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Modification of the Simon-Hoh Instability by the sheath effects in partially magnetized E × B plasmas

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Effects of dissipation on the gradient drift modes in partially magnetized $\mathbf{E}\times\mathbf{B}$ plasmas are studied with emphasis on the sheath effects. It is shown that the dissipation induced instabilities driven by the density gradient and $\mathbf{E}\times\mathbf{B}$ drifts persist in conditions where the criteria for standard Simon-Hoh instability in $\mathbf{E}\times\mathbf{B}$ plasmas are not satisfied. *Published by AIP Publishing.*
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Plasma discharges based on $\mathbf{E}\times\mathbf{B}$ fields are used in a variety of applications for electric propulsion and plasma processing. In applications such as Hall thruster, Penning traps, and magnetron devices, the plasmas are only partially magnetized; the electrons are magnetized while the ions are not. In such plasmas, the electric field perpendicular to the external magnetic field is often used to confine electrons and support the discharge. When the plasma density is inhomogeneous across the magnetic field, as is expected for magnetically confined electrons, the electron drift together with the inertial response of unmagnetized ions results in a peculiar eigen-mode of partially magnetized plasmas: the so-called "anti-drift" mode.[1] This mode is the basis for various extensions and instabilities existing in a partially magnetized plasma, which are generically referred here as gradient-drift modes.[2–4] The simplest expression for the anti-drift mode frequency is given by

$$\omega \equiv \frac{k^2 c_s^2}{\omega_*} = -k_y L_n \omega_{ci}, \tag{1}$$

where k_y refers to the perpendicular wave-vector, $c_s = \sqrt{T_e/m_i}$, $\omega_* = -c_s^2 k_y / L_n \omega_{ci}$, $\omega_{ci} = eB_0/m_i c$ is the ion cyclotron frequency (assuming singly-charge ions), and $L_n^{-1} = (dn/dx)/n$ is the inverse density gradient length scale.

For the Hall thruster geometry, consider a small region of the plasma which is approximately planar (i.e., we set the radius of curvature characteristic of the system to be very large) and we set the axial direction to be the **x**-direction, the **y**-direction to be along the azimuthal direction, and the **z**-direction to be in the radial direction. Then, k_y is the component of the wave-vector along the azimuthal direction which is periodic. Similar approximations can be made for the magnetron and Penning discharge geometries, so we assume that the y-direction is always periodic, z-direction is along the magnetic field, and x-direction is the direction of the density gradient and the electric field. It should be noted that this mode occurs for $k_z = 0$.[5]

The $\mathbf{E}\times\mathbf{B}$ plasma discharges are supported by the energy input from the externally imposed electric field $\mathbf{E}_0 = E_0\mathbf{x}$ which causes the equilibrium electron $\mathbf{E}_0\times\mathbf{B}_0$-drift where $\mathbf{B}_0 = B_0\mathbf{z}$. The Doppler shift due to the electron drift, $\mathbf{v}_0 = cE_0/B_0\mathbf{y}$, results in the modification of the electron response with $\omega \to \omega - k_y v_0$, giving the dispersion relation

$$\frac{k^2 c_s^2}{\omega^2} = \frac{\omega_*}{\omega - \omega_0}, \tag{2}$$

and, therefore, producing the reactive instability of the anti-drift mode if the condition $\omega_0/\omega^* > 0$ is met. This implies a condition on the density gradient and externally applied electric field, $E_0(dn/dx) > 0$. This instability, referred to as the collisionless Simon-Hoh instability, is one of the several instabilities existing in Hall plasmas[3,6] In the simplest case, this instability and resulting plasma dynamics are often considered in neglect of the electron motion along the magnetic field or assuming periodicity in this direction. It is our goal to consider finite and bounded plasmas where the magnetic field lines are intercepted by material walls. In this case, the sheaths formed at the boundaries become important and constrain the parallel plasma motion. It will be seen that this induces a dissipative-type instability.

That there exist instabilities enabled by the sheath with wavelengths of the order of the system length was first shown in Refs. 7 and 8. Shear flow and temperature-gradient instability driven by the sheath were studied in application to the divertor and scrape-off layer in tokamaks. The sheath driven instability due to the electron secondary emission was shown in Ref. 9. All of these works were performed under the assumption of fully magnetized plasmas. Sheath effects on the modes in partially magnetized plasma were investigated in Ref. 4 but in neglect of the plasma density gradient.

It turns out that the effect of the sheath is similar in structure to the effect of electron motion along the magnetic field in the presence of the electron-neutral collisions. Thus, we will review this case first. Then, the appropriate boundary conditions for our problem will be presented and used to derive the dispersion equation for the case of the collisionless, bounded plasma. The following paragraph will demonstrate the modification of the condition for the classical

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Simon-Hoh instability due to the presence of finite resistivity and then expose the two main limiting cases (e.g., $v_0 = 0$ and $v_* = 0$). Finally, the system with both electron drift and ion density gradient will be analyzed, and it will be shown that the growth rate (as well as the angular frequency of the plasma oscillation modes) is significantly modified by the presence of finite resistivity, would it be due to neutral-electron collisions or due to the conservation of current at the bounding walls.

The equation of motion for the electrons, neglecting electron inertia, is given by

$$0 = -e\left(-\nabla\Phi + \frac{\mathbf{v}_e}{c}\times\mathbf{B}\right) - \frac{\nabla(n_e T_e)}{n_e} - \nu m_e v_{ez}\mathbf{z}. \quad (3)$$

Linearizing Eq. (3) for small perturbations in the electric potential, number density, and velocity yields the set of equations

$$\frac{\nu}{v_{Te}^2}\tilde{v}_{ez} = \frac{\partial}{\partial z}\left(\frac{e\tilde{\Phi}}{T_e} - \frac{\tilde{n}_e}{n_0}\right), \quad (4)$$

$$\tilde{\mathbf{v}}_{e\perp} = \frac{c\mathbf{b}\times\nabla_\perp\tilde{\Phi}}{B_0} - \frac{cT_e\mathbf{b}\times\nabla_\perp\tilde{n}_e}{n_0 e B_0}. \quad (5)$$

Assuming, because of the periodicity in **y** and the infinite extend in **x**, the form $(\tilde{\Phi},\tilde{n}) \sim \exp\left(-i\omega t + i\mathbf{k}_\perp\cdot\mathbf{r}_\perp\right)$, it is easy to show that

$$(\omega-\omega_0)\frac{\tilde{n}_e}{n_0} - \omega_*\frac{e\tilde{\Phi}}{T_e} + i\frac{\partial\tilde{v}_{ez}}{\partial z} = 0. \quad (6)$$

The equation of motion for the unmagnetized ions is given by

$$\frac{d\mathbf{v_i}}{dt} + (\vec{v}_i\cdot\nabla)\mathbf{v}_i = -\frac{e}{m_i}\nabla\Phi. \quad (7)$$

Then, to first-order in the perturbation $\tilde{\Phi} = \tilde{\Phi}(z)e^{-i\omega t + i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}$, we have

$$\tilde{v}_{iz} = \frac{e}{i\omega m_i}\frac{\partial\tilde{\Phi}}{\partial z}, \quad (8)$$

$$\tilde{\mathbf{v}}_{i\perp} = \frac{e}{m_i\omega}\mathbf{k}_\perp\tilde{\Phi}. \quad (9)$$

The continuity equation for the ions then yields

$$-i\omega\frac{\tilde{n}_i}{n_0} + i\mathbf{k}_\perp\cdot\tilde{\mathbf{v}}_{i\perp} + \frac{\partial\tilde{v}_{iz}}{\partial z} = 0. \quad (10)$$

Assuming quasi-neutrality as well as taking $\partial/\partial z \to ik_z$, one finds the dispersion equation

$$\frac{\omega_* + ik_z^2 v_{Te}^2/\nu}{\omega - \omega_0 + ik_z^2 v_{Te}^2/\nu} = \frac{k^2 c_s^2}{\omega^2}, \quad (11)$$

which was shown for a plasma of unmagnetized ions and non-vanishing k_z.[1,6] It should be noted that a similar problem was considered in Refs. 10 and 11, but the parallel electron velocity was taken in the form of an ambipolar diffusion flux rather than in the form of Eqs. (4) and (5). In fact, the self-consistent density response is constrained by the quasi-neutrality condition where both parallel and perpendicular electron velocities are responsible for maintaining quasi-neutrality.

In many physical situations, it is interesting to look at large scale perturbations along the magnetic field lines since these are the modes that will *see* the presence of the bounding walls. Then, k_z is much smaller than k_y (it is implicitly assumed that perturbations in the x-direction are similar to the length scale of the system [i.e., $k_x^2 \sim k_z^2 \ll k_y^2$)]. Since the **z**-direction is finite, the eigen-functions of the system will be the solution of a differential equation in z; it is then necessary to determine the boundary condition along the magnetic field.

This boundary condition is determined by the dynamics of the electrons and ions in the sheath region. The standard sheath model for a planar, quasi-neutral plasma will be assumed. Therefore, the walls will be fully absorbing, the plasma-sheath boundary will be defined as the location where the ions reach Bohm's velocity c_s, and the electron will be distributed isothermally (i.e., Boltzmann distribution) along the magnetic field being mostly reflected by the strong electric field in the sheath region. Since, at steady-state, there is no net current entering the sheath, the net current density vanishes

$$J_{sh,0} = 0 = J_{0i} + J_{0e} = en_0 c_s - \frac{en_0 v_{Te}}{\sqrt{2\pi}}e^{-e\phi_{sh}/T_e}, \quad (12)$$

where $e\phi_{sh}/T_e > 0$ is the equilibrium plasma potential at the sheath entrance (with respect to the wall potential). It is straightforward to solve for the normalized electric potential, the result being, for a singly-charged Xenon plasma, $-e\phi_{sh}/T_e = ln(2\pi m_e/m_i)/2 \approx 5$. Since the plasma density and electric potential perturbations will perturb the electron current entering the sheath, one finds for small perturbations

$$\tilde{J}_{ez}(L) = en_0 c_s\left(\frac{e\tilde{\Phi}(L)}{T_e} - \frac{\tilde{n}_e(L)}{n_0}\right). \quad (13)$$

This provides us with a boundary condition on the electron current density at the plasma-sheath interface ($z \approx L$). Using the ion density response and quasi-neutrality in the bulk plasma, one finds the simple form for Eq. (13)

$$\tilde{J}_{ez}(L) = en_0 c_s\frac{e\tilde{\Phi}(L)}{T_e}\left(1 - \frac{k_y^2 c_s^2}{\omega^2}\right). \quad (14)$$

This boundary condition can be implemented in Eq. (6) by averaging over z from $-L$ to L, whilst assuming the density perturbation is even in z

$$(\omega-\omega_0)\frac{\tilde{n}_e(L)}{n_0} - \omega_*\frac{e\tilde{\Phi}(L)}{T_e} = i\frac{1}{en_0 L}\tilde{J}_{ez}(L). \quad (15)$$

Again, using quasi-neutrality as well as the ion density, we have

$$\left((\omega-\omega_0)\frac{k_y^2 c_s^2}{\omega^2} - \omega_*\right)\frac{e\tilde{\Phi}(L)}{T_e} = i\frac{c_s}{L}\frac{e\tilde{\Phi}(L)}{T_e}\left(1 - \frac{k_y^2 c_s^2}{\omega^2}\right). \quad (16)$$

It should be noted that in the long wavelength approximation $k_z^2 \ll k_y^2$, the perturbed ion current in the **z**-direction, $\tilde{v}_z/\tilde{v}_y \sim k_z/k_y$, can be neglected so that the ion density response [used above in Eqs. (14) and (15)] is only determined by the ion transverse current. The final dispersion relation can be transformed to the form

$$\frac{k_y^2 c_s^2}{\omega^2} = \frac{\omega_* + i c_s/L}{\omega - \omega_0 + i c_s/L}, \tag{17}$$

which is similar in structure to the dissipative instability due to electron-neutral collisions seen in Eq. (11).

Properties of the dispersion Eqs. (11) and (17) can be presented in the same form using the notation $\nu_\parallel = c_s/L$ as a characteristic parallel flow frequency in the sheath bound plasma or with $\nu_\parallel = k_z^2 v_{Te}^2/\nu$ as typical parallel diffusion frequency for collisional plasmas. The relative importance of the sheath resistivity over the parallel collisional diffusion for $k_z \simeq 1/L$ is given by the standard condition[12,13]

$$\lambda/L > \sqrt{m_e/m_i}, \tag{18}$$

where $\lambda \equiv v_{Te}/\nu$ is the electron mean free path.

Also, we are interested in low frequency long wavelength instabilities with $\omega < \omega_{LH} = \sqrt{\omega_{ce}\omega_{ci}}$ ($\approx 488\omega_{ci}$ for a xenon plasma) so that the effects of the lower hybrid mode seen in Ref. 3 are neglected. We consider modes with low m (azimuthal mode number), roughly corresponding to the wavelengths of the order of the device radius. For estimates in what follows, we use $k_y\rho_s \simeq 1$.

Before presenting some results for different limiting cases as well as the modification of the Simon-Hoh instability, it is worth revisiting the physical mechanism behind the resistive instability. In the presence of cross electric and magnetic fields, the electrons will drift along the perpendicular direction to both of these fields. Then, if a small perturbation exists as well as an ion density gradient along this direction, there will be periodic regions of lower and higher density of electrons (relative to the ions) and therefore a fluctuating electric potential will be generated. In a perfectly conducting plasma, the electrons are free to move to lower density regions and thus short-circuit the induced electric field; the drift wave is then purely oscillatory. However, when some agent restricts the free motion of electrons, would it be electron-neutral collisions or the presence of a plasma sheath which restricts the electron current along the magnetic field;[7] there will be a phase lag between the electric potential and the density fluctuations. In such a case, the induced $\mathbf{E}\times\mathbf{B}$ drift will actually increase the density fluctuations, exasperating the difference between high and low density regions. The main results which follow provide a quantitative description of the growth rate of this resistive drift instability in different limits.

The collisionless Simon-Hoh instability is recovered in the limit $\nu_\parallel \to 0$. The instability then requires the standard condition $4v_0v_* > c_s^2$. However, for non-zero $\nu_\parallel \neq 0$, the condition is modified, becoming $4(v_0v_* + \nu_\parallel/k_y^2) > c_s^2$, and the dissipation leads to the disappearance of the threshold in v_0 resulting in a weak instability occurring for lower values of v_0 and even in the limit of $v_0 \to 0$ (see Fig. 1).

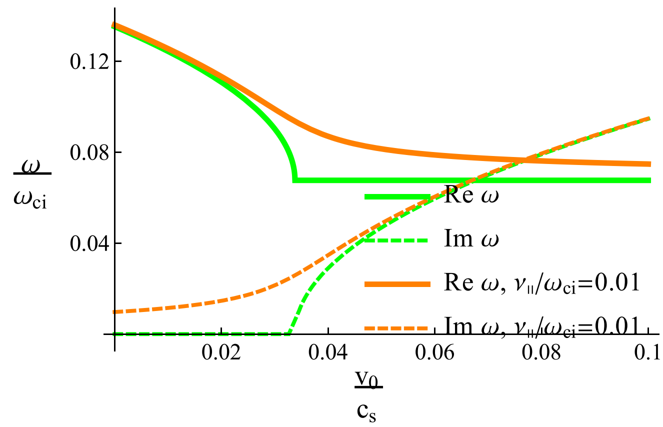

FIG. 1. The collisionless Simon-Hoh instability and its modification due to the sheath resistivity with $\nu_\parallel/\omega_{ci} = 0.01$ and $k_y\rho_s = 1$ when the threshold for the instability disappears.

It is of interest to note the instability in the absence of the electron drift, $v_0 = 0$, which occurs for finite values of the $\nu_\parallel$ parameter. In this case, the Simon-Hoh instability is not present and the anti-drift mode is destabilized by the dissipation, either from collisions or from the sheath resistivity (see Fig. 2). This anti-drift-dissipative mode is the partially magnetized counterpart to the drift-dissipative instabilities existing in fully magnetized plasmas.[7,14] In the limit of small $\nu_\parallel$, the growth rate vanishes and the real part of the mode frequency converges to $k_\perp c_s/\omega_*$ as expected from Eqs. (11) and (17). For larger values of $\nu_\parallel \gg (\omega_*, \omega_0)$ the dispersion Eq. (17) predicts the weakly unstable ion sound mode $\omega^2 \simeq k_y^2 c_s^2$ with the growth rate decaying as $\gamma \sim \nu_\parallel^{-1}$.

Alternatively, in the case of vanishing $v_* = 0$, one finds the instability which is driven by the electron drift and the finite resistivity. The growth rate decays for large $\nu_\parallel$ and vanishes for $\nu_\parallel \to 0$ as expected. Note that the instability exists only for $v_0 > c_s$ while the mode is stable (see Fig. 3) when $v_0 < c_s$.

Similar trends are observed in the general case when both ω_0 and ω^* are finite. These are illustrated in Figs. 4–6 for the case when ω_0 and ω^* are of the same sign, corresponding to the situation of the unstable, collisionless Simon-Hoh instability. For the strongly unstable case, $v_0/c_s > 1$, the instability exists even for $\nu_\parallel = 0$ (this is the collisionless

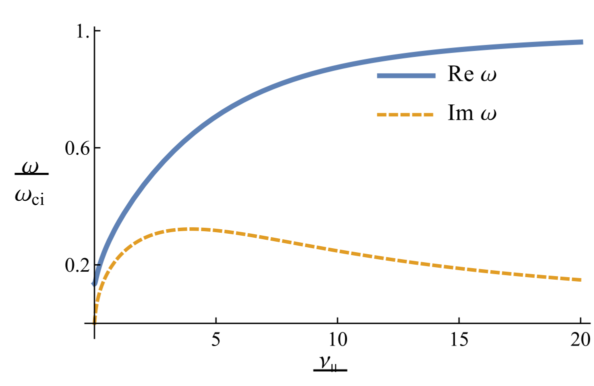

FIG. 2. The real and imaginary parts of the mode frequency as a function of $\nu_\parallel/\omega_{ci}$ for $v_0 = 0$. The maximum value of the growth rate, $\gamma \approx 0.322\omega_{ci}$, occurs at $\nu \sim \omega^*/2$. For $\nu_\parallel \to 0$, $\omega \to k_\perp^2 c_s^2/\omega_*^2$, and for $\nu_\parallel \to \infty$, $\omega = k_y c_s$.

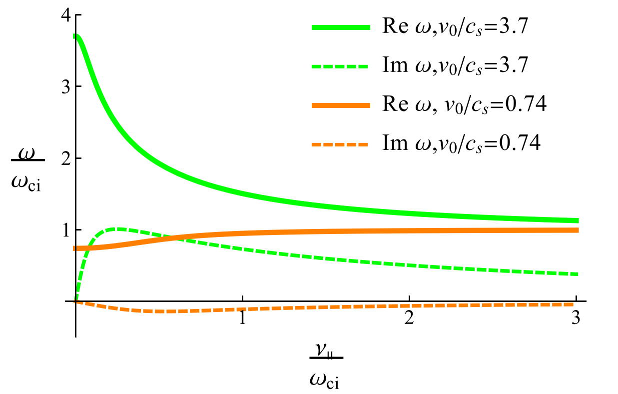

FIG. 3. ω/ω_{ci} as a function of $\nu_{\parallel}/\omega_{ci}$ for $k_y\rho_s = 1$ and vanishing v_*. In the opposite limit of $\nu_{\parallel} \to \infty$, one finds $\omega = \omega_{ci}$ as expected. The maximum value of the growth rate, $\gamma \approx \omega_0/3$, occurs at $\nu/\omega_{ci} \approx c_s/v_0$. In the limit of vanishing $\nu_{\parallel} \to 0$, $\omega = \omega_0$.

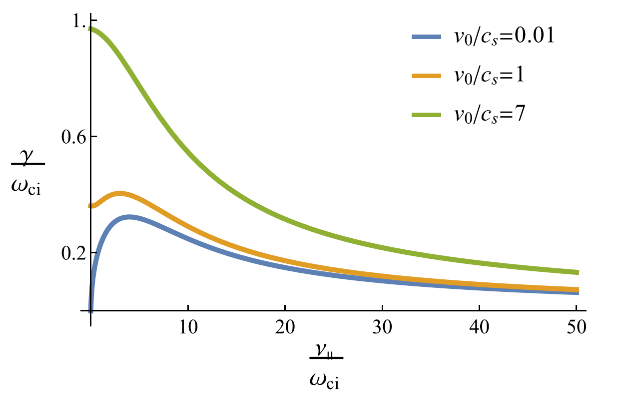

FIG. 4. γ/ω_{ci} as a function of $\nu_{\parallel}/\omega_{ci}$ for $k_y\rho_s = 1$ and $\omega^*/\omega_{ci} \sim 7$. The maximum of the growth rate increases with ω_0.

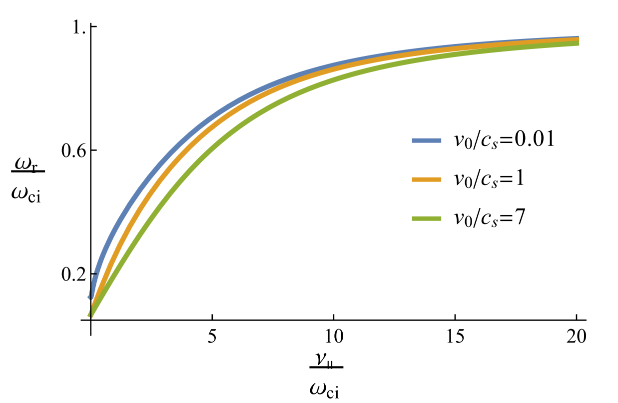

FIG. 5. ω_r/ω_{ci} as a function of $\nu_{\parallel}/\omega_{ci}$ for $k_y\rho_s = 1$.

Simon-Hoh instability). In this case, the sheath dissipation (finite $\nu_{\parallel}$) is stabilizing: the growth rate decreases as $\nu_{\parallel}$ increases. With the decrease in the ratio v_0/c_s, the mode goes from the weakly unstable regime to the stable regime. However as the instability drive from finite v_0/c_s is decreasing, the effect of the finite ω^* becomes more important and the mode is destabilized due to a finite $\nu_{\parallel}$: this is the dissipating instability of the anti-drift mode, similar the case shown in Fig. 2.

The collisionless Simon-Hoh instability occurs for $v_*v_0 > c_s^2/4$. This condition is not met if $v_*v_0 < 0$ or positive

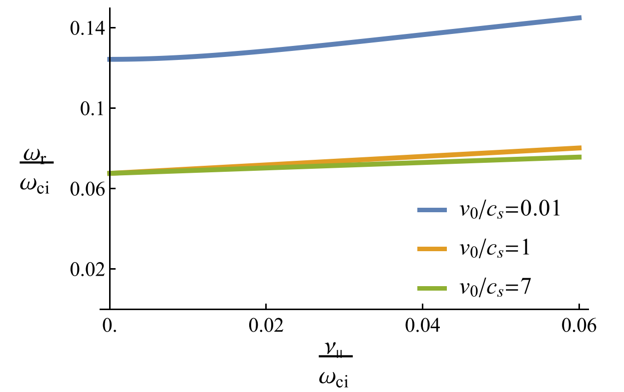

FIG. 6. ω_r/ω_{ci} as a function of (small) $\nu_{\parallel}/\omega_{ci}$ for $k_y\rho_s = 1$.

but too small. Then, the Simon-Hoh instability is absent, but an instability remains due to a finite $\nu_{\parallel}$; this situation is illustrated in Fig. 7. For large $\nu_{\parallel}$, the mode goes into the weakly unstable ions sound. In the limit of small $\nu_{\parallel}$, the growth rate is linearly proportional to $\nu_{\parallel}$ and is given by the approximate expression

$$\gamma \approx \frac{\nu_{\parallel} k_y^2 c_s^2}{2\omega_*^2}\left(-1 \pm \sqrt{1 - \frac{4\omega_0\omega_*}{k_y^2 c_s^2}\left[\frac{2\omega_*(\omega_* - \omega_0)}{k_y^2 c_s^2 - 4\omega_0\omega_*} - 1\right]}\right). \tag{19}$$

For one of the roots (shown in Fig. 7), the maximum growth rate decreases with an increase in the ratio of the absolute value v_0/v_* when it remains negative. On the other hand, the other root has negative growth rate for the ratio $\omega_0/\omega^* > -1$ and increases with the increase in the ratio of the absolute value of ω_0/ω^* when it remains negative. For large values, the behavior of the growth rate of this root is similar to the case $\omega^* = 0$ shown in Fig. 3.

Therefore, various regimes of dissipative instabilities of the gradient drift mode can be realized depending on typical values of plasma parameters which vary between various types of $\mathbf{E} \times \mathbf{B}$ devices (e.g., Hall thruster, Penning discharge, and magnetron). Plasma parameters may also vary for different regions in the same device and/or the devices of the same type (e.g., for different realizations of the Hall thrusters). For the most common types of Hall thrusters, the value of the $\mathbf{E} \times \mathbf{B}$-drift velocity v_0 may range from 10^8 cm/s

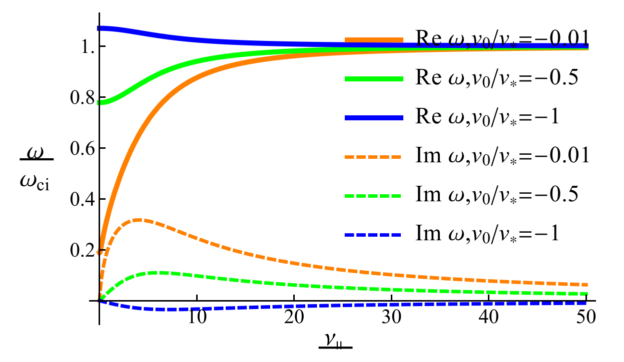

FIG. 7. ω/ω_{ci} as a function of $\nu_{\parallel}/\omega_{ci}$.

TABLE I. Penning discharge and magnetrons parameters in different regimes. The plasma length along the magnetic field in Ref. 11 ($L = 0.4$ cm) is an estimate and we have estimated that $\nu_{en} \sim 10^6$ s^{-1} for the electron-neutrals collision frequency.

Parameter	References 16 and 17	Reference 18	Reference 11
T_e (eV)	10	5.5	3.1
Gas	Xe	Ar	Ar
B_0 (G)	100	100	6000
E_0 (V/cm)	1	20	116
L_n (cm)	–5	–1.2	–0.2
L (cm)	10	170	0.4
v_0/c_s	3.7	55	7.1
v_*/c_s	7.4	13	0.95
ω_{ci} (s^{-1})	7.3×10^3	24×10^3	1440×10^3
$k_z^2 v_{Te}^2/\nu_{en}\omega_{ci}$	2.4×10^4	14	2.4×10^4
$c_s/L\omega_{ci}$	3.7	0.089	0.47

to vanishingly small and negative values near the anode; the plasma density gradient length scale may be in the range $L_n = 0.2$–1 cm or larger and the electron temperature T_e from a few eV to several tens of eV (see Ref. 15 and references therein for a more detailed description of various Hall thruster parameters). The wide range of parameters for some $\mathbf{E} \times \mathbf{B}$ devices are given in Table I. Due to the wide range of plasma parameters in various situations, one can expect that different regimes exist in the dispersion Eqs. (11) and (17) and shown in Fig. 2–7 that may occur in different devices and different operational regimes.

In summary, in this note, we have investigated the role of the sheath plasma boundaries on the gradient drift instabilities in partially magnetized plasmas with $\mathbf{E} \times \mathbf{B}$-fields. It was found that the sheath resistivity results in a dispersion equation analogous to the collisional plasma, but the resistive diffusion frequency parameter $k_z^2 v_{Te}^2/\nu_{en}$ is replaced with the sheath resistivity parameter c_s/L. The sheath dissipation is more important than the standard collisional resistivity when $\lambda/L > \sqrt{m_e/m_i}$. We have shown that the sheath resistivity may result in long wavelength instabilities which are driven either by the $\mathbf{E} \times \mathbf{B}$-drift or the density gradient drifts alone and in situations when the collisionless Simon-Hoh instability is not operative; this is the most important result of our study.

The partially magnetized plasmas, as found in Hall thrusters and magnetrons, exhibit a variety of fluctuations driven by a variety of mechanisms,[3] but the exact nature of the instabilities is still unknown. Large scale structures (such as spokes), often observed in these devices, can be driven directly via linear large scale instability(s), such as discussed in this paper, for example, or via the secondary nonlinear instabilities, or inverse cascade.[3] Our results suggest that the effects of sheath dissipation also need to be included in the modeling of $\mathbf{E} \times \mathbf{B}$ plasma devices.

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