_{\rm P} = \frac{\kappa\bar F}{qR_0^2\bar\rho_0}, \qquad \bar\Omega_{\rm T} = \Omega + q\bar\Omega_{\rm P}, \qquad \bar\omega_{\rm A}^2 = \frac{\bar c_{\rm A}^2}{q^2R_0^2}, \qquad \bar c_{\rm A}^2 = \frac{\bar F^2}{4\pi\bar\rho_0R_0^2},$$
$$\bar\omega_{\rm s} = \frac{\bar c_{\rm s}}{R_0}. \tag{34}$$

Solving equations (32) and (33), we obtain

$$\lambda_\rho = \frac{1}{D}\left[\bar\omega_{\rm A}^2\left(\frac{\bar\Omega_{\rm T}^2}{2} - q\bar\Omega_{\rm P}\bar\Omega_{\rm T} + q^2\bar\Omega_{\rm P}^2\right) - \frac{\bar\Omega_{\rm T}^2}{2}\bar\Omega_{\rm P}^2\right], \tag{35}$$

$$\lambda_F = \frac{1}{D}\left[\bar\omega_{\rm s}^2\left(\frac{\bar\Omega_{\rm P}\bar\Omega_{\rm T}}{q} - \bar\Omega_{\rm P}^2\right) - \frac{\bar\Omega_{\rm T}^2}{2}\bar\Omega_{\rm P}^2\right], \tag{36}$$

$$D = \bar\omega_{\rm A}^2(\bar\omega_{\rm s}^2 - q^2\bar\Omega_{\rm P}^2) - \bar\Omega_{\rm P}^2\bar\omega_{\rm s}^2 \neq 0. \tag{37}$$

In the case, when $D = 0$, the set of equations (32) and (33) has no solutions unless

$$\bar\omega_{\rm A}^2\left(\frac{\bar\Omega_{\rm T}^2}{2} - q\bar\Omega_{\rm P}\bar\Omega_{\rm T} + q^2\bar\Omega_{\rm P}^2\right) - \frac{\bar\Omega_{\rm T}^2}{2}\bar\Omega_{\rm P}^2 = 0. \tag{38}$$

In the latter case, these equations have an infinite family of solutions.

We will restrict ourselves to the equilibria, for which $D \neq 0$. Expressions (35) and (36) are simplified, if we take into account the above assumptions of small β and of slow plasma rotation. Then we obtain

$$\lambda_\rho = \frac{M_{\rm P}^2 - M_{\rm P}M_{\rm T} + M_{\rm T}^2/2}{1 - M_{\rm P}^2}, \tag{39}$$

$$\lambda_F = \frac{\bar\omega_{\rm s}^2}{q^2\bar\omega_{\rm A}^2}\left(\frac{M_{\rm T}^2}{2} - \frac{M_{\rm P}^2 - M_{\rm P}M_{\rm T} + M_{\rm T}^2/2}{1 - M_{\rm P}^2}\right), \tag{40}$$

where $M_{\rm T} \equiv \bar\Omega_{\rm T}/\bar\omega_{\rm s}$ and $M_{\rm P} \equiv q\bar\Omega_{\rm P}/\bar\omega_{\rm s}$ are the toroidal and poloidal Mach number, respectively.

Now we take into account that, according to the definition,

$$\lambda_\rho \simeq \frac{R_0}{r}\frac{1}{\bar\rho_0}\left|\frac{\partial\tilde\rho}{\partial\theta}\right| \simeq \frac{R_0}{r}\frac{|\tilde\rho_0|}{\bar\rho_0}. \tag{41}$$

Then it follows that the above assumption $\tilde\rho_0 \sim \epsilon\bar\rho_0$ is valid if

$$\left|1 - M_{\rm P}^2\right| \geqslant \left|\frac{M_{\rm T}^2}{2} - M_{\rm P}M_{\rm T} + M_{\rm P}^2\right|. \tag{42}$$

Hereafter in this paper we assume that condition (42) is satisfied. Then, according to equation (40), $\lambda_F \sim O(\beta) \ll 1$. This means that, up to the terms of order β, the poloidal current stream function F is a function of poloidal flux ψ only. We neglect these corrections of

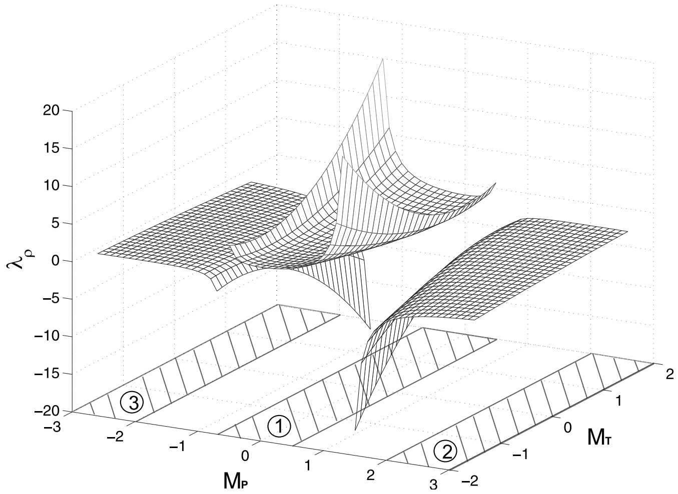

Figure 1. The coefficient λ_ρ as the function of two variables M_P and M_T. The domains in (M_P, M_T)-plane, where the assumptions made are justified, are shaded.

order β and put $\lambda_F = 0$. The coefficient λ_ρ from equation (39) as a function of two variables—the toroidal and poloidal Mach numbers—is represented in figure 1. The poloidal oscillations of the equilibrium mass density are small, $\tilde{\rho}_0 \sim \epsilon\bar{\rho}_0$, in three domains marked by numbers from 1 to 3. The dependence of λ_ρ on the poloidal Mach number M_P for different values of M_T is given in figure 2. Expression (39) is valid in the domains 1, 2 and 3.

3. GAMs and ZFs in rotating plasma

3.1. The equations of electrostatic axisymmetric perturbations

We consider the perturbations of the above equilibrium and assume

$$\rho = \rho_0 + \rho', \qquad \boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{v}', \qquad p = p_0 + p', \qquad \boldsymbol{j} = \boldsymbol{j}_0 + \boldsymbol{j}'. \tag{43}$$

We consider the electrostatic perturbations and neglect the perturbation of the magnetic field, $\boldsymbol{B}' = 0$. Then linearizing the set of MHD equations (1)–(4) we have

$$\rho_0 \left(\frac{\partial}{\partial t} + \boldsymbol{v}_0 \cdot \nabla\right) \boldsymbol{v}' + \rho_0 \left(\boldsymbol{v}' \cdot \nabla\right) \boldsymbol{v}_0 + \rho' \left(\boldsymbol{v}_0 \cdot \nabla\right) \boldsymbol{v}_0 = -\nabla p' + \frac{1}{c}\boldsymbol{j}' \times \boldsymbol{B}_0, \tag{44}$$

$$\boldsymbol{v}' \times \boldsymbol{B}_0 = c\nabla\phi', \tag{45}$$

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot \left(\rho_0 \boldsymbol{v}' + \rho' \boldsymbol{v}_0\right) = 0, \tag{46}$$

$$\left(\frac{\partial}{\partial t} + \boldsymbol{v}_0 \cdot \nabla\right) \frac{p' - c_s^2 \rho'}{\rho_0^\Gamma} + \left(\boldsymbol{v}' \cdot \nabla\right) \frac{p_0}{\rho_0^\Gamma} = 0, \tag{47}$$

$$\nabla \cdot \boldsymbol{j}' = 0. \tag{48}$$

Here ρ' is the perturbation of mass density, $\boldsymbol{v}'$ the perturbation of velocity, p' the perturbation of pressure, $\boldsymbol{j}'$ is the perturbed current density and ϕ' is the perturbed electrostatic potential. We assume, that the perturbations are axisymmetric, and take their spatio-temporal dependence in

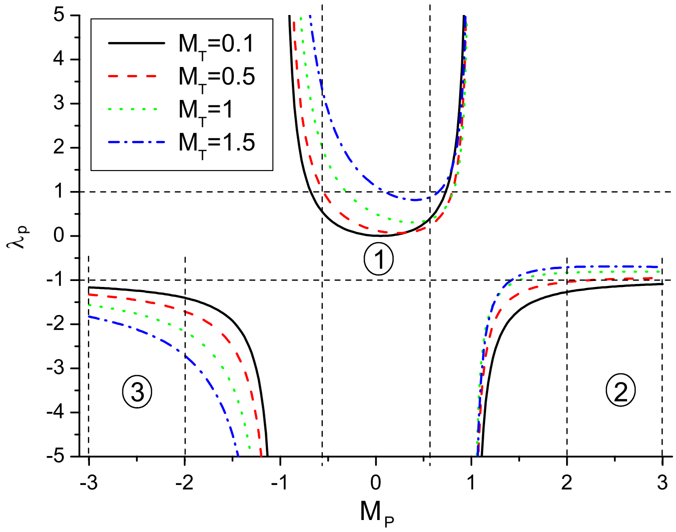

Figure 2. The dependence of λ_ρ on M_{P} for different M_{T}.

the form $f' = f'(\psi, \theta)\exp(-\mathrm{i}\omega t)$, where $f'(\psi, \theta)$ is a periodic function of θ: $f'(\psi, \theta+2\pi) = f'(\psi, \theta)$. It follows from equation (45) that $\phi' = \phi'(\psi)$, and therefore the perturbation of velocity lies on the magnetic surfaces of the tokamak ($\boldsymbol{v}' \cdot \nabla\psi = 0$) and takes the form

$$\boldsymbol{v}' = \frac{c}{B_0^2}\boldsymbol{B}_0 \times \nabla\phi' + \frac{v'_\parallel}{B_0}\boldsymbol{B}_0 \equiv \left(\frac{v'_\parallel}{B_0} - \frac{cF}{B_0^2}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right)\boldsymbol{B}_0 + cR^2\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\nabla\varphi, \tag{49}$$

where $v'_\parallel$ is the perturbation of velocity along the magnetic field. Then the perturbed equation of state can be rewritten as

$$\left(-\mathrm{i}\omega + \Omega_{\mathrm{P}}\frac{\partial}{\partial\theta}\right)\frac{p' - c_{\mathrm{s}}^2\rho'}{\rho_0^\Gamma} = 0, \tag{50}$$

and we obtain

$$p' = c_{\mathrm{s}}^2\rho'. \tag{51}$$

This means that the entropy is not perturbed by the axisymmetric electrostatic perturbation. Since the perturbed pressure is related to the perturbed mass density by algebraic equation (51), which does not contain ω, the order of the resulting dispersion relation for the continuum spectrum decreases by one. In this sense the case, when the entropy is uniform on the magnetic surface and the electrostatic, axisymmetric perturbations are described by the adiabatic equation of state, is degenerate. We will return to this point below, when we discuss the dispersion relation.

Substituting equation (49) into the continuity equation (46), we get

$$-\mathrm{i}\omega\rho' + \boldsymbol{B}_0 \cdot \nabla\left\{\rho_0\left(\frac{v'_\parallel}{B_0} - \frac{cF}{B_0^2}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right) + \frac{\kappa}{\rho_0}\rho'\right\} = 0. \tag{52}$$

The component of the equation of motion (44) along the magnetic field yields the equation

$$-\mathrm{i}\omega v'_\parallel B_0 + \boldsymbol{B}_0 \cdot \nabla\left\{\frac{\kappa}{\rho_0}\left(v'_\parallel B_0 - cF\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right) - c\Omega R^2\frac{\mathrm{d}\phi'}{\mathrm{d}\psi} + c_{\mathrm{s}}^2\frac{\rho'}{\rho_0}\right\} = 0. \tag{53}$$

While deriving equation (53) the equilibrium balance of forces along the magnetic field and equation (51) were taken into account. Details of the derivation of equation (53) can be found in appendix A.

It follows from equations (52) and (53) that

$$\langle \rho' \rangle = 0, \qquad \langle v'_\parallel B_0 \rangle = 0, \tag{54}$$

where $\langle \cdots \rangle$ is the average over the magnetic surface:

$$\langle f \rangle \equiv \frac{\displaystyle\int_0^{2\pi} f \, \mathrm{d}\theta / J}{\displaystyle\int_0^{2\pi} \mathrm{d}\theta / J}. \tag{55}$$

It was taken into account in deriving equation (54) that $\langle \boldsymbol{B}_0 \cdot \nabla f \rangle = 0$ for any f, which is a periodic function of poloidal angle θ. So, ρ' and V' are oscillating functions of θ to the main order in ϵ.

According to equation (44), the perturbation of the current density is defined by the equation

$$\boldsymbol{j}' = \frac{c}{B_0^2} \boldsymbol{B}_0 \times \boldsymbol{\mathcal{F}} + \frac{j'_\parallel}{B_0} \boldsymbol{B}_0, \tag{56}$$

where

$$\boldsymbol{\mathcal{F}} = \rho_0 [-\mathrm{i}\omega \boldsymbol{v}' + (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}' + (\boldsymbol{v}' \cdot \nabla)\boldsymbol{v}_0] + \rho'(\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_0 + \nabla p'. \tag{57}$$

Substituting equation (56) into quasineutrality condition (48), we obtain

$$\nabla \cdot \left(\frac{c}{B_0^2} \boldsymbol{B}_0 \times \boldsymbol{\mathcal{F}} \right) + \boldsymbol{B}_0 \cdot \nabla \left(\frac{j'_\parallel}{B_0} \right) = 0. \tag{58}$$

Averaging equation (58), we arrive at the equation

$$\left\langle \nabla \cdot \left(\mathcal{F}_\perp \frac{\nabla \psi}{|\nabla \psi|^2} \right) \right\rangle = 0, \tag{59}$$

where

$$\mathcal{F}_\perp \equiv \frac{\boldsymbol{\mathcal{F}} \cdot (\boldsymbol{B}_0 \times \nabla \psi)}{B_0^2}. \tag{60}$$

After simple manipulations equation (60) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}\psi} \left(\int_0^{2\pi} \frac{\mathcal{F}_\perp \mathrm{d}\theta}{J} \right) = 0. \tag{61}$$

Calculating $\mathcal{F}_\perp$ (details are presented in appendix B), we obtain the following equation:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}\psi} \Bigg\{ \int_0^{2\pi} \frac{\mathrm{d}\theta}{J} \Bigg(& -\mathrm{i}\omega \frac{c\rho_0 |\nabla\psi|^2}{B_0^2} \frac{\mathrm{d}\phi'}{\mathrm{d}\psi} + p' \boldsymbol{B}_0 \cdot \nabla \left(\frac{F}{B_0^2} \right) \\ & + \frac{\rho_0}{B_0^2} v'_\parallel B_0 \left[\left(\Omega + \frac{\kappa B_0^2}{\rho_0 F} \right) \boldsymbol{B}_0 \cdot \nabla R^2 - \frac{\kappa B_0^2}{\rho_0 F} \boldsymbol{B}_0 \cdot \nabla \left(\frac{|\nabla\psi|^2}{B_0^2} \right) \right] \\ & + \frac{\rho' F}{2B_0^2} \left[\left(\Omega + \frac{\kappa B_0^2}{\rho_0 F} \right)^2 \boldsymbol{B}_0 \cdot \nabla R^2 - \frac{\kappa^2 B_0^4}{\rho_0^2 F^2} \boldsymbol{B}_0 \cdot \nabla \left(\frac{|\nabla\psi|^2}{B_0^2} \right) \right] \Bigg) \Bigg\} = 0. \end{aligned} \tag{62}$$

The set of equations (51), (52), (53) and (62) describes the electrostatic axisymmetric perturbations of the rotating plasma in tokamaks. These equations are the *exact* equations. Below they will be simplified to the case of a large-aspect-ratio tokamak.

3.2. Mass density and parallel velocity perturbations

To the main order all equilibrium functions (mass density, pressure, magnetic field magnitude B_0, the Jacobian J, the poloidal and toroidal plasma velocities) are constant on the magnetic surfaces in the case of large-aspect-ratio tokamaks, $\epsilon \ll 1$. Their poloidal angle dependences appear as the corrections of order ϵ to the main order functions. Also, we have $B_0^2 = (F^2/R^2)(1 + O(\epsilon^2))$. This ordering allows us to simplify the equations of mass density and parallel velocity perturbations, which have been shown to be poloidal angle dependent functions to the main order in ϵ. Then equations (52) and (53) are reduced to the form

$$\left(-\mathrm{i}\omega + \bar{\Omega}_{\mathrm{P}}\frac{\partial}{\partial\theta}\right)\rho' + \frac{\bar{\rho}_0\bar{J}}{\bar{B}_0^2}\frac{\partial V'}{\partial\theta} = \frac{c\bar{\rho}_0}{\bar{F}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}(1+\lambda_\rho)\,\boldsymbol{B}_0\cdot\nabla R^2 \tag{63}$$

$$\left(-\mathrm{i}\omega + \bar{\Omega}_{\mathrm{P}}\frac{\partial}{\partial\theta}\right)V' + \frac{\bar{c}_{\mathrm{s}}^2\bar{J}}{\bar{\rho}_0}\frac{\partial\rho'}{\partial\theta} = c\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}[\bar{\Omega}_{\mathrm{T}} - q\bar{\Omega}_{\mathrm{P}}(1+\lambda_\rho)]\,\boldsymbol{B}_0\cdot\nabla R^2, \tag{64}$$

where $V' \equiv v'_{\parallel}B_0$, and the overbar denotes the poloidal angle independent part of the corresponding function. To the main order in ϵ we look for the solutions of the set of equations (63) and (64) in the form

$$\rho' = \rho_1\exp(\mathrm{i}\theta) + \rho_{-1}\exp(-\mathrm{i}\theta), \qquad V' = V_1\exp(\mathrm{i}\theta) + V_{-1}\exp(-\mathrm{i}\theta). \tag{65}$$

In the large-aspect-ratio tokamak with circular magnetic surfaces, defined by equation (26), the poloidal angle dependent function on the rhs's of equations (63) and (64) can be represented to the main order in ϵ as

$$\boldsymbol{B}_0\cdot\nabla R^2 \simeq \xi_1\exp(\mathrm{i}\theta) + \xi_{-1}\exp(-\mathrm{i}\theta). \tag{66}$$

Substituting equations (65) and (66) into the set of equations (63) and (64), we obtain

$$(\omega \mp \bar{\Omega}_{\mathrm{P}})\rho_{\pm1} \mp \frac{\bar{\rho}_0\bar{J}}{\bar{B}_0^2}V_{\pm1} = \mathrm{i}\frac{c\bar{\rho}_0}{\bar{F}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}(1+\lambda_\rho)\xi_{\pm1}, \tag{67}$$

$$(\omega \mp \bar{\Omega}_{\mathrm{P}})V_{\pm1} \mp \frac{\bar{c}_{\mathrm{s}}^2\bar{J}}{\bar{\rho}_0}\rho_{\pm1} = \mathrm{i}c\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}[\bar{\Omega}_{\mathrm{T}} - q\bar{\Omega}_{\mathrm{P}}(1+\lambda_\rho)]\xi_{\pm1}. \tag{68}$$

It is important to note here that the Doppler shift of the frequency has opposite signs for the $m = \pm1$ harmonics. It is this effect that will result in the appearance of the new type of modes induced by poloidal rotation.

Solving the set of equations (67) and (68), we find

$$\rho_{\pm1} = \frac{\mathrm{i}c\bar{\rho}_0}{D_{\pm1}\bar{F}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\left[\pm\frac{\bar{\Omega}_{\mathrm{T}}}{q} + (\omega \mp 2\bar{\Omega}_{\mathrm{P}})(1+\lambda_\rho)\right]\xi_{\pm1}, \tag{69}$$

$$V_{\pm1} = \frac{\mathrm{i}c}{D_{\pm1}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\left[(\omega \mp \bar{\Omega}_{\mathrm{P}})\left(\bar{\Omega}_{\mathrm{T}} - q\bar{\Omega}_{\mathrm{P}}(1+\lambda_\rho)\right) \pm \frac{\bar{\omega}_{\mathrm{s}}^2}{q}(1+\lambda_\rho)\right]\xi_{\pm1}, \tag{70}$$

$$D_{\pm1} = (\omega \mp \bar{\Omega}_{\mathrm{P}})^2 - \bar{\omega}_{\mathrm{s}}^2/q^2. \tag{71}$$

Now we can proceed to derive a single equation for ϕ', which will define the axisymmetric electrostatic mode under consideration.

3.3. Dispersion relation for the GAM continuum

In the case of large-aspect-ratio tokamaks equation (62) can be simplified and written as

$$\frac{\mathrm{d}}{\mathrm{d}\psi}\left\{-\mathrm{i}\omega\frac{c\bar{\rho}_0 r^2}{q^2\bar{J}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi} + \frac{\bar{\rho}_0\bar{\Omega}_{\mathrm{T}}}{\bar{B}_0^2\bar{J}}\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\theta\,V'\,\boldsymbol{B}_0\cdot\nabla R^2 \right.$$
$$\left. + \frac{R_0^2}{\bar{F}\bar{J}}\left(\bar{\omega}_{\mathrm{s}}^2 + \frac{\bar{\Omega}_{\mathrm{T}}^2}{2}\right)\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\theta\rho'\,\boldsymbol{B}_0\cdot\nabla R^2\right\} = 0. \tag{72}$$

In deriving equation (72) we have taken into account equation (51). Keeping in mind equations (65) and (66), we find from equation (72) that

$$\frac{\mathrm{d}}{\mathrm{d}\psi}\left\{-\mathrm{i}\omega\frac{c\bar{\rho}_0 r^2}{q^2\bar{J}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}+\left[\frac{\bar{\rho}_0\bar{\Omega}_{\mathrm{T}}}{2\bar{B}_0^2\bar{J}}\left(\frac{V_1}{\xi_1}+\frac{V_{-1}}{\xi_{-1}}\right)\right.\right.$$
$$\left.\left.+\frac{R_0^2}{2\bar{F}\bar{J}}\left(\bar{\omega}_{\mathrm{s}}^2+\frac{\bar{\Omega}_{\mathrm{T}}^2}{2}\right)\left(\frac{\rho_1}{\xi_1}+\frac{\rho_{-1}}{\xi_{-1}}\right)\right]\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\theta\,(\boldsymbol{B}_0\cdot\nabla R^2)^2\right\}=0. \quad (73)$$

Now we calculate the terms depending on $\rho_{\pm 1}$ and $V_{\pm 1}$. Using equations (69) and (70) we find

$$\frac{V_1}{\xi_1}+\frac{V_{-1}}{\xi_{-1}}=\frac{\mathrm{i}c}{D_1D_{-1}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\left\{\omega(D_1+D_{-1})[\bar{\Omega}_{\mathrm{T}}-q\bar{\Omega}_{\mathrm{P}}(1+\lambda_\rho)]\right.$$
$$\left.+(D_{-1}-D_1)\left[-\bar{\Omega}_{\mathrm{P}}\bar{\Omega}_{\mathrm{T}}+q\bar{\Omega}_{\mathrm{P}}^2(1+\lambda_\rho)+\frac{\bar{\omega}_{\mathrm{s}}^2}{q}(1+\lambda_\rho)\right]\right\}, \quad (74)$$

$$\frac{\rho_1}{\xi_1}+\frac{\rho_{-1}}{\xi_{-1}}=\frac{\mathrm{i}c\bar{\rho}_0}{D_1D_{-1}\bar{F}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\left\{\omega(D_1+D_{-1})(1+\lambda_\rho)\right.$$
$$\left.+\left(\frac{\bar{\Omega}_{\mathrm{T}}}{q}-2\bar{\Omega}_{\mathrm{P}}(1+\lambda_\rho)\right)(D_{-1}-D_1)\right\}. \quad (75)$$

It follows from equation (71) that

$$D_1+D_{-1}=2\hat{\omega}^2, \qquad D_{-1}-D_1=4\omega\bar{\Omega}_{\mathrm{P}}, \qquad \hat{\omega}^2\equiv\omega^2+\bar{\Omega}_{\mathrm{P}}^2-\bar{\omega}_{\mathrm{s}}^2/q^2,$$
$$D_1D_{-1}=\hat{\omega}^4-4\omega^2\bar{\Omega}_{\mathrm{P}}^2\equiv\hat{\omega}^4-4\hat{\omega}^2\bar{\Omega}_{\mathrm{P}}^2+4\bar{\Omega}_{\mathrm{P}}^2(\bar{\Omega}_{\mathrm{P}}^2-\bar{\omega}_{\mathrm{s}}^2/q^2). \quad (76)$$

The terms $D_{-1}-D_1$ are not equal to zero because of the Doppler shift of frequency, which has opposite signs for the $m=\pm 1$ harmonics. It is this effect which is responsible for the appearance of a new type of GAM.

We take into account equation (26) to obtain

$$\frac{1}{2\pi}\int_0^{2\pi}\mathrm{d}\theta\,(\boldsymbol{B}_0\cdot\nabla R^2)^2\simeq\frac{1}{2\pi}\int_0^{2\pi}4r^2R_0^2\bar{J}^2\sin^2\theta\,\mathrm{d}\theta=2r^2\bar{F}^2/q^2R_0^2. \quad (77)$$

Then, substituting equations (74)–(77) into equation (73), we finally arrive at the equation

$$\frac{\mathrm{d}}{\mathrm{d}\psi}\left[\frac{\omega\bar{\rho}_0r^2}{q^2\bar{J}}\frac{\Lambda}{D_1D_{-1}}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right]=0, \quad (78)$$

where

$$\Lambda=\hat{\omega}^4-2a\bar{\omega}_{\mathrm{s}}^2\hat{\omega}^2-b\bar{\omega}_{\mathrm{s}}^4,$$
$$a=1+\lambda_\rho+\frac{M_{\mathrm{T}}^2}{2}(3+\lambda_\rho)-M_{\mathrm{P}}M_{\mathrm{T}}(1+\lambda_\rho)+\frac{2M_{\mathrm{P}}^2}{q^2},$$
$$b=\frac{4M_{\mathrm{P}}}{q^2}\left\{M_{\mathrm{P}}\left(\frac{1}{q^2}-2(1+\lambda_\rho)\right)+M_{\mathrm{T}}(2+\lambda_\rho)-\frac{M_{\mathrm{P}}^3}{q^2}\right.$$
$$\left.-M_{\mathrm{P}}M_{\mathrm{T}}^2(2+\lambda_\rho)+\frac{M_{\mathrm{T}}^3}{2}+M_{\mathrm{P}}^2M_{\mathrm{T}}(1+\lambda_\rho)\right\}. \quad (79)$$

The term in Λ, which does not contain $\hat{\omega}^2$, is related to the poloidal plasma rotation. Its origin is in the opposite signs of the Doppler frequency shift in the equations for the $m=1$ and $m=-1$ harmonics of parallel velocity and of mass density.

The continuum spectrum of axisymmetric electrostatic perturbations is defined by zeros of the coefficient $\omega\Lambda/D_1D_{-1}$ in equation (78), i.e. by the equation $\omega\Lambda/D_1D_{-1}=0$. Let us note that Λ and $D_{\pm 1}$ are invariant with respect to the transformation $M_{\mathrm{P}}\to -M_{\mathrm{P}}$, $M_{\mathrm{T}}\to -M_{\mathrm{T}}$. So, the continuum spectrum of electrostatic axisymmetric perturbations is invariant with respect to this transformation. Always one of the solutions of dispersion relation is $\omega = 0$. In the case of purely toroidal rotation in a tokamak with isothermal magnetic surfaces it turns into the low-frequency GAM induced by rotation [11–13]. The transformation is caused by the term proportional to $\Gamma - 1$ in the dispersion relation derived in [11–13]. This term originates from the last term on the lhs of equation (47), if we assume that $p_0/\rho_0 \equiv T(\psi)$, and 1 in $\Gamma - 1$ is related to the isothermic equilibrium. It disappears when the magnetic surfaces are isentropic.

Below we will study another solution of the dispersion relation.

In the case of purely toroidal plasma rotation, $M_{\rm P} = 0$, we have $\lambda_\rho = M_{\rm T}^2/2$ and

$$D_1 = D_{-1} = \hat{\omega}^2 \equiv \omega^2 - \bar{\omega}_{\rm s}^2/q^2, \qquad \Lambda = \hat{\omega}^2\left[\hat{\omega}^2 - \bar{\omega}_{\rm s}^2\left(2 + 4M_{\rm T}^2 + \frac{M_{\rm T}^4}{2}\right)\right]. \tag{80}$$

As a result, there is a cancellation of the multiplier $\hat{\omega}^2$ in the dispersion relation $\Lambda/D_1 D_{-1} = 0$. Then we obtain the quadratic equation which gives the mode

$$\omega^2 = \omega_{\rm s}^2\left(2 + \frac{1}{q^2} + 4M_{\rm T}^2 + \frac{M_{\rm T}^4}{2}\right). \tag{81}$$

This mode is an ordinary GAM with the frequency shift due to the toroidal rotation. No new branch of low-frequency GAMs due to toroidal rotation arises in the case of equilibrium with isentropic magnetic surfaces considered here for the reason discussed above. In the previous studies of GAMs in toroidally rotating plasmas the cases of plasma equilibria, in which either the plasma temperature [11–13] or the mass density [16] is uniform on the magnetic surfaces, were considered. In the former case, in addition to a frequency modification of ordinary GAM frequency, a new branch of low-frequency GAMs due to toroidal rotation was found [11–13]. In the latter case, where the equilibrium mass density was assumed to be poloidal angle independent, a new branch of aperiodic, unstable perturbations induced by toroidal rotation was found [16]. Thus, the properties of GAMs in toroidally rotating plasmas strongly depend on the plasma equilibrium.

It should be noted that we have found no extra branch of the solutions related to the sound continuum. Such a branch exists if $m, n \neq 0$ and $m + nq \ll 1$ [14, 24], but it completely disappears in our case, when $m = 0$ and $n = 0$. This is evidence of degeneracy of the considered axisymmetric electrostatic perturbations with $(m, n) = 0$.

When the poloidal plasma rotation takes place ($\bar{\Omega}_{\rm P} \neq 0$), the dispersion relation is $\Lambda = 0$. As a result, we have the dispersion relation of the fourth order. Its roots are defined by the equations

$$\omega_1^2 = \bar{\omega}_{\rm s}^2\left[\frac{1}{q^2}(1 - M_{\rm P}^2) + a + \sqrt{a^2 + b}\right], \tag{82}$$

$$\omega_2^2 = \bar{\omega}_{\rm s}^2\left[\frac{1}{q^2}(1 - M_{\rm P}^2) + a - \sqrt{a^2 + b}\right]. \tag{83}$$

In the case when both toroidal and poloidal rotations are slow compared with $\bar{\omega}_{\rm s}$, so that $(M_{\rm P}, M_{\rm T}) \ll 1$, with an accuracy up to quadratic terms with respect to these small parameters, we have

$$\lambda_\rho = \frac{M_{\rm T}^2}{2} - M_{\rm P} M_{\rm T} + M_{\rm P}^2. \tag{84}$$

Then from equations (82) and (83) we obtain two continuum spectra

$$\omega_1^2 = \bar{\omega}_{\rm s}^2\left[2 + \frac{1}{q^2} + 4M_{\rm T}^2 - 4M_{\rm P}M_{\rm T}\left(1 - \frac{1}{q^2}\right) + M_{\rm P}^2\left(2 - \frac{1}{q^2} + \frac{2}{q^4}\right)\right], \tag{85}$$

$$\omega_2^2 = \frac{\bar{\omega}_{\rm s}^2}{q^2}\left[1 - 4M_{\rm P}M_{\rm T} + M_{\rm P}^2\left(3 - \frac{2}{q^2}\right)\right]. \tag{86}$$

The first mode is the ordinary GAM modified by plasma rotation. Another mode has a lower frequency which is close to the sound frequency $\bar{\omega}_s/q$. The new mode is induced by poloidal plasma rotation (despite its weak dependence on poloidal angular velocity). It can be called a rotation-induced acoustic mode. This mode does not exist in the case of purely toroidal rotation, $\bar{\Omega}_P = 0$, due to the previously discussed degeneracy of the considered axisymmetric electrostatic perturbations with $(m, n) = 0$, which results in the above-mentioned cancellation of the multiplier $\omega^2 - \bar{\omega}_s^2/q^2$ in the dispersion relation. The mode appears as a consequence of the Doppler-shifted response of the side-bands of plasma density, pressure and parallel velocity perturbations to the electrostatic potential perturbation, driven by the curvature of magnetic field lines and by the effect of non-uniformity of equilibrium plasma density and pressure on magnetic flux surfaces created by the centrifugal forces. The Doppler shift of frequency is caused by poloidal rotation and has opposite signs for the $m = 1$ and $m = -1$ side-bands.

The dispersion relation can also be significantly simplified in the limiting case of slow poloidal plasma rotation $M_P \ll 1$, but sufficiently fast, sonic toroidal rotation. In this case from equation (39) we have

$$\lambda_\rho = \frac{M_T^2}{2} - M_P M_T + O(M_P^2). \tag{87}$$

Substituting equation (87) in equations (79), (82) and (83), we obtain two continuum spectra, which are described by the following expressions:

$$\omega_1^2 = \bar{\omega}_s^2 \left[2 + \frac{1}{q^2} + 4M_T^2 + \frac{M_T^4}{2} - 2M_P M_T \left(2 + M_T^2\right) \left(1 - \frac{1}{q^2(1 + 2M_T^2 + M_T^4/4)}\right)\right], \tag{88}$$

$$\omega_2^2 = \frac{\bar{\omega}_s^2}{q^2} \left[1 - \frac{2M_P M_T(2 + M_T^2)}{1 + 2M_T^2 + M_T^4/4}\right]. \tag{89}$$

Again, the first of these modes is an ordinary GAM with the frequency shifted up by the effect of toroidal plasma rotation. Another mode is a rotation-induced acoustic mode. Its frequency is close to the sound frequency. The small frequency shift from the sound frequency $\bar{\omega}_s/q$ is proportional to the poloidal Mach number M_P.

In the general case using formulae (82) and (83) we have numerically studied the solutions of the dispersion relation under conditions typical of the tokamak edge, choosing $q = 3$. We have separately considered three domains shown in figures 1 and 2.

Figure 3 shows the squares of mode frequencies ω_1^2 and ω_2^2 as 2D functions of M_P and M_T in domain 1, corresponding to sufficiently slow poloidal rotation, $|M_P| \leqslant 0.6$. Both ω_1^2 and ω_2^2 are positive, which means that two branches of stable continuum modes exist. In the absence of rotation one of these modes (ω_1) transforms into an ordinary GAM. Another one (ω_2) is the new found mode, which is induced by poloidal plasma rotation. It always has a lower frequency than the first mode. In the case of very slow poloidal rotation, $M_P \ll 1$, it transforms into the rotation-induced acoustic mode and its frequency is defined by equation (86).

In figure 4 we have presented ω_1^2 and ω_2^2 as 2D functions of M_P and M_T in domain 2, where the poloidal Mach number M_P is a sufficiently large, positive number. The poloidal angular velocity in this domain exceeds the ion sound frequency, $\bar{\Omega}_P > \bar{c}_s/qR_0$. The situation is qualitatively different from domain 1. The mode described by ω_1 remains stable ($\omega_1^2 > 0$), but in some subdomain ω_2^2 becomes negative. This means that the mode is aperiodically unstable. In this case the mode can be identified as an unstable ZF. According to the figure, the most unstable are the flows with negative M_T, such that $M_P \cdot M_T < 0$.

In figure 5 the dependence of ω_2^2 on the poloidal Mach number M_P is given for different values of M_T. When the toroidal plasma rotation is sufficiently fast ($M_T \simeq 1$) the mode

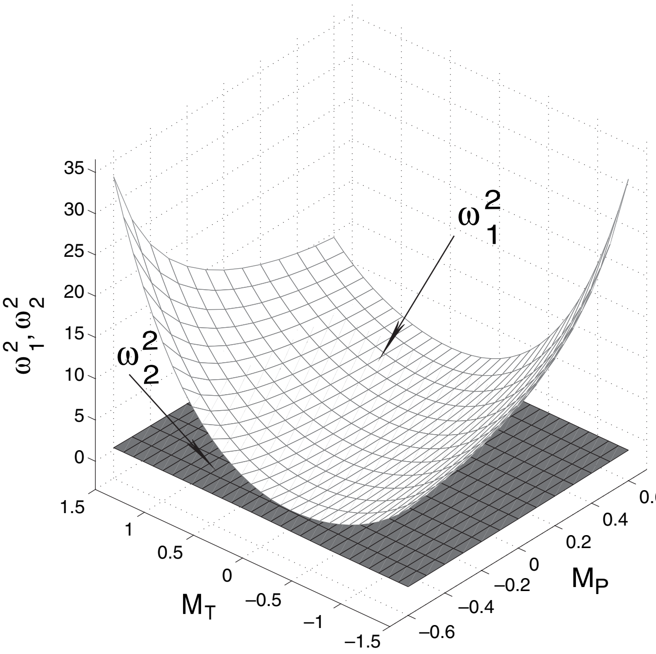

Figure 3. The squares of mode frequencies ω_1^2 and ω_2^2 as functions of the poloidal and toroidal Mach numbers M_P and M_T in domain 1 for $q = 3$. The frequencies are normalized to $\bar{\omega}_\mathrm{s}$.

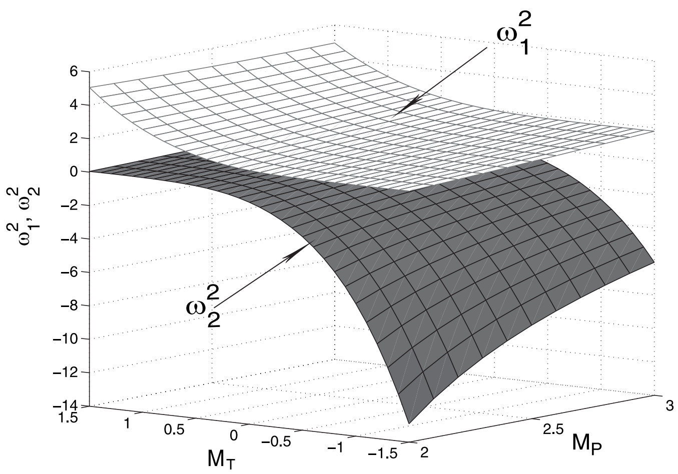

Figure 4. The squares of mode frequencies ω_1^2 and ω_2^2 as functions of the poloidal and toroidal Mach numbers M_P and M_T in domain 2 for $q = 3$. The frequencies are normalized to $\bar{\omega}_\mathrm{s}$.

described by ω_2 is stable and can be identified as the GAM induced by the poloidal plasma rotation. The instability of this mode appears when the toroidal rotation is relatively slow, so that $M_\mathrm{T} \leqslant 0.78$. The function ω_2^2 grows with M_P. If it is negative for some M_T in the interval $2.0 < M_\mathrm{P} < M_0$, it passes through zero at some point $M_\mathrm{P} = M_0(M_\mathrm{T})$, which depends on M_T. At this point the mode transforms into the stable ZF. With a further increase in M_P the mode transforms into a stable, oscillating mode, which can be identified as the GAM induced by the poloidal plasma rotation. Its frequency is smaller than the frequency of the ordinary GAM.

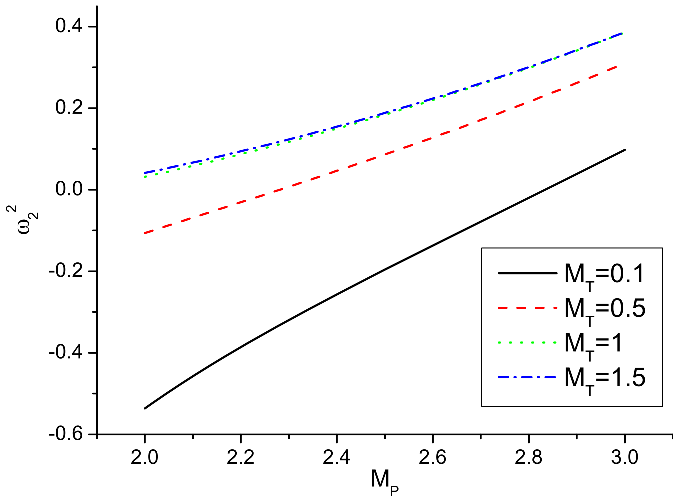

Figure 5. The square of ω_2^2 as a function of the poloidal Mach number M_P in domain 2 for $q = 3$ and for different M_T . The frequency is normalized to $\bar{\omega}_\mathrm{s}$.

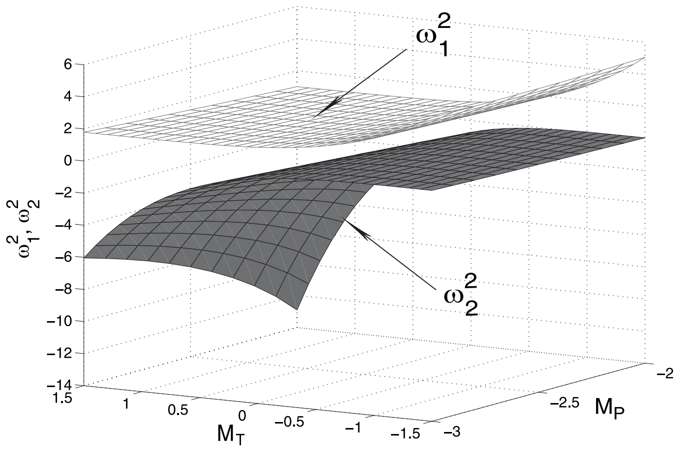

Figure 6. The squares of mode frequencies ω_1^2 and ω_2^2 as functions of the poloidal and toroidal Mach numbers M_P and M_T in domain 3 for $q = 3$. The frequencies are normalized to $\bar{\omega}_\mathrm{s}$.

Figures 6 and 7 give the results of numerical analysis of the mode frequencies in domain 3. In this domain the poloidal Mach number M_P is a sufficiently large, negative number. As in domain 3, the magnitude of poloidal angular velocity in this domain exceeds the ion sound frequency, $|\bar{\Omega}_\mathrm{P}| > \bar{c}_\mathrm{s}/qR_0$. In figure 6 ω_1^2 and ω_2^2 are given as 2D functions of M_P and M_T. The mode described by ω_1 is always stable. At the same time in some range of parameters the mode described by ω_2 corresponds to an unstable ZF. According to this figure the most unstable with respect to ZF perturbations are the flows with positive M_T, such that again $M_\mathrm{P} \cdot M_\mathrm{T} < 0$. In figure 7 the dependence of ω_2^2 on the poloidal Mach number M_P is given for different non-negative values of M_T. In the range of parameters considered the mode ω_2 is unstable. An additional analysis shows that for fixed M_T it is stabilized, when the poloidal rotation is faster (when $M_\mathrm{P} < -3.0$). The larger the M_T, the larger the $|M_\mathrm{P}| = M_0$ required to stabilize the mode ω_2. To the left from the point $M_\mathrm{P} = -M_0$ the mode ω_2 becomes the stable, oscillating mode and can be identified as the GAM induced by the poloidal rotation.

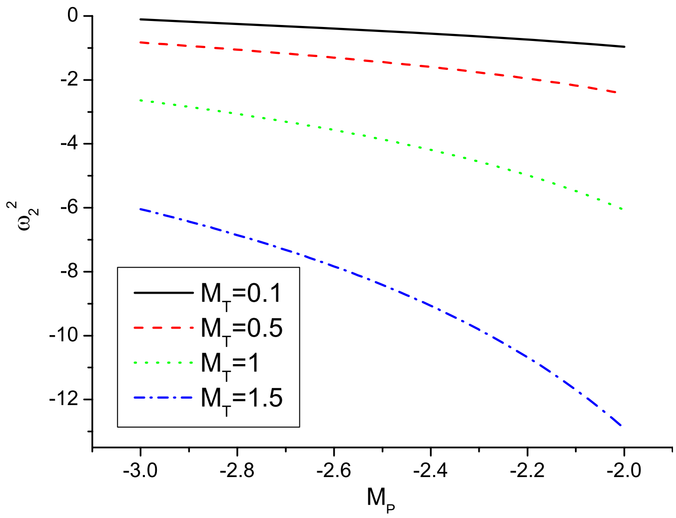

Figure 7. The square of ω_2^2 as a function of the poloidal Mach number $M_{\rm P}$ in domain 3 for $q = 3$ and for different $M_{\rm T}$. The frequency is normalized to $\bar{\omega}_{\rm s}$.

4. Conclusion

In this paper, a theory of low-frequency, electrostatic ideal modes—GAMs and zonal flows—in tokamaks is developed to include the effects of equilibrium plasma flows. Unlike the previous studies [11–15], where the toroidal rotation effect on GAMs and Alfvén waves was considered, the case of general plasma equilibrium, including both toroidal and poloidal plasma flows, is studied here. The analysis is based on ideal MHD equations with the adiabatic equation of state. The assumptions involved are that the aspect ratio of tokamak is large ($R_0/a \gg 1$), that the pressure is low ($\beta \sim \epsilon^2$), that the poloidal plasma rotation is relatively slow ($\Omega_{\rm P} \leqslant c_{\rm s}/R_0$) and that the magnetic surfaces of the tokamak are circular. We have restricted ourselves to the poloidal flows such that $\Omega_{\rm P} \neq c_{\rm s}/qR_0$, $\Omega_{\rm P} - c_{\rm s}/qR_0 \sim (\Omega_{\rm P}, \Omega_{\rm T})$. In this case the non-uniformity of plasma mass density and pressure on the magnetic surfaces of the tokamak is sufficiently small ($\tilde{\rho}_0 \sim \epsilon\bar{\rho}_0$, $\tilde{p}_0 \sim \epsilon\bar{p}_0$) and can be calculated analytically (see equation (39)). The electrostatic, axisymmetric perturbations of the above plasma equilibrium are considered. The set of equations (50), (52), (53) and (62) is derived, which shows that the perturbations are characterized by the electrostatic potential, which is uniform on the magnetic surfaces (the principal harmonic), and the mass density, pressure and parallel velocity, which are oscillating functions of the poloidal angle in the main order in ϵ (the side-bands). The responses of plasma mass density, pressure and parallel velocity to the electrostatic potential perturbation are driven by two effects: by the curvature of the magnetic field lines (which is responsible for the ordinary GAMs) and by non-uniformity of plasma mass density and pressure on the magnetic surfaces, created by the centrifugal forces. Due to the poloidal rotation the frequencies of the responses of side-bands to the electrostatic potential perturbation are shifted (the Doppler effect). It is important that the Doppler shift has opposite signs for the $m = \pm 1$ harmonics. This difference in the Doppler shift of side-band frequencies results in a new type of GAMs—zonal flows.

The normal mode equation in terms of the electrostatic potential (see equation (78)) is derived. Zeros of the coefficient in front of the derivative of perturbed electrostatic potential give the dispersion relation, defining the continuum spectrum. In the limiting case of purely toroidal rotation ($M_{\rm P} = 0$) the continuum spectrum is described by equation (81). It gives the ordinary GAM with the frequency shifted up by the toroidal rotation. Unlike the earlier results obtained for the plasma equilibria with toroidal rotation, in which either the temperature [11–13] or the mass density [16] was assumed uniform on the magnetic surfaces, no new types of GAMs or zonal flows arise in the case of isentropic magnetic surfaces. When the poloidal rotation of plasma takes place, along with an ordinary GAM a new type of continuum modes appears. In the case of slow plasma rotation $(M_\mathrm{P}, M_\mathrm{T}) \ll 1$ its frequency is close to the acoustic mode frequency c_s/qR_0 (see equation (86)). This frequency is lower than the frequency of the ordinary GAM. The frequency of the latter is shifted up by the plasma rotation effect (see equation (85)). The new mode is intrinsically related to poloidal plasma rotation (despite its weak dependence on poloidal angular velocity!), and it can be called the rotation-induced acoustic mode. It originates from the different Doppler frequency shifts of the side-band harmonics $m = 1$ and $m = -1$ of plasma density, pressure and parallel velocity perturbations to the electrostatic potential perturbation. This branch of continuum spectrum disappears in the case of purely toroidal plasma rotation due to degeneracy of axisymmetric electrostatic perturbations with $(m, n) = 0$. In this case both the numerator and the denominator of the coefficient Λ/D_1D_{-1} in equation (78) contain the multiplier $\omega^2 - \bar{\omega}_\mathrm{s}^2/q^2$, which is canceled. So, the coefficient Λ/D_1D_{-1} in front of the highest derivative in equation (78) is not equal to zero at the point defined by $\omega^2 - \bar{\omega}_\mathrm{s}^2(r)/q^2(r) = 0$. In the case of sonic toroidal rotation $M_\mathrm{T} \simeq 1$ and slow poloidal one ($M_\mathrm{P} \ll 1$) the difference between the frequency of the rotation-induced acoustic mode and the sound frequency $\bar{\omega}_\mathrm{s}/q$ is proportional to the poloidal Mach number M_P (equation (89)).

A numerical analysis of the general expressions for the frequencies of two branches of continuum spectrum (82) and (83) is performed for $q = 3$. It shows that in the case of relatively slow poloidal rotation such that $\bar{\Omega}_\mathrm{P} \leqslant \bar{c}_\mathrm{s}/qR_0$ both branches of the continuum spectrum are stable. The branch, which is an ordinary GAM modified by plasma rotation, has a higher frequency compared with the new found mode induced by poloidal rotation (figure 3). When the angular velocity of poloidal rotation is large, $|M_\mathrm{P}| \geqslant 2$, the situation becomes qualitatively different. While the branch, corresponding to the standard GAM, remains stable and its frequency is mainly a growing function of $|M_\mathrm{P}|$ for $M_\mathrm{T} = \mathrm{const}$, a new branch of the continuum modes becomes unstable in some range of the poloidal and toroidal Mach numbers. The instability is aperiodic, so that the modes are non-oscillating and in a natural way are identified as unstable zonal flows. The most unstable (with larger growth rates) are the flows with $M_\mathrm{P} \cdot M_\mathrm{T} < 0$. For $M_\mathrm{T} = \mathrm{const}$ the instability is suppressed with an increase in the poloidal Mach number $|M_\mathrm{P}|$. As a result, passing through some point $M_\mathrm{P} = M_0$, the mode transforms into the oscillating mode—the marginally stable GAM.

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Appendix A. Perturbed parallel force balance (52)

We scalarly multiply equation (44) by $\boldsymbol{B}_0$. Then, taking into account equation (49) and the equilibrium parallel force balance $\rho_0\boldsymbol{B}_0 \cdot (\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}_0 = -\boldsymbol{B}_0 \cdot \nabla p_0$, we obtain

$$-\mathrm{i}\omega\rho_0 v'_\parallel B_0 + \rho_0\boldsymbol{B}_0 \cdot \left[(\boldsymbol{v}_0 \cdot \nabla)\boldsymbol{v}' + (\boldsymbol{v}' \cdot \nabla)\boldsymbol{v}_0\right] - \frac{\rho'}{\rho_0}\boldsymbol{B}_0 \cdot \nabla p_0 = -\boldsymbol{B}_0 \cdot \nabla p'. \quad (90)$$

The term in square brackets in equation (90) can be transformed as follows:

$$\begin{aligned}\boldsymbol{B}_0\cdot\left[(\boldsymbol{v}_0\cdot\nabla)\boldsymbol{v}'+(\boldsymbol{v}'\cdot\nabla)\boldsymbol{v}_0\right] &= \boldsymbol{B}_0\cdot\nabla(\boldsymbol{v}_0\cdot\boldsymbol{v}')\\ &\quad+(\boldsymbol{v}'\times\boldsymbol{B}_0)\cdot\nabla\times\boldsymbol{v}_0+(\boldsymbol{v}_0\times\boldsymbol{B}_0)\cdot\nabla\times\boldsymbol{v}'\\ &=\nabla\cdot\left[\boldsymbol{v}'(\boldsymbol{v}_0\cdot\boldsymbol{B}_0)+\boldsymbol{v}_0(\boldsymbol{v}'\cdot\boldsymbol{B}_0)\right]-\boldsymbol{B}_0\cdot\nabla(\boldsymbol{v}_0\cdot\boldsymbol{v}')\\ &\equiv\boldsymbol{B}_0\cdot\nabla\left\{\frac{\kappa}{\rho_0}\left(v'_\parallel B_0-cF\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right)-c\Omega R^2\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right\}.\end{aligned}\qquad(91)$$

The pressure terms in equation (90) are transformed using equations (23) and (51). Then we obtain

$$\begin{aligned}\frac{1}{\rho_0}\left(\boldsymbol{B}_0\cdot\nabla p'-\frac{\rho'}{\rho_0}\boldsymbol{B}_0\cdot\nabla p_0\right)&=\frac{1}{\rho_0}\boldsymbol{B}_0\cdot\nabla(c_\mathrm{s}^2\rho')+\rho'c_\mathrm{s}^2(\boldsymbol{B}_0\cdot\nabla)\frac{1}{\rho_0}\\ &=(\boldsymbol{B}_0\cdot\nabla)\left(c_\mathrm{s}^2\frac{\rho'}{\rho_0}\right).\end{aligned}\qquad(92)$$

Substituting equations (91) and (92) into equation (90), we arrive at equation (53).

Appendix B. Derivation of equation (62)

We represent $\mathcal{F}_\perp$ in the form

$$\mathcal{F}_\perp=\mathcal{F}_\perp^{(1)}+\mathcal{F}_\perp^{(2)}+\mathcal{F}_\perp^{(3)}+\mathcal{F}_\perp^{(4)}\qquad(93)$$

and calculate its corresponding pieces step by step. The first two terms in equation (93) are easily calculated

$$\mathcal{F}_\perp^{(1)}\equiv-\mathrm{i}\omega\frac{\rho_0}{B_0^2}\boldsymbol{v}'\cdot(\boldsymbol{B}_0\times\nabla\psi)=-\mathrm{i}\omega\frac{c\rho_0|\nabla\psi|^2}{B_0^2}\frac{\mathrm{d}\psi'}{\mathrm{d}\psi},\qquad(94)$$

$$\begin{aligned}\mathcal{F}_\perp^{(2)}&\equiv\frac{1}{B_0^2}(\boldsymbol{B}_0\times\nabla\psi)\cdot\nabla p'=-\frac{F}{B_0^2}\boldsymbol{B}_0\cdot\nabla p'\\ &=-\boldsymbol{B}_0\cdot\nabla\left(\frac{p'F}{B_0^2}\right)+p'\boldsymbol{B}_0\cdot\nabla\left(\frac{F}{B_0^2}\right).\end{aligned}\qquad(95)$$

The first term in equation (95) makes no contribution to equation (62), because its average over the magnetic surface is equal to zero.

The last two pieces are related to the plasma rotation. For $\mathcal{F}_\perp^{(3)}$ we have

$$\begin{aligned}\mathcal{F}_\perp^{(3)}&\equiv\frac{\rho_0}{B_0^2}(\boldsymbol{B}_0\times\nabla\psi)\cdot[(\boldsymbol{v}_0\cdot\nabla)\boldsymbol{v}'+(\boldsymbol{v}'\cdot\nabla)\boldsymbol{v}_0]\\ &=\frac{\rho_0}{B_0^2}(\boldsymbol{B}_0\times\nabla\psi)\cdot[\nabla(\boldsymbol{v}_0\cdot\boldsymbol{v}')+(\nabla\times\boldsymbol{v}_0)\times\boldsymbol{v}'+(\nabla\times\boldsymbol{v}')\times\boldsymbol{v}_0]\\ &=-\frac{\rho_0}{B_0^2}[F\boldsymbol{B}_0\cdot\nabla(\boldsymbol{v}_0\cdot\boldsymbol{v}')+(\boldsymbol{v}'\cdot\boldsymbol{B}_0)\nabla\cdot(\boldsymbol{v}_0\times\nabla\psi)\\ &\quad+(\boldsymbol{v}_0\cdot\boldsymbol{B}_0)\nabla\cdot(\boldsymbol{v}'\times\nabla\psi)].\end{aligned}\qquad(96)$$

Using equations (9), (14) and (49), we find

$$\boldsymbol{v}_0\cdot\boldsymbol{B}_0=F\left(\Omega+\frac{\kappa B_0^2}{\rho_0 F}\right),\qquad \boldsymbol{v}_0\cdot\boldsymbol{v}'=\frac{v'_\parallel F}{B_0}\left(\Omega+\frac{\kappa B_0^2}{\rho_0 F}\right)+\frac{c|\nabla\psi|^2}{B_0^2}\Omega\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}.\qquad(97)$$

Also, using equations (14) and (49), we obtain

$$\nabla\cdot(\boldsymbol{v}_0\times\nabla\psi)=-\boldsymbol{B}_0\cdot\nabla\left(R^2\Omega+\frac{\kappa F}{\rho_0}\right),\qquad(98)$$

$$\nabla\cdot(\boldsymbol{v}'\times\nabla\psi)=-\boldsymbol{B}_0\cdot\nabla\left(F\frac{v'_\parallel}{B_0}+\frac{c|\nabla\psi|^2}{B_0^2}\frac{d\phi'}{d\psi}\right).\qquad(99)$$

Substituting equations (97)–(99) into equation (96), we arrive at

$$\mathcal{F}_\perp^{(3)} = \frac{\rho_0 v_\parallel'}{B_0}\left[\left(\Omega + \frac{\kappa B_0^2}{\rho_0 F}\right)\boldsymbol{B}_0\cdot\nabla R^2 - \frac{\kappa B_0^2}{\rho_0 F}\boldsymbol{B}_0\cdot\nabla\left(\frac{|\nabla\psi|^2}{B_0^2}\right)\right] + \boldsymbol{B}_0\cdot\nabla\left(c\kappa\frac{|\nabla\psi|^2}{B_0^2}\frac{\mathrm{d}\phi'}{\mathrm{d}\psi}\right). \tag{100}$$

The last term of equation (100) does not contribute to equation (62).

The last piece of $\mathcal{F}_\perp$ is related to the perturbed mass density and equilibrium plasma rotation

$$\mathcal{F}_\perp^{(4)} \equiv \frac{\rho'}{B_0^2}(\boldsymbol{B}_0\times\nabla\psi)\cdot(\boldsymbol{v}_0\cdot\nabla)\boldsymbol{v}_0 = \frac{\rho'}{B_0^2}(\boldsymbol{B}_0\times\nabla\psi)\cdot\left((\nabla\times\boldsymbol{v}_0)\times\boldsymbol{v}_0 + \frac{1}{2}\nabla v_0^2\right)$$
$$= -\frac{\rho'}{B_0^2}\left(\frac{F}{2}\boldsymbol{B}_0\cdot\nabla v_0^2 + (\boldsymbol{v}_0\cdot\boldsymbol{B}_0)\nabla\cdot(\boldsymbol{v}_0\times\nabla\psi)\right). \tag{101}$$

Substituting equations (97) and (98) into equation (101) we obtain

$$\mathcal{F}_\perp^{(4)} = \frac{\rho' F}{2B_0^2}\left[\left(\Omega + \frac{\kappa B_0^2}{\rho_0 F}\right)^2\boldsymbol{B}_0\cdot\nabla R^2 - \frac{\kappa^2 B_0^4}{\rho_0^2 F^2}\boldsymbol{B}_0\cdot\nabla\left(\frac{|\nabla\psi|^2}{B_0^2}\right)\right]. \tag{102}$$

Finally, we substitute equations (93)–(95), (100) and (102) into equation (61) and arrive at equation (62).

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