PHYSICAL REVIEW E **84**, 016407 (2011)

# Resonant transparency of a two-layer plasma structure in a magnetic field

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(Received 21 March 2011; published 25 July 2011)

Transparency of a two-layer plasma structure in an external steady-state magnetic field, perpendicular to the wave incidence plane, is studied. The case of the p-polarized electromagnetic wave is considered. The electromagnetic wave is obliquely incident on the two-layer structure and is evanescent in both layers. The conditions for total transparency of the two-layer structure are found. The parametric dependencies of the transparency coefficient on the plasma slab widths, the magnitude of the wave number component, as well on the magnetic field magnitude are obtained.

DOI: 10.1103/PhysRevE.84.016407 PACS number(s): 52.25.Os, 52.25.Xz, 52.35.Lv

## I. INTRODUCTION

The problem of the interaction of electromagnetic radiation with overcritical plasmas with $\omega < \omega_{pe}$, where $\omega$ and $\omega_{pe}$ are the wave and plasma frequencies, respectively, is of interest for a number of areas in magnetically confined and laser plasmas, in particular for plasma heating, plasma diagnostics, radio communications, and radar applications. Recently, there has been a growing interest in this subject in new areas of photonic technologies such as plasmonics [1–4].

Normally, an overcritical density plasma layer is opaque to electromagnetic waves. However, the transparency can be achieved by including on each side of the overcritical density plasma with negative permittivity ($\varepsilon < 0$) a boundary layer of plasma with positive permittivity $0 < \varepsilon < 1$. The resonant transmission of a p-polarized electromagnetic wave through a symmetrical three-layer structure composed of a medium with negative dielectric permittivity, that was placed between layers with positive permittivity, was demonstrated both theoretically and experimentally [5]. Lately [6], it was shown that symmetry of the system is not a necessary condition, and resonant transparency of an asymmetric two-layer system is also possible. In these configurations, transparency of a dense plasma slab is possible due to resonant excitation of surface waves, which supports energy transport through the opaque region [5].

The general conditions for resonant wave transmission through two- and three-layer configurations were analyzed in [7,8]. It was shown that the resonant transparency can also be achieved by placing the dense plasma layer between two boundary layers of a material with a dielectric permittivity $\varepsilon > 1$ [9]. In this case, resonant wave transmission takes place without the surface mode excitation but by exploiting the standing-wave resonances [9]. Anomalous plasma transparency can also be achieved due to nonlinear interactions [10]. The effects of electron temperature on the transparency of an overcritical density plasma layer were studied in [7]. It was demonstrated that in warm plasmas the excitation of the propagating longitudinal electrostatic modes is possible that facilitates the total transparency of an opaque plasma slab by creating additional resonances in the system [7].

Resonant transmission is strongly affected by absorption effects. A detailed study of the effects of absorption in transition metamaterials was carried out in [11]. It was shown that resonant absorption can be controlled by changing the parameters of the transition layer and more generally by spatial profiles of the material parameters. In [12], general expressions for the reflection, transmission, and heating coefficients were obtained for a plane electromagnetic wave obliquely incident on a warm bounded plasma by solving the coupled Vlasov-Maxwell set of equations.

There has been a growing recent interest in resonant transmission through composite metamaterial structures; e.g., the transmission response of a subwavelength metal mesh structure symmetrically cladded between the dielectric layers has been experimentally measured and explained using a combination of numerical and analytical modeling [13]. The experimentally observed broad, high-transmission band was shown to be due to the superposition of two noninteracting modes. The transmission of the s-polarized electromagnetic waves through different metal, dielectric, and plasma structures has been also studied by many authors [1,14,15]. While it was shown that incident p-polarized photons can resonantly tunnel through a silver film with narrow-grooved zero-order gratings on both sides via exciting standing-wave surface plasmon-polariton modes, for the s-polarized photons the film acted as a nearly perfect mirror [16].

The effects of an external magnetic field on the transparency of plasmas were also studied in various contexts [17–21]. In [17], the absorption, reflection, and transmission of electromagnetic waves by a nonuniform plasma slab immersed in uniform magnetic field were investigated. It was shown that more than 90% of the electromagnetic wave power can be absorbed in high-density and high-collision plasma. The reflection, absorption, and transmission of microwaves by a magnetized, steady-state, two-dimensional, nonuniform plasma slab were studied in [18]. The total reflected, absorbed, and transmitted powers were calculated and their functional dependencies on the number density, collision frequency, and propagation angle were obtained [18]. In [19], it was shown that the transmission and reflection coefficients, and the distribution of the electromagnetic field in a magnetized plasma layer, can be found from a two-point boundary value problem. The characteristics of electromagnetic wave propagation through a magnetized plasma slab with linear electron density profile were also analyzed in [20]. The effects of the external magnetic field on radio wave transmission were studied with application to radio communication through the overdense plasmas [21]. Most of the authors, however, dealt with single-layer plasma structures.

In this paper, we study the propagation of a p-polarized electromagnetic wave through a two-layer plasma structure immersed in the external magnetic field. We analyze dispersion properties of surface waves localized at the interface between two semi-infinite layers of the magnetized plasmas. These properties are fundamental to the transparency of the two-layer structure. We determine the conditions for the resonant transparency and investigate the effects of the magnetic field magnitude and the slab width. The transmission characteristics and the dispersion properties are studied by using the impedance matching technique described in [8].

## II. BASIC EQUATIONS

Consider a two-layer plasma structure in a vacuum (Fig. 1). The structure is immersed in the external steady-state magnetic field $\mathbf{H_0}$ directed along the $z$ axis. It is assumed that the density of the first plasma slab layer Pl1 is small ($0 < \varepsilon_{10} < 1$, where $\varepsilon_{10}$ is the dielectric permittivity of the layer in the absence of a magnetic field), while the second slab Pl2 is dense with $\varepsilon_{20} < 0$ (here $\varepsilon_{20}$ is the dielectric permittivity of the second layer at $\mathbf{H_0} = 0$).

An electromagnetic wave is obliquely incident from a semi-infinite vacuum (air) region V1 at the plasma layer Pl1 on the left. The transmitted wave propagates into a semi-infinite vacuum (air) region V2 on the right. The wave is assumed to be p-polarized with the wave vector $\mathbf{k} = k_x\mathbf{e_x} + k_y\mathbf{e_y}$; i.e., the electric field vector $\mathbf{E}$ is in the incidence $xy$ plane. In general, there are incident (with $k_x > 0$) and reflected (with $k_x < 0$) waves in the region V1, but there is no reflected wave in the region V2. In the plasma regions Pl1 and Pl2, which have widths $a_1$ and $a_2$, correspondingly, the waves are evanescent (with $\mathrm{Re}k_x = 0$). It is assumed that the plasma slabs are uniform in $y$ and $z$ directions. We neglect the nonlinear effects; i.e., it is assumed that the wave phase velocity is larger than the electron oscillatory velocity. This condition translates into the condition of the intensity of the electric wave field $\mathbf{E}$: $|\mathbf{E}| \ll m_e\omega^2/(ke)$, where $e$ and $m_e$ are the electron charge and mass, respectively.

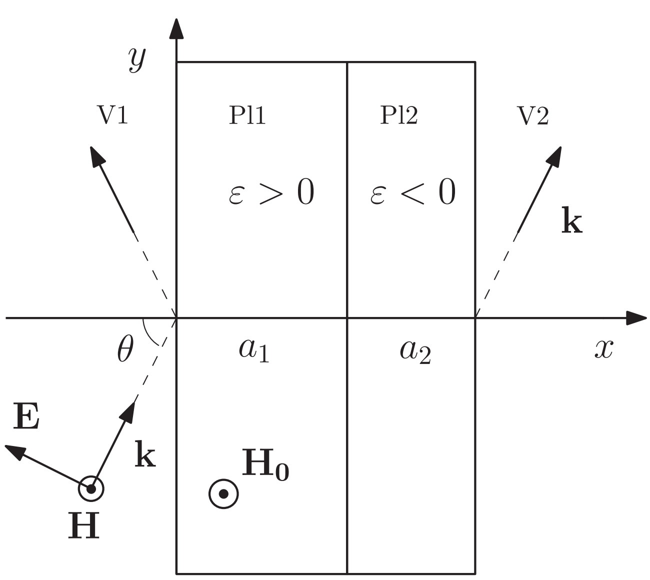


FIG. 1. Schematic representation of the propagation of an electromagnetic wave through the two-layer structure.

We assume that ions are immobile, the magnetic permittivity of a plasma slab is $\mu = 1$, the phase velocity of the electromagnetic wave is larger than the electron thermal velocity, and the electron collision frequency is smaller than the wave frequency. Taking into account these assumptions, the dielectric permittivity tensor in the external steady-state magnetic field has the following nonzero components:

$$\varepsilon_{11} = \varepsilon_{22} = \varepsilon = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2},$$

$$\varepsilon_{12} = -\varepsilon_{21} = ig = \frac{i\omega_c\omega_p^2}{\omega(\omega^2 - \omega_c^2)},$$

$$\varepsilon_{33} = 1 - \frac{\omega_p^2}{\omega^2},$$

where $\omega_c$ is the electron cyclotron frequency.

The electromagnetic field of the p-polarized wave is represented by

$$\mathbf{E} = (E_x(x), E_y(x), 0)\exp(ik_y y - i\omega t),$$

$$\mathbf{H} = (0, 0, H_z(x))\exp(ik_y y - i\omega t).$$

From Maxwell's equations, one finds the following equations for the field components $E_x$, $E_y$, and $H_z$ in the Voigt geometry:

$$E_x(x) = -\frac{1}{k(\varepsilon^2 - g^2)}\left(k_y\varepsilon H_z + g\frac{dH_z}{dx}\right), \quad (1)$$

$$E_y(x) = -\frac{i}{k(\varepsilon^2 - g^2)}\left(k_y g H_z + \varepsilon\frac{dH_z}{dx}\right), \quad (2)$$

$$\frac{d^2H_z}{dx^2} + \kappa^2 H_z = 0, \quad (3)$$

where $\kappa^2 = k_y^2 - (\varepsilon^2 - g^2)k^2/\varepsilon$, $k = \omega/c$, and $c$ is the speed of light.

The local wave impedance is defined as

$$Z(x) = \frac{E_y(x)}{H_z(x)}. \quad (4)$$

Using Eqs. (2) and (3), one finds the wave field components $E_y$ and $H_z$ in the vacuum region V1:

$$E_y = \frac{k_x A_v}{k}[\exp(ik_x x) - \Gamma_v\exp(-ik_x x)], \quad (5)$$

$$H_z = A_v[\exp(ik_x x) + \Gamma_v\exp(-ik_x x)], \quad (6)$$

where the first terms in the brackets on right-hand side of Eqs. (5) and (6) account for the incident wave and the second terms account for the reflected wave, $A_v$ is the amplitude of the incident wave, and $\Gamma_v$ is the reflection coefficient.

Then the local impedance for the electromagnetic wave field in the region V1 is

$$Z_{v1}(x) = Z_v\frac{\exp(ik_x x) - \Gamma_v\exp(-ik_x x)}{\exp(ik_x x) + \Gamma_v\exp(-ik_x x)}, \quad (7)$$

where $Z_v = k_x/k$ is the characteristic impedance of the vacuum region.

Using Eq. (7), one finds that the reflection coefficient $\Gamma_v$ is determined by the following relation:

$$\Gamma_v = \frac{Z_v - Z_{v1}(0)}{Z_v + Z_{v1}(0)}, \tag{8}$$

where $Z_{v1}(0)$ is the impedance at the plasma-vacuum interface ($x = 0$).

Since there is only a transmitted ($k_x > 0$) wave in the vacuum region V2, the wave impedance in the region is independent of spatial coordinates and is equal to the characteristic impedance

$$Z_{v2} = Z_v.$$

For the electromagnetic wave field determined by Eqs. (1)–(3), the local wave impedance in the plasma layer is

$$Z = \frac{E_y}{H_z} = -\frac{i}{k(\varepsilon^2 - g^2)}\left(k_y g + \varepsilon \frac{1}{H_z}\frac{dH_z}{dx}\right). \tag{9}$$

Solving Eq. (3), one finds that the magnetic field in a plasma layer can be represented in the form

$$H_z = A[\exp(-\kappa x) + \Gamma \exp(\kappa x)], \tag{10}$$

and the wave impedance becomes

$$Z(x) = -i\psi + i\xi \frac{\exp(-\kappa x) - \Gamma \exp(\kappa x)}{\exp(-\kappa x) + \Gamma \exp(\kappa x)}, \tag{11}$$

where

$$\xi = \frac{\kappa \varepsilon}{k(\varepsilon^2 - g^2)},$$
$$\psi = \frac{k_y g}{k(\varepsilon^2 - g^2)},$$

$A$ is the amplitude of the wave, which is incident on a plasma slab, and $\Gamma$ is its reflection coefficient.

It is convenient to introduce $Z(x_0)$ and $Z(x_a)$ as the impedances at the left ($x = x_0$) and right ($x = x_a$) boundaries of the plasma layer. From Eq. (11) one has

$$Z(x_0) = -i\psi + i\xi \frac{\exp(-\kappa x_0) - \Gamma \exp(\kappa x_0)}{\exp(-\kappa x_0) + \Gamma \exp(\kappa x_0)}, \tag{12}$$

$$Z(x_a) = -i\psi + i\xi \frac{\exp(-\kappa x_a) - \Gamma \exp(\kappa x_a)}{\exp(-\kappa x_a) + \Gamma \exp(\kappa x_a)}. \tag{13}$$

We can eliminate the reflection coefficient $\Gamma$ in Eqs. (12) and (13), expressing the impedance $Z(x_a)$ in terms of the impedance $Z(x_0)$ (or vice versa):

$$Z(x_a) + i\psi = i\xi \frac{(Z(x_0) + i\psi) - i\xi L}{i\xi - (Z(x_0) + i\psi)L}, \tag{14}$$

$$Z(x_0) + i\psi = i\xi \frac{(Z(x_a) + i\psi) + i\xi L}{i\xi + (Z(x_a) + i\psi)L}, \tag{15}$$

where $L = \tanh(\kappa a)$.

Equations (14) and (15) provide us with the relations between the impedances $Z_1(a_1)$ and $Z_1(0)$ for the first plasma layer Pl1, as well as the relations between $Z_2(a_1 + a_2)$ and $Z_2(a_1)$ for the second layer Pl2.

Since the tangential electric and magnetic field components are continuous at interfaces, the wave impedance is also a continuous function. Matching the impedances at each interface, we get

$$Z_{v1}(0) = Z_1(0), \tag{16}$$
$$Z_1(a_1) = Z_2(a_1), \tag{17}$$
$$Z_2(a_1 + a_2) = Z_v. \tag{18}$$

Using the boundary conditions (16) and (18) and the relations (14) and (15), one can obtain the impedance $Z_{v1}(0)$ at the first plasma-vacuum interface, $x = 0$. Then, substituting $Z_{v1}(0) = Z_1(0)$ in Eq. (8), one calculates the reflection coefficient

$$\Gamma_v = \frac{Z_v - Z_1(0)}{Z_v + Z_1(0)}, \tag{19}$$

where

$$\begin{aligned} Z_1(0) &= -i\psi_1 + i\xi_1 \frac{[Z_2(a_1) + i\psi_1] + i\xi_1 L_1}{i\xi_1 + [Z_2(a_1) + i\psi_1]L_1}, \\ Z_2(a_1) &= -i\psi_2 + i\xi_2 \frac{(Z_v + i\psi_2) + i\xi_2 L_2}{i\xi_2 + (Z_v + i\psi_2)L_2}. \end{aligned} \tag{20}$$

The transmission coefficient $T$ is determined from the reflection coefficient:

$$T = 1 - \Gamma_v^2. \tag{21}$$

The total transparency of the two-layer structure is achieved for $\Gamma_v = 0$. In this case, the impedance at $x = 0$ is precisely the vacuum impedance; i.e., $Z_1(0) = Z_v$. From this condition and Eq. (21), we obtain the following condition for the resonant transparency of the two-layer structure:

$$\psi_1 - \xi_1 \frac{Z_v + i(\psi_1 - \xi_1 L_1)}{i\xi_1 - (Z_v + i\psi_1)L_1} = \psi_2 - \xi_2 \frac{Z_v + i(\psi_2 + \xi_2 L_2)}{i\xi_2 + (Z_v + i\psi_2)L_2}, \tag{22}$$

where $L_1 = \tanh(\kappa_1 a_1)$ and $L_2 = \tanh(\kappa_2 a_2)$. This complex equation is equivalent to the system of two real equations

$$\begin{aligned} &Z_v^2 L_2(L_1\psi_1 + \xi_1) - L_1(\psi_1^2 - \xi_1^2)(L_2\psi_2 + \xi_2) \\ &\quad = Z_v^2 L_1(L_2\psi_2 - \xi_2) - L_2(\psi_2^2 - \xi_2^2)(L_1\psi_1 - \xi_1), \end{aligned} \tag{23}$$

$$\begin{aligned} &(L_1\psi_1 + \xi_1)(L_2\psi_2 + \xi_2) + L_1 L_2(\psi_1^2 - \xi_1^2) \\ &\quad = (L_1\psi_1 - \xi_1)(L_2\psi_2 - \xi_2) + L_1 L_2(\psi_2^2 - \xi_2^2). \end{aligned} \tag{24}$$

The system of equations (23) and (24) determines the conditions for the resonant transparency ($T = 1$) of the two-layer structure.

In the case of thick layers, $L_i \approx 1 - \exp(-2\kappa_i a_i)$, where $i = 1,2$, and condition (22) reduces to

$$\begin{aligned} (\psi_1 + \xi_1) - (\psi_2 - \xi_2) = &-2\xi_1 \frac{\psi_1 + \xi_1 - iZ_v}{\psi_1 - \xi_1 - iZ_v} e^{-2\kappa_1 a_1} \\ &- 2\xi_2 \frac{\psi_2 - \xi_2 - iZ_v}{\psi_2 + \xi_2 - iZ_v} e^{-2\kappa_2 a_2}. \end{aligned} \tag{25}$$

In the limit of a thick layer, the complex relation (25) is equivalent to the system of two real equations

$$(\psi_1+\xi_1)-(\psi_2-\xi_2) = -2\xi_1\frac{\psi_1^2-\xi_1^2+Z_v^2}{(\psi_1-\xi_1)^2+Z_v^2}e^{-2\kappa_1 a_1} -2\xi_2\frac{\psi_2^2-\xi_2^2+Z_v^2}{(\psi_2+\xi_2)^2+Z_v^2}e^{-2\kappa_2 a_2}, \quad (26)$$

$$\frac{\xi_1^2}{(\psi_1-\xi_1)^2+Z_v^2}e^{-2\kappa_1 a_1}-\frac{\xi_2^2}{(\psi_2+\xi_2)^2+Z_v^2}e^{-2\kappa_2 a_2}=0. \quad (27)$$

## III. DISPERSION PROPERTIES OF SURFACE WAVES AT THE INTERFACE BETWEEN TWO SEMI-INFINITE MAGNETIZED PLASMAS

In the lowest order ($\kappa_{1,2}a_{1,2}\to\infty$), one can neglect the right-hand side of Eq. (25), in which case it simplifies to

$$\psi_1+\xi_1=\psi_2-\xi_2. \quad (28)$$

Note that the resonance condition (28) for the case of infinitely thick layers coincides with the dispersion relation for the surface waves propagating in the $y$ direction at the interface between two semi-infinite magnetized plasma layers.

Equation (28) may be represented in more explicit form:

$$\frac{k_y g_1+\varepsilon_1\kappa_1}{\varepsilon_1^2-g_1^2}=\frac{k_y g_2-\varepsilon_2\kappa_2}{\varepsilon_2^2-g_2^2}. \quad (29)$$

Dispersion of the surface waves at the plasma-plasma interface has some interesting features. Contrary to the case of surface waves propagating at plasma-vacuum and dielectric interfaces [22,23], where the surface waves are always slow ($v_{\rm ph}<c$, where $v_{\rm ph}$ is the wave phase velocity), propagation of both fast ($v_{\rm ph}>c$) and slow ($v_{\rm ph}<c$) waves is possible at the interface of plasmas with $\epsilon<1$ [24]. Thus, in the plasma layers Pl1 and Pl2, the fast surface modes can couple to the incident or reflected electromagnetic waves, which are evanescent in the plasmas.

The surface waves in magnetized plasmas are nonreciprocal; i.e., the wave frequency depends on sign of the wave vector component $k_y$. We term the wave with $k_y>0$ a positive branch and the wave with $k_y<0$ a negative branch. The positive and negative branches exist in different frequency ranges (Fig. 2).

To find the upper limit of the frequency range for both positive and negative branches, we let $|k_y|\gg k$, and Eq. (29) gives us

$$g_1\mp\varepsilon_1=g_2\pm\varepsilon_2, \quad (30)$$

$$2-\frac{\omega_{p1}^2}{(\omega\mp\omega_c)\omega}-\frac{\omega_{p2}^2}{(\omega\pm\omega_c)\omega}=0. \quad (31)$$

Here, the upper and lower signs are for the positive and negative branches, correspondingly. In the case of a weak magnetic field ($\omega_c\ll\omega_{p1,2}$), Eq. (30) has the following solution:

$$\omega_\infty=\omega_\infty^{(0)}\pm\frac{\omega_c}{2}\frac{\omega_{p1}^2-\omega_{p2}^2}{\omega_{p2}^2+\omega_{p1}^2}, \quad (32)$$

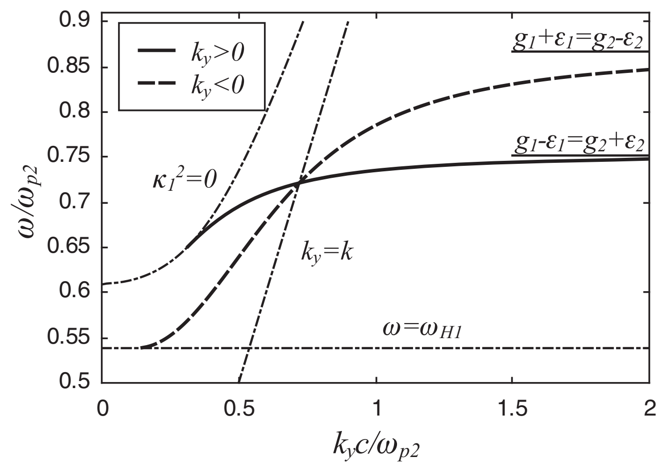


FIG. 2. Dispersion of the surface waves at the plasma-plasma interface. The curves are obtained for $\omega_{p1}/\omega_{p2}=0.5$ and $\omega_c/\omega_{p2}=0.2$.

where $\omega_\infty^{(0)}=\sqrt{(\omega_{p1}^2+\omega_{p2}^2)/2}$ is the asymptotic frequency (at $|k_y|\gg k$) for the case of nonmagnetized plasma.

The amplitude of the electromagnetic field of the surface mode decays away from the plasma-plasma interface; i.e., the decay constant $\kappa$ is a real number. Requiring $\kappa^2>0$, one determines the onset frequency. For the wave propagating in the positive direction, the onset frequency can be found from the inequality $k_y\geqslant k\sqrt{\epsilon_{V1}}$, where $\epsilon_{V1}=(\varepsilon_1^2-g_1^2)/\varepsilon_1$ is the Voigt dielectric constant for the plasma slab Pl1. The negative branch starts at the hybrid frequency $\omega_{H1}=\sqrt{\omega_{p1}^2+\omega_c^2}$ (see Fig. 2), which is smaller than the onset frequency for the positive branch. Below $\omega_{H1}$, the Voigt dielectric constant is large and positive, implying that $\kappa_1^2<0$ for a finite propagation vector; i.e., no surface magnetoplasmon is allowed.

## IV. TRANSPARENCY OF THE TWO-LAYER STRUCTURE

In this section we study the dependence of transmission properties of the two-layer structure as a function of the wave number $k_y$, the electron cyclotron frequency $\omega_c$, and the slab widths. First, consider the case without a magnetic field. For $\mathbf{H_0}=0$, the resonance transparency condition (22) yields the system of two equations [6]

$$\frac{\kappa_1}{\varepsilon_{10}}=-\frac{\kappa_2}{\varepsilon_{20}}, \quad (33)$$

$$\kappa_1 a_1=\kappa_2 a_2. \quad (34)$$

Equation (33) coincides with the dispersion relation for the surface waves propagating at the interface between two semi-infinite unmagnetized plasmas. Equation (34) determines the relationship between the slab widths. Under this condition the amplification of the field amplitude in one layer is offset by the attenuation of the field in the other layer [6]. Under these resonance conditions, one gets the absolute transparency ($T=1$) of the two-layer structure.

The transparency of an unmagnetized two-layer structure depends on the wave number component $k_y$. In Fig. 3, the transparency coefficient $T$ as a function of the normalized wave vector component $k_y/k$ is shown. The solid curve in Fig. 3 is

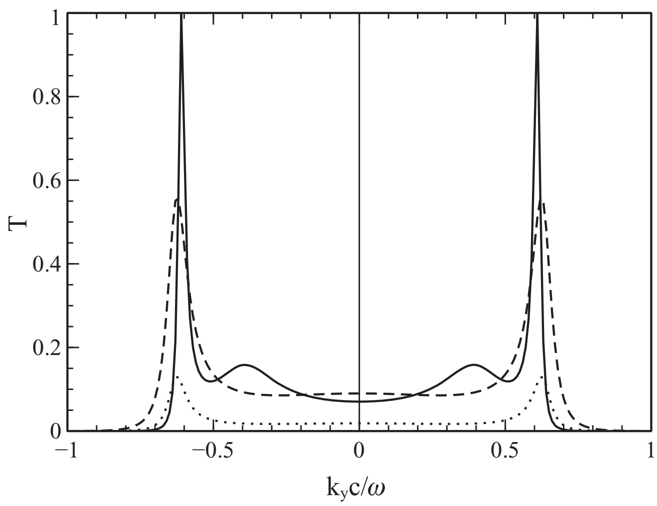


FIG. 3. The dependence of the transparency coefficient on the normalized wave vector for $\mathbf{H_0} = 0$ and different layer widths: $a_1 = 11.5401\delta$, $a_2 = 2\delta$ (solid line); $a_1 = 5.8264\delta$, $a_2 = 2\delta$ (dashed line); $a_1 = 11.5401\delta$, $a_2 = 3\delta$ (dotted line), where $\delta = c/\omega_{p2}$. Here, $\omega/\omega_{p2} = 0.6007$ and $\omega_{p1}/\omega_{p2} = 0.5$.

obtained for the normalized wave frequency $\omega/\omega_{p2} = 0.6007$, the ratio of plasma frequencies $\omega_{p1}/\omega_{p2} = 0.5$, and the widths of the plasma layers $a_1 = 11.5401\delta$, $a_2 = 2.0\delta$, where $\delta = c/\omega_{p2}$. In the absence of the magnetic field, the surface waves are reciprocal, and the transmission coefficients $T$ for the positive ($k_y > 0$) and negative ($k_y > 0$) branches are identical. Therefore, the transparency dependence is symmetrical with respect to the axis, $k_y/k = 0$.

If the widths of the plasma slabs do not satisfy the resonance condition (34), then the transparency of the system is reduced. [See the dashed and dotted curves in Fig. 3, which are obtained for the same external conditions as the solid curve in Fig. 3, except for a different width of the first layer in the dashed curve case ($a_1 = 5.8264\delta$), and a different width of the second layer in the dotted curve case ($a_2 = 3.0\delta$).] The transparency coefficient decreases with increasing deviation of the layer width from the resonant value (see Fig. 4).

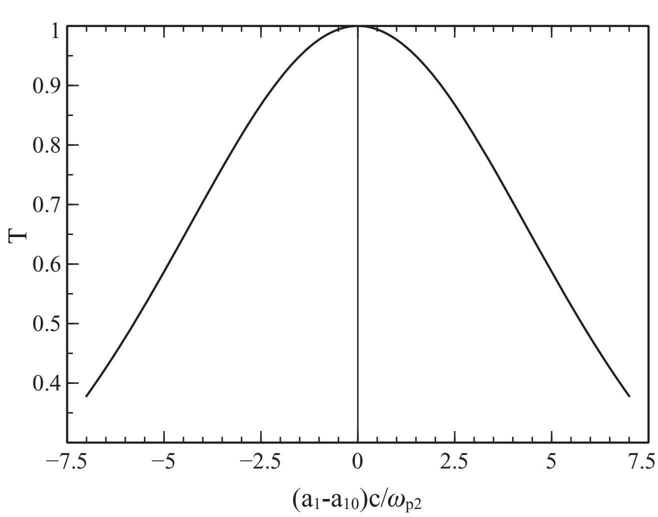


FIG. 4. The dependence of the transparency coefficient on the deviation of the first layer width $a_1$ from the resonant width $a_{10}$ $(=11.5401\delta)$ for $\mathbf{H_0} = 0$, $k_y/k = 0.6095$, and $a_2 = 2\delta$. The other parameters are the same as in Fig. 3.

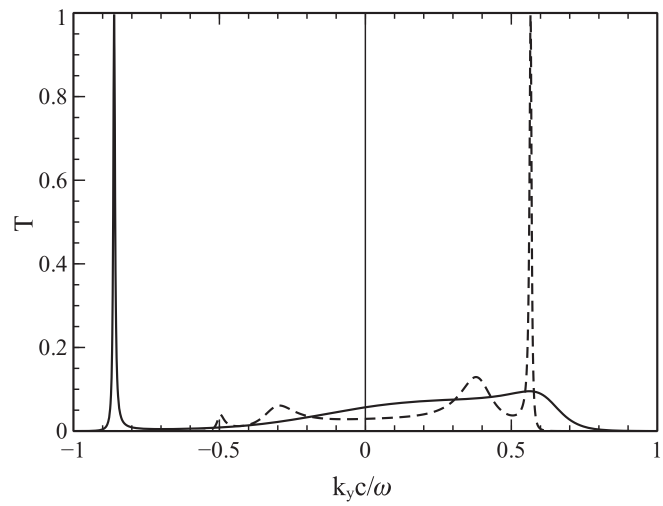


FIG. 5. The transparency coefficient dependence for $k_y < 0$, $a_1 = 5.717\delta$ (solid curve) and $k_y > 0, a_1 = 19.3824\delta$ (dashed curve). The other parameters are $a_2 = 2.985\delta$, $\omega/\omega_{p2} = 0.67$, $\omega_{p1}/\omega_{p2} = 0.5$, and $\omega_c/\omega_{p2} = 0.2$.

By eliminating the inverse skin depth parameters $\kappa_1$ and $\kappa_2$ from Eqs. (33) and (34), the resonance condition equations can be obtained in the form

$$k_y = \pm\sqrt{\frac{\varepsilon_{10}\varepsilon_{20}}{\varepsilon_{10} + \varepsilon_{20}}}, \tag{35}$$

$$\bar{\varepsilon} = \varepsilon_{10}a_1 + \varepsilon_{20}a_2 = 0, \tag{36}$$

where $\bar{\varepsilon}$ is the effective dielectric width for the two-layer structure at $\mathbf{H_0} = 0$.

Now, we analyze the effects of the magnetic field on the transparency of the two-layer structure. In the presence

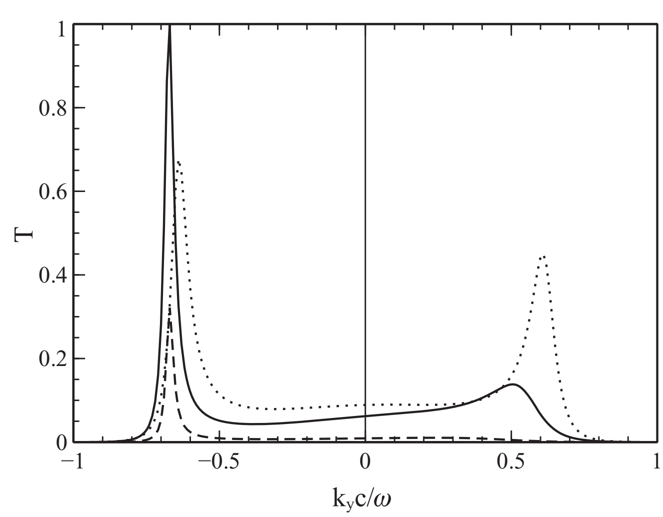


FIG. 6. The dependence of the transparency coefficient on the normalized wave vector for different values of the magnetic field: $\omega_c/\omega_{p2} = 0.1$ (solid line), $\omega_c/\omega_{p2} = 0.2$ (dashed line), and $\omega_c/\omega_{p2} = 0.02$ (dotted line). The other parameters are the same as those corresponding to the dashed line in Fig. 3.

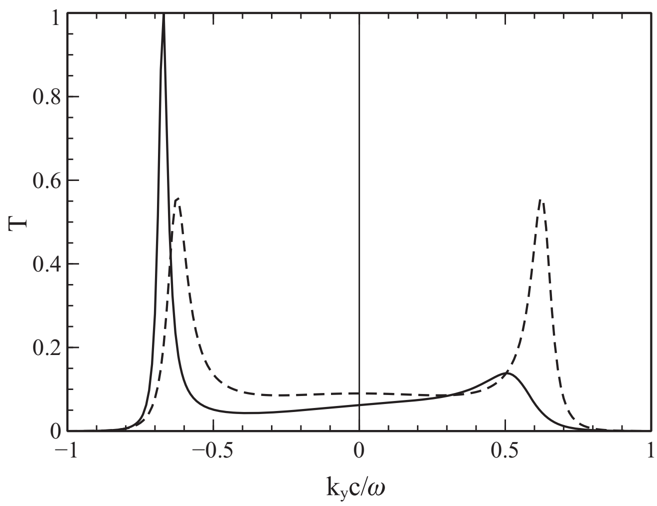


FIG. 7. The dependence of the transparency coefficient on the normalized wave vector. Solid and dashed lines are for $\omega_c/\omega_{p2} = 0.1$ and $\omega_c/\omega_{p2} = 0$, correspondingly. The other parameters correspond to the dashed curve in Fig. 3.

of the external steady-state magnetic field, the resonance conditions are described by Eqs. (23) and (24), which are different from those for the case $\mathbf{H_0} = 0$ [see Eqs. (33) and (34)]. At $\mathbf{H_0} \neq 0$, the transparency coefficient dependence on $k_y/k$ is asymmetrical with respect to the axis $k_y/k = 0$ (see Fig. 5). The asymmetry is due to nonreciprocity of the waves propagating in magnetized plasmas (see Sec. III). The resonance conditions and transparency of the two-slab system depend on sign of $k_y$. At $\mathbf{H_0} \neq 0$, it is impossible to get the resonance transparency for both positive and negative $k_y$ and the same slab parameters simultaneously (see Fig. 5). However, the resonance transparency may be reached for the $k_y > 0$ and $k_y < 0$ cases at different sets of the parameters, for example at different widths of the first slab (Fig. 5).

The transparency coefficient is sensitive to the magnitude of the external magnetic field. To show this, we calculated the transparency coefficient dependence of the normalized wave vector component $k_y/k$ for different cyclotron frequencies. In Fig. 6, the $T$ dependencies are presented for $\omega_c/\omega_{p2} = 0.1$ (solid line), 0.2 (dashed line), and 0.02 (dotted line). Note that, applying the external magnetic field, one can control the transparency of the two-layer structure. The transparency can even be improved for cases when the slab parameters initially are not chosen to satisfy resonant conditions (see, for example, Fig. 7).

## V. CONCLUSIONS

We have studied the transparency of two-layer plasma structures in the external steady-state magnetic field $\mathbf{H_0}$. The case of the electromagnetic wave obliquely incident on this structure has been considered with an electromagnetic field which is evanescent in both layers. The wave has been assumed to be p-polarized, and the external magnetic field is perpendicular to the wave incidence plane (Voigt geometry).

Conditions (23) and (24) for total transparency of the two-layer structure have been found. The effects of the plasma layer widths, the magnitude of the wave number component $k_y$, as well as the magnetic field magnitude (Fig. 6) on the transparency have been studied. It has been found that the difference in the two peaks of the transmission coefficient $T$ becomes larger if the magnetic field increases (see Fig. 6). It has been also shown that the structure, which is not totally transparent in the absence of a magnetic field, can become totally transparent at $\mathbf{H_0} \neq 0$ (see Fig. 7).

The dispersion properties of the surface waves are crucial for the phenomenon of resonant transparency studied in this paper. It is worth noting that, in the absence of the magnetic field, one of the resonant transparency conditions coincides with the dispersion relation for the surface waves propagating at the interface of two semi-infinite plasmas. For a finite magnetic field, the surface waves are nonreciprocal. The frequency ranges for both positive ($k_y > 0$) and negative ($k_y < 0$) branches have been found. Due to the nonreciprocity, the dependence of the transparency coefficient on the wave number is different for positive ($k_y > 0$) and negative ($k_y < 0$) branches, contrary to the case of $\mathbf{H_0 = 0}$.

For thick plasma layers, one of the conditions for total transparency at $\mathbf{H_0 \neq 0}$ is the same as the dispersion relation for the surface waves propagating at the interface between two semi-infinite regions. However, for finite widths, general Eqs. (23) and (24) have to be used instead of the dispersion relation (28).

Note that in our study we have used some simplifications. In particular, the plasma densities have been assumed to be uniform. However, in many laboratories and in naturally occurring plasmas, the electron density is nearly uniform only in central regions, not near boundaries. Therefore, the plasma uniformity assumption is applied only if the plasma nonuniformity scale, which in laboratory low-pressure plasmas is about a few Debye radii, is smaller than the skin depths $1/\kappa_1$ and $1/\kappa_2$. The nonuniformity effects may increase energy dissipation in the structure [22,25]. In our study, however, we have neglected all dissipation effects, which may affect the transparency of the system. Moreover, we also neglected nonlinear effects, as well as the thermal motion of electrons. These effects can influence the transparency of the structure [7,10,26], even increasing it at certain conditions [10].

In conclusion, we have shown that a two-slab structure of optically opaque material can be totally transparent to a p-polarized obliquely incident electromagnetic wave in the external magnetic field. The anomalously high transparency is explained through the energy transport by two (decaying and growing) evanescent waves which are excited by the incident electromagnetic wave. Two waves are efficiently excited at the resonance in the two-layer structure. If the layers are thick or an external magnetic field is absent, the waves in the layers are the surface waves with the dispersion described by the dispersion relation for surface waves propagating at the interface of two semi-infinite plasmas. At $\mathbf{H_0 \neq 0}$ and arbitrary plasma widths, the dispersion properties of the waves in the slabs are different from those at the interface of the semi-infinite plasmas. The results obtained in this paper can be used in laboratory plasma experiments, as well as in plasmonic and communications applications.

## ACKNOWLEDGMENT

This work was supported by NATO Collaborative Linkage Grant No. CBP.NUKR.CLG.983378.

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