Plasmon resonances, anomalous transparency, and reflectionless absorption in overdense plasmas

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Plasmon resonances, anomalous transparency, and reflectionless absorption in overdense plasmas

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The structure of the surface and standing wave resonances and their coupling in the configuration of the overdense plasma slab with a single diffraction grating are studied, using impedance matching techniques. Analytical criteria and exact expressions are obtained for plasma and diffraction grating parameters which define resonance conditions for absolute transparency in the ideal plasma and reflectionless absorption in a plasma with dissipation. *Published by AIP Publishing.*
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I. INTRODUCTION

Propagation of electromagnetic waves through overdense plasmas has been of great interest for quite some time due to its practical importance for communications, radars, plasma heating, and other applications. The anomalous (up to 100%) energy transmission through an overdense plasma region of negative permittivity ($\varepsilon_p < 0$) can be achieved via superposition of decaying and growing evanescent waves (tunneling).[1–4] In general, the excitation and amplification of evanescent waves can be supported by coupling to various resonances of inhomogeneous plasma structures which may include both standing wave resonances of propagating waves[5] and surface wave (plasmon) resonances of evanescent waves.[3] Surface wave (or polariton/plasmon[6]) eigenmodes exist at the interface of overdense plasma ($\varepsilon_p < 0$) and vacuum, or other plasma (dielectric) regions with positive permittivity ($\varepsilon > 0$).[7] Such eigen-modes propagate along the interface and are localized in the perpendicular direction (across the interface). More general types of plasmon modes, including periodic or individual subwavelength structures (such as aperture/slit), can be found in 2D and 3D geometries.[8–10] The crucial role of surface wave resonances for phenomena of extraordinary transmission and absorption has been established for a number of models and supported by experiments.[2,11,12] Indeed, in some plasma applications, excitation of surface modes has explained the total absorption of electromagnetic waves in overdense plasmas.[13] Furthermore, surface mode resonances play an important role in heating of microwave plasmas, see Ref. 14 and references therein. Plasmon resonances also define unique properties of plasma-based metamaterials.[15]

An electromagnetic wave propagating in vacuum cannot be matched (coupled) with a surface wave localized at the interface between a vacuum region and a region of negative permittivity. Such coupling, however, can be achieved in configurations of double- or multi-layer structures which include an intermediate plasma (dielectric) region with positive permittivity.[16,17] This principle serves as a basis for the so called zero-ε_{eff} structures[3,18,19] with $\varepsilon_{eff} = \varepsilon_a a + \varepsilon_p d = 0$, where a is the width of the intermediate layer with $\varepsilon_a > 0$ and d is the width of the plasma layer with $\varepsilon_p < 0$. In the ideal case without dissipation, such structures have 100% transmissivity for the resonant values of the incidence angle.

In another approach, the plasmon evanescent surface wave mode can be coupled to the propagating vacuum wave $\sim\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$ via scattering on a periodic subwavelength structure $\sim\exp(i\mathbf{q}\cdot\mathbf{r})$. Such scattering will generate sideband evanescent harmonics, $\pm\mathbf{k}+\mathbf{q}$, that can be matched to the surface mode eigen-mode. Periodic spatial modulations can be created via modulation of plasma parameters (e.g., density) with external waves or laser/electron beams.[10,20] Intrinsic, nonlinear wave generation was also suggested as a possible mechanism.[15,21–23] Theoretical models for light tunneling via periodically structured metal films have been presented in Refs. 26–30. Alternatively, an external periodic metal diffraction grating can be used,[24,25] which appears as a more practical solution for plasma applications.[13,31,32]

In general, the full solution of the transmission and/or reflection problem is obtained by solving a system of coupled linear equations for incident, reflected, and transmitted propagating waves, as well as for standing and evanescent modes inside the plasma layer and the vacuum/dielectric layer structures. Formally, it is a straightforward linear problem, but due to a large number of variables, solving it analytically is technically cumbersome, and the solution is therefore often obtained numerically. As a result, the nature of the involved resonance modes and their coupling is not easily understood. Furthermore, conditions and geometric parameters for optimizing the transmissivity and/or absorption cannot be expressed analytically. Based on the idea of critical coupling in optical waveguides,[33] a phenomenological model has been proposed to find a quantitative description of resonance coupling and conditions for transmission and absorption in overdense plasmas.[2,13,32] The plasmon resonator with evanescent eigen-modes has special features, which distinguishes it from standard waveguide type resonators. One important difference is that the energy transport by evanescent modes in tunneling regimes is not described by the group velocity of the wave packets. Instead, one can use the energy flow velocity, which is very different from the group velocity. Typically in tunneling regimes, the energy flow velocity is very small[34] while the group velocity is not even defined. It is worthwhile to note that the slow energy flow velocity for tunneling structures is the basis for "slow" or "frozen" light concepts and devices. Another feature of the plasmon eigen-modes is that such modes are not really stationary modes in finite size configurations. In configurations with plasma layers of finite widths, the modes are quasistationary, slowly decaying in time, "leaking" energy into the outside region ("leaking" modes).[35] These are precisely the modes that can be coupled to the propagating mode in free space and amplify the evanescent modes, leading to anomalous transmission and absorption.

In this paper, we present a simple framework for the description of resonance coupling, mode amplification, and transmission in overdense plasma with a single diffraction grating structure. The method we use is based on the standard wave impedance concept and allows for a compact formulation of wave propagation in a wide class of problems with complex multi-layer structures which include overdense plasma layers. It enables us to clarify the nature of resonances and their coupling, and to obtain relatively simple expressions for optimal conditions for wave transmission and absorption in the overdense plasma layer. It allows for easy consecutive calculations of the transmission and reflection coefficients and can be easily generalized for multiple layers and continuous profiles.

II. BASIC MODEL FOR THE PLASMA LAYER WITH A DIFFRACTION GRATING

We consider the electromagnetic wave incident on the plasma layer preceded by the diffraction grating which is placed at a distance a from the plasma boundary as shown in Fig. 1. The transverse magnetic TM or p-polarization is assumed, so that the electromagnetic field has the components $\mathbf{E} = (E_x, E_y, 0)$ and $\mathbf{B} = (0, 0, B_z)$.

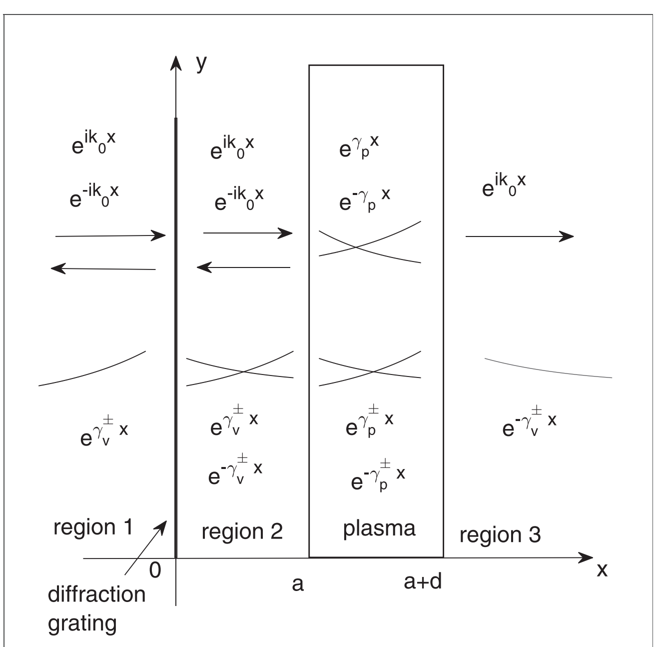

FIG. 1. Geometry of the problem.

The magnetic field is described by the following equation:

$$\varepsilon \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon}\frac{\partial B_z}{\partial x}\right) - k_y^2 B_z + \frac{\omega^2}{c^2}\varepsilon B_z + \frac{\omega^2}{c^2}\varepsilon_g h_g[\mu_0 + \mu_1 \cos(qy)]\delta(x) B_z = 0. \tag{1}$$

The regions 1, 2, and 3 have vacuum permittivity $\varepsilon = 1$. The region "p" is a plasma layer of thickness d and permittivity $\varepsilon_p = 1 - \omega_{pe}^2/\omega^2$, where ω_{pe} is the electron plasma frequency. The last term in Eq. (1) describes the diffraction grating at $x = 0$, where q is the wave vector of the grating, μ_1 is the modulation parameter, and μ_0 is the mean (average) transparency of the grating.[24] This simple model was suggested in Ref. 24, and it is the simplest version of a more general model used in Ref. 25.

Equation (1) is used in the neighborhood of the diffraction grating to obtain the following boundary condition across the interface at $x = 0$:

$$\left[\frac{dB_z}{dz}\right]_{-\delta}^{\delta} = \frac{dB_z}{dz}(\delta) - \frac{dB_z}{dz}(-\delta) = -\frac{\omega^2}{c^2}\varepsilon_g h_g[\mu_0 + \mu_1 \cos(qy)] B_z|_{x=0}. \tag{2}$$

The magnetic field is continuous at the grating, $[B_z]_{-\delta}^{\delta} = 0$, where

$$[B_z] \equiv B_z(\delta) - B_z(-\delta). \tag{3}$$

We will use the notation in (3) to describe a discontinuity at any interface.

The diffraction grating leads to the generation of sideband harmonics with shifted wave vectors $k_y \pm q$. In general, there are multiple harmonics coupled via Eq. (2). Here, similarly to other works,[24,27,29,30] we neglect the higher order side-bands and consider the analytical solution in the form

$$B_z(x, y, t) = \left[B^0(x) e^{ik_y y} + B^+(x) e^{i(k_y+q)y} + B^-(x) e^{i(k_y-q)y}\right] \exp(-i\omega t). \tag{4}$$

Condition (2) generates the following relationships for the principal, B^0, and the sideband, $B^{\pm}$, harmonics:

$$\left[\frac{dB^0}{dx}\right]_{-\delta}^{\delta} = -k_{g0} B^0|_{x=0} - \frac{k_{g1}}{2}(B^+ + B^-)|_{x=0}, \tag{5}$$

$$\left[\frac{dB^{\pm}}{dx}\right]_{-\delta}^{\delta} = -k_{g0} B^{\pm}|_{x=x_g} - \frac{k_{g1}}{2} B^0|_{x=x_g}, \tag{6}$$

where $k_{g0} = \omega^2 \varepsilon_g h_g \mu_0 / c^2$ and $k_{g1} = \omega^2 \varepsilon_g h_g \mu_1 / c^2$.

It follows from (1) that the derivative of the magnetic field is discontinuous at the plasma-vacuum interfaces. The relevant boundary condition is

$$\left[\frac{1}{\varepsilon}\frac{dB_z}{dx}\right] = 0. \tag{7}$$

The magnetic field is continuous at all interfaces, $[B_z] = 0$.

The boundary condition (7) is most conveniently written in terms of the impedances, Appendix A. The matching conditions (5,6) can be written in terms of the impedances as well by noting that

$$\frac{1}{B_0}\left[\frac{dB^0}{dx}\right] = -k_{g0} - \frac{k_{g1}}{2}\left(\frac{B^+ + B^-}{B^0}\right), \tag{8}$$

$$\frac{1}{B_0}\left[\frac{dB^0}{dx}\right] = -k_{g0} - \frac{k_{g1}}{2}\frac{B^0}{B^\pm}, \tag{9}$$

where the right-hand sides in these equations are evaluated at $x = 0$.

In this paper, we will consider only the case of normal incidence when $k_y = 0$ in Eq. (4). Then, the $(\pm)$ side-bands become symmetric, $B^+ = B^- \equiv B^\pm$, and one of the matching conditions at the diffraction grating can be written as

$$\left(\frac{1}{B_0}\left[\frac{dB^0}{dx}\right] + k_{g0}\right)\left(\frac{1}{B^+}\left[\frac{dB^+}{dx}\right] + k_{g0}\right) = \frac{k_{g1}^2}{2}. \tag{10}$$

Using the definition of the impedance and the corresponding techniques described in Appendixes A and B, we find

$$\frac{1}{B_0}\left[\frac{dB^0}{dx}\right] = \frac{i\omega\varepsilon}{c}\left[Z_2^0(0) - Z_1^0(0)\right], \tag{11}$$

$$\frac{1}{B^+}\left[\frac{dB^+}{dx}\right] = \frac{i\omega\varepsilon}{c}\left[Z_2^+(0) - Z_1^0(0)\right]. \tag{12}$$

The matching condition at the diffraction grating then becomes

$$\left(Z_2^0(0) - Z_1^0(0) - \frac{ik_{g0}c}{\omega}\right)\left(Z_2^+(0) - Z_1^+(0) - \frac{ik_{g0}c}{\omega}\right) = -\frac{k_{g1}^2c^2}{2\omega^2}. \tag{13}$$

This equation shows the coupling of the principal component and the side-bands at the diffraction grating. As in Ref. 24, to simplify the presentation, we set $h_g = k_g = 0$ in Eq. (13). Note that a finite h_g does not affect the surface wave resonances but introduces additional resonances of Fabry-Perot type which were considered in Ref. 3.

A. The structure of the electromagnetic field and transmission coefficient

In this section, we describe the structure of the electromagnetic field in each region, formulate the matching conditions, and outline the calculation of the transmission coefficient. We will use the lower indices 1, 2, p, and 3 to define the relevant quantities in the corresponding region; we will use the upper indices $(0, \pm)$ to define the principal component and the sidebands.

1. Vacuum region 1

In the outmost left vacuum region, we have the incident and reflected waves in the main harmonic

$$B_1^0 = \exp(ik_0x) + \Gamma_1^0\exp(-ik_0x), \tag{14}$$

where k_0 is the wave vector of the wave propagating in vacuum, $k_0^2 = \omega^2/c^2$. Here, the amplitude of the incident was normalized to unity. Then, the running value of the impedance in this region is

$$Z_1^0(x) = \frac{\exp(ik_0x) - \Gamma_1^0\exp(ik_0x)}{\exp(ik_0x) + \Gamma_1^0\exp(ik_0x)}. \tag{15}$$

The amplitude of the reflected wave Γ_1^0 can be related to the value of the impedance for the main harmonic in region 1 at the diffraction grating $Z_1^0(0) \equiv Z_1^0(0-\delta), \delta \to 0$

$$\Gamma_1^0 = \frac{1 - Z_1^0(0)}{1 + Z_1^0(0)}. \tag{16}$$

The characteristic impedance of vacuum for the principal harmonic is $\kappa_v^0 \equiv k_0c/\omega = 1$.

The side-bands are evanescent in vacuum and are localized near the considered structure. Thus, in region 1, the side-bands are decaying as $x \to -\infty$ and have the form

$$B_1^+ = A_1^+\exp(\gamma_v^+x), \tag{17}$$

where $\gamma_v^{+2} = q^2 - k_0^2 > 0, \gamma_v^+ > 0$. The impedance of the sidebands in this region is constant

$$Z_1^+(x) \equiv Z_1^+ = -\frac{ic\gamma_v^+}{\omega} = -\kappa_v^+ = const, \tag{18}$$

where κ^+ is the characteristic impedance of vacuum for the evanescent side-band harmonics.

B. Vacuum region 2

In the vacuum region 2, between the diffraction grating and plasma layer, the main and the side-bands harmonics have the following incident and the reflected components:

$$B_2^0 = A_2^0\left[\exp(ik_0x) + \Gamma_2^0\exp(-ik_0x)\right], \tag{19}$$

$$B_2^+ = A_2^+\left[\exp(-\gamma_v^+x) + \Gamma_2^+\exp(\gamma_v^+x)\right]. \tag{20}$$

The corresponding expressions for the local values of the impedances are

$$Z_2^0(x) = \frac{\exp(ik_0x) - \Gamma_2^0\exp(ik_0x)}{\exp(ik_0x) + \Gamma_2^0\exp(ik_0x)}, \tag{21}$$

$$Z_2^+(x) = \kappa_v^+\frac{\exp(-\gamma_v^+x) - \Gamma_2^+\exp(\gamma_v^+x)}{\exp(-\gamma_v^+x) + \Gamma_2^+\exp(\gamma_v^+x)}. \tag{22}$$

Using (A5) and (A6), one can extend the impedances at the left and right boundaries in this region via the transformations

$$Z_2^0(0) = \frac{Z_2^0(a) - i\tan(k_0 a)}{1 - iZ_2^0(a)\tan(k_0 a)}, \tag{23}$$

$$Z_2^+(0) = \kappa_v^+ \frac{Z_2^+(a) + \kappa_v^+ \tanh\left(\gamma_v^+ a\right)}{\kappa_v^+ + Z_2^+(a)\tanh\left(\gamma_v^+ a\right)}. \tag{24}$$

At the right boundary of region 2, across the vacuum-plasma interface, the impedances are continuous. Therefore, one can relate the impedances $Z_2^0(a)$, $Z_2^+(a)$ in the vacuum region 2 to the impedances $Z_p^+(a), Z_p^0(a)$ in the plasma region as follows:

$$Z_2^+(a) = Z_p^+(0), \tag{25}$$

$$Z_2^0(a) = Z_p^0(0). \tag{26}$$

C. Plasma layer region

In the plasma layer, principal components and both side-bands are evanescent

$$B_p^0 = A_p^0\left[\exp(-\gamma_p x) + \Gamma_p^0 \exp(\gamma_p x)\right], \tag{27}$$

$$B_p^+ = A_p^+\left[\exp(-\gamma_p^+ x) + \Gamma_p^+ \exp(\gamma_p^+ x)\right], \tag{28}$$

with $(\gamma_p^0)^2 = -k_0^2\varepsilon_p > 0,\ (\gamma_p^+)^2 = q^2 - k_0^2\varepsilon_p > 0,\ \varepsilon_p < 0.$ Note that that the coordinate system inside the plasma layer can be redefined so that the left side boundary of the plasma layer is at $x = 0$.

Then the local values of the impedances are

$$Z_p^+(x) = \kappa_p^+ \frac{\exp\left(-\gamma_p^+ x\right) - \Gamma_p^+ \exp\left(\gamma_p^+ x\right)}{\exp\left(-\gamma_p^+ x\right) + \Gamma_p^+ \exp\left(\gamma_p^+ x\right)}, \tag{29}$$

$$Z_p^0(x) = \kappa_p^0 \frac{\exp\left(-\gamma_p^0 x\right) - \Gamma_p^0 \exp\left(\gamma_p^0 x\right)}{\exp\left(-\gamma_p^0 x\right) + \Gamma_p^0 \exp\left(\gamma_p^0 x\right)}. \tag{30}$$

The impedances at the right and left boundaries of the plasma region are related by the transformations

$$Z_p^+(0) = \kappa_p^+ \frac{Z_p^+(d) + \kappa_p^+ \tanh\left(\gamma_p^+ d\right)}{\kappa_p^+ + Z_p^+(d)\tanh\left(\gamma_p^+ d\right)}, \tag{31}$$

$$Z_p^0(0) = \kappa_p^0 \frac{Z_p^0(d) + \kappa_p^0 \tanh\left(\gamma_p^0 d\right)}{\kappa_p^0 + Z_p^0(d)\tanh\left(\gamma_p^0 d\right)}, \tag{32}$$

where $\kappa_p^\pm = i\gamma_p^\pm c/\omega\varepsilon_p$, $\kappa_p^0 = i\gamma_p^0 c/\omega\varepsilon_p$ are the characteristic wave impedances in this region, and d is the width of the plasma layer.

At the right boundary of the plasma layer, the impedances are continuously matched to the vacuum region 3

$$Z_p^0(d) = Z_3^0, \tag{33}$$

$$Z_p^+(d) = Z_3^+. \tag{34}$$

We have dropped here the arguments for Z_3^0 and Z_3^+ since the impedances in the region 3 are constant (see below).

D. Vacuum region 3

In this region, we have only the transmitted wave propagating to the right and the side-bands that are decaying for $x \to +\infty$

$$B_3^0 = A_3^0 \exp(ik_0 x), \tag{35}$$

$$B_3^+ = A_3^+ \exp(-\gamma_v^+ x). \tag{36}$$

Respectively, the impedances in this region are constant and given by the relations

$$Z_3^0 = \frac{k_0 c}{\omega} = 1, \tag{37}$$

$$Z_3^+ = \frac{i\gamma_v^+ c}{\omega} = \kappa_v^+. \tag{38}$$

The transmission coefficient is given by the amplitude of the transmitted wave: $T = A_3$ (see Appendix C).

III. SURFACE WAVE RESONANCE AND REFLECTIONLESS TRANSMISSION

One of the most fascinating plasmonics phenomena is absolute transparency of an overdense plasma layer. Absolute transparency has been demonstrated in complex multi-layer structures where it can be supported by surface wave resonances as well as by standing wave (Fabry-Perot type) resonances.[3] Some resonances caused by standing waves can be supported by additional modes which exist in warm plasmas.[4] It was proposed in Refs. 2 and 24 that two diffraction gratings on both sides of a plasma layer are required to realize absolute transparency. This conclusion was based on the critical coupling concept.[2,24] Here, we show that full transparency can also be achieved with a single grating.[36] Using the impedance matching formulation, we show the role of the plasmon resonance and of the coupling to the standing wave resonance in region 1.

Full (100%) transmission is achieved when $|T| = 1$, where T is the transmission coefficient. This occurs when the reflected wave is absent in region 1: $\Gamma_1^0 = 0$ and therefore $Z_1^0(0) = 1$. The impedances of the decaying sidebands in the vacuum region 1 are $Z_1^+ = Z_1^- = -\kappa_v^+$. Thus, the matching condition at the diffraction grating (13) has the form

$$1 = Z_2^0(0) - \frac{k_{g1}^2 c^2}{2\omega^2} \frac{1}{Z_2^+(0) + \kappa_v^+}. \tag{39}$$

Note that κ_v^+ is imaginary, and so is $Z_2^+(0)$ for an ideal plasma; the impedance $Z_2^0(0)$ is in general complex. Then, for Eq. (39) to hold, the last term has to cancel out the imaginary part of $Z_2^0(0)$. For weak coupling, and small values of the parameter $k_{g1}^2 c^2/\omega^2$, the resonance occurs near the pole of the last term in (39)

$$Z_2^+(0) + \kappa_v^+ = 0. \tag{40}$$

From this equation, and using Eqs. (24), (25), and (29), one obtains

$$\left[1+L_v^+\right]\left[2\kappa_v^+k_p^+ + k_v^{+2}L_p^+ + k_p^{+2}L_p^+\right] = 0, \tag{41}$$

and the resonant condition is given by

$$2\frac{\kappa_p^+}{\kappa_v^+} + \left(\frac{\kappa_p^+}{\kappa_v^+}\right)^2 L_p^+ + L_p^+ = 0, \tag{42}$$

where $L_p^+ = \tanh(\gamma_p^+ d)$, $L_v^+ \equiv \tanh(\gamma_v^+ a)$. It is easy to see that this is the exact dispersion equation for the surface wave of a plasma layer with a finite thickness d. The roots of this quadratic equation correspond to the symmetric and antisymmetric bonding of the surface waves localized on the opposite boundaries of the layer.

For a large thickness, we have $L_p^+ \to 1$, and (42) yields

$$\kappa_v^+ + k_p^+ = 0, \tag{43}$$

which is the dispersion equation for a surface mode at the interface of the vacuum and a semi-infinite layer with $\varepsilon_p < 0$. From (43), one finds the resonant value of the diffraction grating wave vector

$$q_0^2 = k_0^2 \frac{\varepsilon_p}{\varepsilon_p + 1}. \tag{44}$$

In addition to the surface wave resonance described by Eq. (43), a second condition has to be satisfied for Eq. (39) to hold, namely,

$$Re\left[Z_2^0(0)\right] = 1. \tag{45}$$

This condition reduces to the following equation:

$$\left[1+\beta^2\right]\left[L_p^2 + \beta^2 L_v^2 L_p^2 - 2\beta L_v L_p\right] = 0, \tag{46}$$

or equivalently

$$(1-\beta L_v L_p)^2 = 1 - L_p^2, \tag{47}$$

with $L_v \equiv \tan(ka)$, $L_p = \tanh(\gamma_p d)$, and $\kappa_p = -i\beta$, where $\beta = 1/\sqrt{-\varepsilon_p}$ is a real number. For given values of plasma permittivity ε_p and L_p, Eq. (47) has two roots L_v which define the resonance condition for the distance between the plasma layer and the diffraction grating. For a thick plasma layer, we have $L_p \to 1$, $L_v \simeq 1/\beta$. In this limit, the resonance condition is given by

$$\tan(ka) = \sqrt{-\varepsilon_p}. \tag{48}$$

This condition indicates that the resonances are close to the standing wave resonances in the vacuum region 2: $ka = n\pi$, where $n = 1,2,\ldots$. In the general case

$$L_v = \frac{1}{\beta L_p} \pm \sqrt{\frac{1}{\beta^2 L_p^2} - \frac{1}{\beta^2}}, \tag{49}$$

and the resonance value of the width of the vacuum region 2 is given by

$$a = k_0^{-1}\tan^{-1}(L_v). \tag{50}$$

Note that condition (47) does not depend on the diffraction grating parameters.

For finite values of the parameter $k_{g1}^2 c^2/\omega^2$, the exact conditions for resonant transmission are obtained from (39) in the form

$$iIm\left(Z_2^0(0)\right) = \frac{k_{g1}^2 c^2}{2\omega^2}\frac{1}{Z_2^+(0) + \kappa_v^+}. \tag{51}$$

Conditions (45) and (51) have to be satisfied simultaneously for absolute (100%) transmission with $|T| = 1$.

As a numerical example, we consider the case of plasma parameters which are of interest for the communication blackout problem:[37,38] $\omega = 2\pi \times 10^9$ rad/s, plasma density $n = 4.5 \times 10^{17}$ m^{-3}, and the plasma layer width $d = 0.02$ m. In our calculations, we set $\omega_p = 2\pi \times 6 \times 10^9$ rad/s, which yields $\varepsilon_p = -35$. For these parameters, one finds from (49) and (50) the resonance value for the distance between the diffraction grating and the plasma layer $a = 0.068153$ m. According to Eq. (44), the resonance value of the diffraction grating wave vector q when the surface wave resonance occurs is $q_0 = 21.271$ m^{-1}. The solution of the exact Eq. (51) gives two values for q, as seen in Fig. 2. Those values are slightly different from q_0 due to coupling of plasmon modes in the plasma layer of finite thickness d [see Eq. (42)]. Figure 2 shows the reflection and transmission coefficients, each as a function of the diffraction grating wave vector for $k_{g1}^2 = 4 \times 10^3$ m^{-2}. The decrease in the parameter k_{g1} results in the narrower resonance curves as shown in Fig. 3 for $k_{g1}^2 = 4 \times 10^2$ m^{-2}.

IV. REFLECTIONLESS ABSORPTION IN A DISSIPATIVE PLASMA LAYER

The plasmon resonances that lead to the amplification of evanescent modes and absolute transmission through an overdense plasma layer are quite narrow in frequency (and the diffraction grating wave vector values). As such they are sensitive to dissipation and can be easily destroyed by dissipation in plasmas. At the same time, the same plasmon resonance is responsible for another interesting phenomenon that occurs due to interaction of an electromagnetic wave with an overdense plasma layer with dissipation, namely, the reflectionless absorption of the electromagnetic wave.

When electron collisions are included, the dielectric plasma permittivity becomes complex

$$\varepsilon_p = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu)}. \tag{52}$$

The impedance matching formalism presented in Sec. II still remains valid and the same equations can be used to find the reflection and transmission coefficients as summarized in Appendixes B and C. In the dissipative case, $|\Gamma_1^0|^2 + |T|^2 < 1$ since a fraction of the energy is absorbed inside the plasma. One can introduce the parameter $A = 1 - |\Gamma_1^0|^2 - |T|^2$ to characterize the absorption. Normally, for overdense plasma with relatively weak dissipation, the absorption

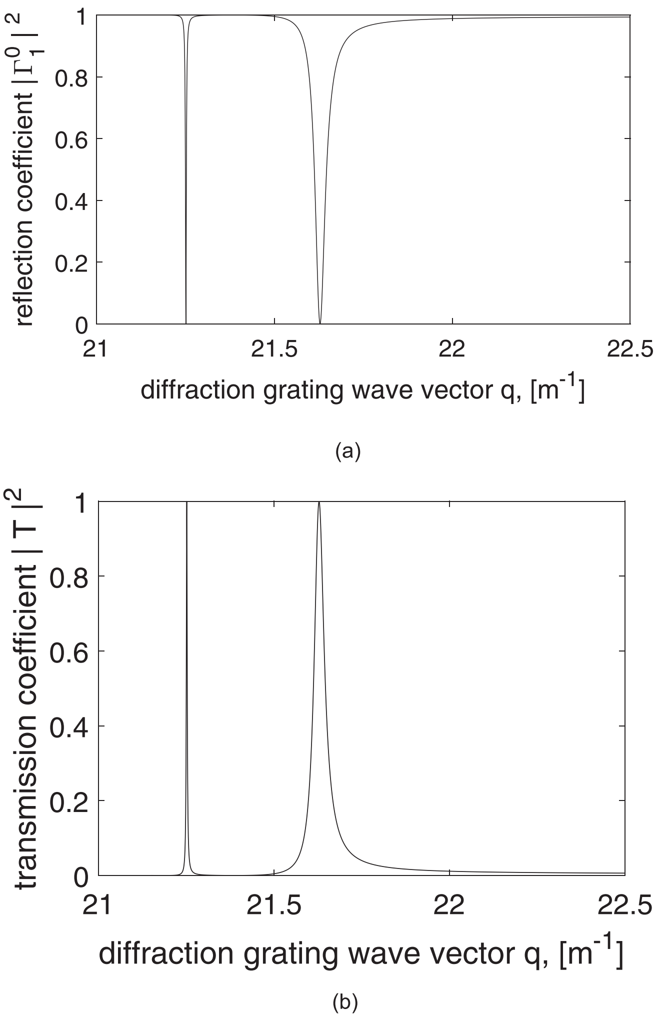

FIG. 2. (a) Reflection, $|\Gamma_1^0|^2$, and (b) transmission, $|T|^2$, coefficients, versus diffraction grating wave vector, q, for $\epsilon_p = -35$, $d = 0.02\,\mathrm{m}$, $a = 0.06815\,\mathrm{m}$, and $k_{g1}^2 = 4000\,\mathrm{m}^{-2}$.

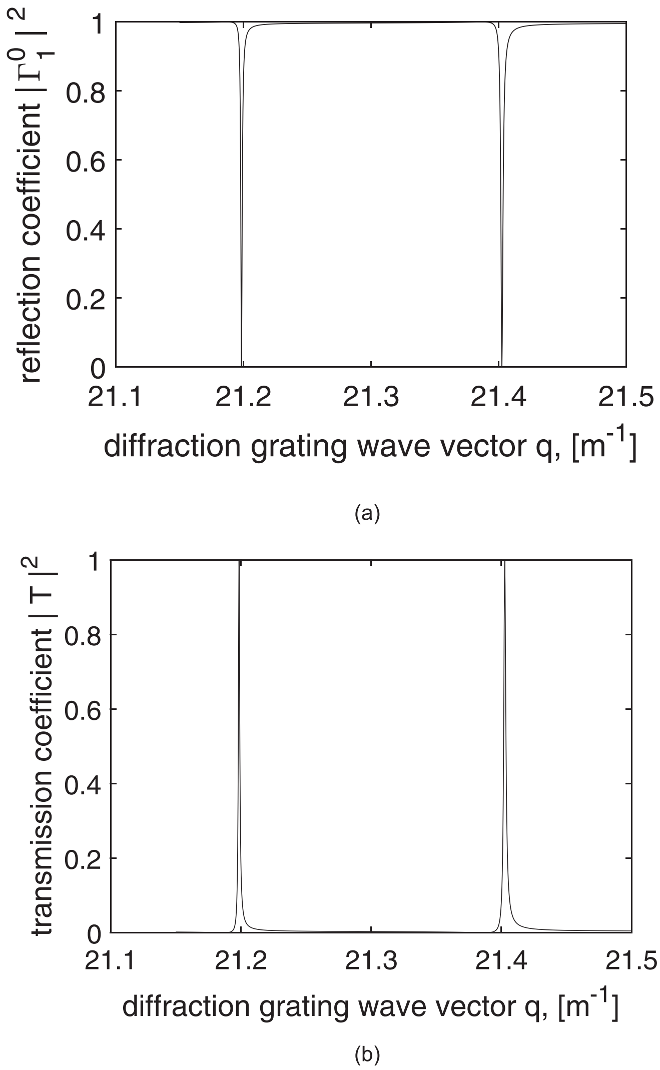

FIG. 3. (a) Reflection, $|\Gamma_1^0|^2$, and (b) transmission, $|T|^2$, coefficients, versus diffraction grating wave vector, q, for $\epsilon_p = -35$, $d = 0.02\,\mathrm{m}$, $a = 0.06815\,\mathrm{m}$, and $k_{g1}^2 = 400\,\mathrm{m}^{-2}$.

is low and most of the energy of the incident wave will be reflected, $|\Gamma_1^0|^2 \simeq 1$. Near the plasmon resonance, however, the reflection can be reduced significantly.[8,13,25,32] Full absorption was shown experimentally in Ref. 13.

The conditions for full absorption can be investigated analytically from results of Sec. II and Appendixes B and C as follows. Consider a sufficiently thick plasma layer so that transmission can be neglected, $T = 0$. In this limit, the entrance impedances of the vacuum region 2 become

$$Z_2^0(0) = \frac{\kappa_p^0 - iL_v}{1 - i\kappa_p^0 L_v}, \tag{53}$$

$$Z_2^+(0) = \kappa_v^+ \frac{\kappa_p^+ + \kappa_v^+ L_v^+}{\kappa_v^+ + \kappa_p^+ L_v^+}. \tag{54}$$

For $T = 0$, the condition for reflectionless absorption is given by Eq. (39). The dominant part of $Z_2^0(0)$ is imaginary, and this part is compensated by the imaginary part of the last term in (39). It is worthwhile to note that in the dissipative case, κ_p^0 acquires a small real part (related to the dissipation in plasma), $\kappa_p^0 = -i\beta + \alpha$, where α and β are real, and $\alpha \ll \beta$. The small perturbation of $Z_2^0(0)$ due to α is crucial as it generates the real part of $Z_2^0(0)$ which is required for the absence of reflection: $Re[Z_2^0(0)] \simeq 1$; otherwise, $Z_2^0(0)$ is imaginary and the reflection is large. The condition $Re[Z_2^0(0)] \simeq 1$ determines the resonant value of the distance from the diffraction grating to the plasma

$$L_v \simeq \frac{1}{\beta} \pm \frac{1}{\beta}\sqrt{\alpha}. \tag{55}$$

Note that the condition $Re[Z_2^0(0)] \simeq 1$ is further modified by the contribution from the last term in (39).

The exact condition for reflectionless absorption that follows from (39) has the form

$$1 = \frac{\kappa_p^0 - iL_v}{1 - i\kappa_p^0 L_v} - K\frac{\left(\kappa_v^+ + \kappa_p L_v^+\right)}{\kappa_v^+(\kappa_v^+ + \kappa_p)(1 + L_v^+)}, \tag{56}$$

where $K = k_{g1}^2 c^2/2\omega^2$. The $(\kappa_v^+ + \kappa_p)$ term in the denominator of this expression emphasizes the role of the plasmon resonance [compare with Eq. (43)]. Equation (56) fully determines the exact conditions for the values of a (width of the vacuum region) and q (diffraction grating wave vector) for reflectionless absorption, when the energy of the incident electromagnetic wave is fully absorbed in the plasma, $T=0, \Gamma_1^0 = 0$. Plasma dissipation results in a slight modification of the resonant value of the diffraction wave vector compared to the values from (44) (see Fig. 3). An approximate value of q is found from Eq. (55). The dependence of the reflection and absorption coefficients as functions of the wave vector q is illustrated in Fig. 4. The decrease in the parameter k_{g1} results in the narrower region of absorption, as can be seen by comparing Figs. 4(a) and 4(b).

V. SUMMARY

The phenomenon of anomalously large transmission of the electromagnetic waves through the media with negative dielectric permittivity such as dense plasmas (or metal films in the visible range) has attracted great interest in the last decade. It is generally understood that such transmission occurs as a result of resonant excitation of plasmon type surface modes. Resonant conditions can be created either with multi-layer zero-ε_{eff} structures or with subwavelength structures. These subwavelength structures can be created by either a single defect such as a subwavelength aperture or with periodic arrays of holes or slits (diffraction grating). Resonant excitation of plasmon modes is a crucial element of many phenomena, such as reflectionless absorption, total backward reflection,[39] negative refraction,[1,2] and other phenomena in plasma based metamaterials.[15,40]

In this paper, we have investigated resonant conditions and coupling of resonances in a system with an overdense plasma layer and a single diffraction grating. We have shown that extraordinary transmission also exists in this configuration, and not only in the two gratings scheme (on both sides of the plasma layer) proposed in Ref. 24. We have presented a simple impedance formulation that allows us to demonstrate clearly the coupling of the plasmon (evanescent) mode inside the plasma layer and standing wave resonances in the vacuum regions. We have demonstrated that those resonances lead to extraordinary transmission and reflectionless absorption. We have determined analytically the resonance value of the width of the vacuum region for full transmission [see Eq. (49)] and for reflectionless absorption [see Eq. (55)]. The exact expressions for the resonant values of the diffraction grating wave vector for full transmission are given by (51) and for reflectionless absorption by (56).

The impedance model presented in this paper is an exact alternative to the phenomenological critical coupling model proposed by others.[13] The impedance model can easily be generalized to configurations with multiple layers and is convenient for numerical calculation of the reflection and transmission coefficients for continuous inhomogeneous plasma profiles.[41–43] As such it is envisaged that the impedance model may become an effective tool for the investigation and design of plasma based metamaterial structures and configurations,[40,44] including multi-layer and 2D structures for broadband applications.[31] Plasma production and heating may be improved in plasma discharges designed with the explicit account of plasmon resonances.

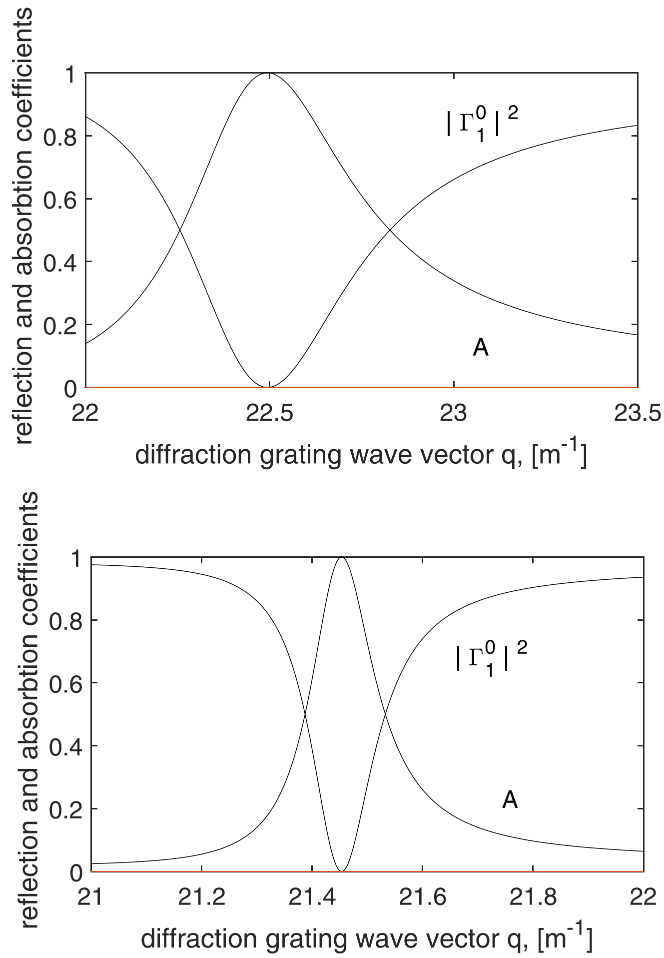

FIG. 4. Reflection and absorption coefficients, $|\Gamma_1^0|^2$ and A, versus diffraction grating wave vector, q, $\epsilon_p = -34.77 + 2.862i, d = 0.06\,\mathrm{m}$, $a = 0.07562\,\mathrm{m}$, and $\nu = 0.08\,\omega$, for (a) $k_{g1}^2 = 4\times 10^3\,\mathrm{m}^{-2}$ and (b) $k_{g1}^{-2} = 400\,\mathrm{m}^{-2}$.

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APPENDIX A: IMPEDANCE

To clarify the notation, we now give the definition of the impedance and the formulas for the impedance transformations in a layer of finite length with permittivity ε. The impedance, Z, is defined by

$$Z \equiv \frac{E_y}{B_z} = -\frac{ic}{\omega\varepsilon}\frac{1}{B_z}\frac{\partial B_z}{\partial x}. \tag{A1}$$

In general, there is an incident wave $\sim\exp(-i\omega t + ikx)$ and a reflected wave $\sim\exp(-i\omega t - ikx)$, so that the magnetic field of the TM wave has the form

$$B_z = B\exp(-i\omega t)\left[\exp(ikx) + \Gamma\exp(-ikx)\right], \tag{A2}$$

where $\omega > 0$ and $k > 0$ are set so that the incident wave propagates to the right, and the reflected wave propagates to the left, with Γ being the reflection coefficient.

Thus, the local impedance is a function of the position x, and is given by

$$Z(x) = \kappa \frac{\exp(ikx) - \Gamma \exp(-ikx)}{\exp(ikx) + \Gamma \exp(-ikx)}, \tag{A3}$$

where κ is the characteristic wave impedance in the layer $\kappa \equiv kc/(\omega\varepsilon)$.

The reflection coefficient is determined by the mismatch between the characteristic impedance κ and the value of the entrance impedance $Z(0)$

$$\Gamma = \frac{\kappa - Z(0)}{\kappa + Z(0)}. \tag{A4}$$

Equations (A3) and (A4) yield the standard transformation of the impedance for a finite interval of length l

$$Z(l) = \kappa \frac{Z(0) + \kappa \tanh(ikl)}{\kappa + Z(0)\tanh(ikl)}. \tag{A5}$$

The inverse transformation is then

$$Z(0) = \kappa \frac{Z(l) - \kappa \tanh(ikl)}{\kappa - Z(l)\tanh(ikl)}. \tag{A6}$$

These expressions can also be used for evanescent waves by setting $k = i\gamma$, with real-valued $\gamma \neq 0$, so that

$$B_z = B\left[\exp(-\gamma x) + \Gamma \exp(\gamma x)\right]. \tag{A7}$$

Then the characteristic impedance $\kappa = i\gamma c/(\omega\varepsilon)$ is imaginary, and the transformation relations (A5) and (A6) become

$$Z(l) = \kappa \frac{Z(0) - \kappa \tanh(\gamma l)}{\kappa - Z(0)\tanh(\gamma l)}, \tag{A8}$$

$$Z(0) = \kappa \frac{Z(l) + \kappa \tanh(\gamma l)}{\kappa + Z(l)\tanh(\gamma l)}. \tag{A9}$$

APPENDIX B: SUMMARY OF THE IMPEDANCE MATCHING EXPRESSIONS

Here we summarize the nomenclature of the impedance matching relations and definitions that allow for sequential calculation of the reflection coefficient which can also be generalized for multi-layer structures. In our notation, the subscripts "1," "2," and "3" refer to the vacuum regions, and the subscript "p" refers to the plasma layer. The superscript "0" refers the principal harmonic, and the superscript "+" refers to the side-band, and the parameters κ_v^0 (κ_p^0) and κ_v^+ (κ_p^+) are the characteristic impedances for the vacuum (plasma) regions for the principal and side-band harmonics.

The impedance matching starts from the outmost right region 3 (vacuum) and proceeds to the left

$$Z_3^0(0) = \kappa_v = 1, \tag{B1}$$

$$Z_3^+(0) = \kappa_v^+ = -i\sqrt{q^2/k_0^2 - 1}, \tag{B2}$$

$$Z_p^0(d) = Z_3^0(0), \tag{B3}$$

$$Z_p^+(d) = Z_3^+(0). \tag{B4}$$

$$\kappa_p^0 = -\frac{i}{\sqrt{-\varepsilon_p}}, \tag{B5}$$

$$\kappa_p^+ = \frac{i}{\varepsilon_p}\sqrt{q^2/k_0^2 - \varepsilon_p}, \tag{B6}$$

$$Z_p^0(0) = \kappa_p^0 \frac{Z_p^0(d) + \kappa_p^0 \tanh(\gamma_p d)}{\kappa_p^0 + Z_p^0(d)\tanh(\gamma_p d)}, \tag{B7}$$

$$Z_p^+(0) = \kappa_p^+ \frac{Z_p^+(d) + \kappa_p^+ \tanh\left(\gamma_p^+ d\right)}{\kappa_p^+ + Z_p^+(d)\tanh(\gamma_p^+ d)}, \tag{B8}$$

$$Z_2^0(a) = Z_p^0(0), \tag{B9}$$

$$Z_2^+(a) = Z_p^+(0), \tag{B10}$$

$$Z_2^0(0) = \frac{Z_2^0(a) - i\tan(k_0 a)}{1 - iZ_2^0(a)\tan(k_0 a)}, \tag{B11}$$

$$Z_2^+(0) = \kappa_v^+ \frac{Z_2^+(a) + \kappa_v^+ \tanh(\gamma_v^+ a)}{\kappa_v^+ + Z_2^+(a)\tanh(\gamma_v^+ a)}, \tag{B12}$$

$$Z_1^+(0) = -\kappa_v^+, \tag{B13}$$

$$Z_1^0(0) = Z_2^0(0) - \frac{k_{g1}^2 c^2}{2\omega^2} \frac{1}{Z_1^+(0) - Z_2^+(0)}. \tag{B14}$$

The reflection coefficient is finally calculated from $Z_1^0(0)$

$$\Gamma_1^0 = \frac{1 - Z_1^0(0)}{1 - Z_1^0(0)}. \tag{B15}$$

For multi-layer structures, this process can be continued to the left. This method also offers a convenient way to calculate the total reflection coefficient in the case of a continuous profile by breaking it down into a number of sub-layers and repeating the cycle of calculations from the right to the left.

APPENDIX C: CALCULATION OF THE TRANSMISSION AND ABSORPTION COEFFICIENTS

The sequence of calculations can be performed from left to right to obtain the transmission coefficient. This sequence is summarized here

$$1 + \Gamma_1^0 = A_2^0(1 + \Gamma_2^0), \tag{C1}$$

$$A_2^0\left[\exp(ik_0 a) + \Gamma_2^0 \exp(-ik_0 a)\right] = A_p^0\left(1 + \Gamma_p^0\right), \tag{C2}$$

$$A_p^0\left[\exp(-\gamma_p d) + \Gamma_p^0 \exp(\gamma_p d)\right] = A_3. \tag{C3}$$

Here, according to (A4)

$$\Gamma_p^0 = \frac{\kappa_p^0 - Z_p^0(0)}{\kappa_p^0 + Z_p^0(0)}, \tag{C4}$$

$$\Gamma_2^0 = \frac{1 - Z_2^0(0)}{1 + Z_2^0(0)}, \tag{C5}$$

where the values of $Z_p^0(0)$ and $Z_2^0(0)$ are calculated with (B7) and (B11). Thus,

$$A_2^0 = \frac{1 + \Gamma_1^0}{1 + \Gamma_2^0}, \tag{C6}$$

$$A_p^0 = \frac{A_2^0}{\left(1 + \Gamma_p^0\right)} \left[\exp\left(ik_0 a\right) + \Gamma_2^0 \exp\left(-ik_0 a\right)\right]. \tag{C7}$$

The transmission coefficient is $T = A_3$, and therefore is given by

$$T = A_p^0 \left[\exp\left(-\gamma_p d\right) + \Gamma_p^0 \exp\left(\gamma_p d\right)\right]. \tag{C8}$$

For ideal plasmas, $|\Gamma_1^0|^2 + |T|^2 = 1$, and it is sufficient to calculate only one coefficient. In plasmas with dissipation, both coefficients Γ_1^0 and T have to be determined independently. The energy absorption can be characterized by the parameter A, defined as $A = 1 - |\Gamma_1^0|^2 - |T|^2$.

[1] A. B. Shvartsburg, "Tunneling of electromagnetic waves: Paradoxes and prospects," Phys.-Usp. **50**(1), 37–51 (2007).
[2] K. Y. Bliokh, Y. P. Bliokh, V. Freilikher, S. Savel'ev, and F. Nori, "Colloquium: Unusual resonators: Plasmonics, metamaterials, and random media," Rev. Mod. Phys. **80**(4), 1201–1213 (2008).
[3] A. I. Smolyakov, E. Fourkal, S. I. Krasheninnikov, and N. Sternberg, "Resonant modes and resonant transmission in multi-layer structures," Prog. Electromagn. Res.-Pier **107**, 293–314 (2010).
[4] E. Fourkal, I. Velchev, C. M. Ma, and A. Smolyakov, "Evanescent wave interference and the total transparency of a warm high-density plasma slab," Phys. Plasmas **13**(9), 092113 (2006).
[5] N. Sternberg and A. Smolyakov, "Resonant transmission of electromagnetic waves in multilayer dense-plasma structures," IEEE Trans. Plasma Sci. **37**(7), 1251–1260 (2009).
[6] In some literature, general electomagnetic case is referred as a polariton, while the short wavelength electrostatic limit as a plasmon. Here, we refer to general electromagnetic case as the plasmon.
[7] O. M. Gradov and L. Stenflo, "Linear-theory of a cold bounded plasma," Phys. Rep.-Rev. Sect. Phys. Lett. **94**(3), 111–137 (1983).
[8] G. Kumar and V. K. Tripathi, "Anomalous absorption of surface plasma wave by particles adsorbed on metal surface," Appl. Phys. Lett. **91**(16), 161503 (2007).
[9] C. S. Liu and V. K. Tripathi, "Excitation of surface plasma waves over metallic surfaces by lasers and electron beams," IEEE Trans. Plasma Sci. **28**(2), 353–358 (2000).
[10] G. Kumar and V. K. Tripathi, "Excitation of a surface plasma wave over a plasma cylinder by a relativistic electron beam," Phys. Plasmas **15**(7), 073504 (2008).
[11] I. I. Smolyaninov and Y. J. Hung, "Enhanced transmission of light through a gold film due to excitation of standing surface-plasmon Bloch waves," Phys. Rev. B **75**(3), 033411 (2007).
[12] T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi, T. Thio, and P. A. Wolff, "Extraordinary optical transmission through sub-wavelength hole arrays," Nature **391**(6668), 667–669 (1998).
[13] Y. P. Bliokh, J. Felsteiner, and Y. Z. Slutsker, "Total absorption of an electromagnetic wave by an overdense plasma," Phys. Rev. Lett. **95**(16), 165003 (2005).
[14] Y. M. Aliev, A. V. Maximov, U. Kortshagen, H. Schluter, and A. Shivarova, "Modeling of microwave discharges in the presence of plasma resonances," Phys. Rev. E **51**(6), 6091–6103 (1995).
[15] O. Sakai, S. Yamaguchi, A. Bambina, A. Iwai, Y. Nakamura, Y. Tamayama, and S. Miyagi, "Plasma metamaterials as cloaking and nonlinear media," Plasma Phys. Controlled Fusion **59**(1), 014042 (2017).
[16] R. Dragila, B. Lutherdavies, and S. Vukovic, "High transparency of classically opaque metallic-films," Phys. Rev. Lett. **55**(10), 1117–1120 (1985).
[17] R. R. Ramazashvili, "Total transmission of electromagnetic-waves through slabs of plasmas and plasma-like media upon the excitation of surface-waves," JETP Lett. **43**(5), 298–301 (1986).
[18] A. Alu and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: Resonance, tunneling and transparency," IEEE Trans. Antennas Propag. **51**(10), 2558–2571 (2003).
[19] A. Alu, M. G. Silveirinha, and N. Engheta, "Transmission-line analysis of epsilon-near-zero-filled narrow channels," Phys. Rev. E **78**(1), 016604 (2008).
[20] P. Kumar, M. Kumar, and V. K. Tripathi, "Excitation of terahertz plasmons eigenmode of a parallel plane guiding system by an electron beam," J. Appl. Phys. **108**(12), 123303 (2010).
[21] L. Stenflo and M. Y. Yu, "Oscillons at a plasma surface," Phys. Plasmas **10**(3), 912–913 (2003).
[22] O. M. Gradov and L. Stenflo, "On the parametric transparency of a magnetized plasma slab," Phys. Lett. A **83**(6), 257–258 (1981).
[23] O. M. Gradov and L. Stenflo, "Anomalous transmission of electromagnetic energy through a plasma slab," Phys. Scr. **25**(5), 631–631 (1982).
[24] Y. P. Bliokh, "Plasmon mechanism of light transmission through a metal film or a plasma layer," Opt. Commun. **259**(2), 436–444 (2006).
[25] C. S. Liu, V. K. Tripathi, and R. Annou, "Resonant reduction in microwave reflectivity from an overdense plasma with the employment of a parallel metal grating," Phys. Plasmas **15**(6), 062103 (2008).
[26] A. V. Kats and A. Y. Nikitin, "Nonzeroth-order anomalous optical transparency in modulated metal films owing to excitation of surface plasmon polaritons: An analytic approach," JETP Lett. **79**(12), 625–631 (2004).
[27] A. M. Dykhne, A. K. Sarychev, and V. M. Shalaev, "Resonant transmittance through metal films with fabricated and light-induced modulation," Phys. Rev. B **67**(19), 195402 (2003).
[28] I. S. Spevak, A. Y. Nikitin, E. V. Bezuglyi, A. Levchenko, and A. V. Kats, "Resonantly suppressed transmission and anomalously enhanced light absorption in periodically modulated ultrathin metal films," Phys. Rev. B **79**(16), 161406 (2009).
[29] S. A. Darmanyan, M. Neviere, and A. V. Zayats, "Analytical theory of optical transmission through periodically structured metal films via tunnel-coupled surface polariton modes," Phys. Rev. B **70**(7), 075103 (2004).
[30] S. A. Darmanyan and A. V. Zayats, "Light tunneling via resonant surface plasmon polariton states and the enhanced transmission of periodically nanostructured metal films: An analytical study," Phys. Rev. B **67**(3), 035424 (2003).
[31] Y. P. Bliokh, Y. L. Brodsky, K. B. Chashka, J. Felsteiner, and Y. Z. Slutsker, "Broad-band polarization-independent absorption of electromagnetic waves by an overdense plasma," Phys. Plasmas **17**(8), 083302 (2010).
[32] Y. Wang, J. X. Cao, G. Wang, L. Wang, Y. Zhu, and T. Y. Niu, "Total absorption of electromagnetic radiation in overdense plasma," Phys. Plasmas **13**(7), 073301 (2006).
[33] A. Yariv, "Universal relations for coupling of optical power between microresonators and dielectric waveguides," Electron. Lett. **36**(4), 321–322 (2000).
[34] W. Frias, A. Smolyakov, and A. Hirose, "Non-local energy transport in tunneling and plasmonic structures," Opt. Express **19**(16), 15281–15296 (2011).
[35] O. M. Gradov and L. Stenflo, "Solitary leaking waves in a plasma slab," Contrib. Plasma Phys. **25**(6), 593–596 (1985).
[36] N. Sternberg and A. I. Smolyakov, "Resonant transmission through dense plasmas via amplification of evanescent mode," PIERS Online **5**(8), 781–785 (2009).
[37] R. J. Vidmar, "On the use of atmospheric-pressure plasmas as electromagnetic reflectors and absorbers," IEEE Trans. Plasma Sci. **18**(4), 733–741 (1990).
[38] M. Kim, M. Keidar, and I. D. Boyd, "Electrostatic manipulation of a hypersonic plasma layer: Images of the two-dimensional sheath," IEEE Trans. Plasma Sci. **36**(4), 1198–1199 (2008).
[39] Y. M. Aliev, A. A. Frolov, G. Brodin, and L. Stenflo, "Total backward reflection of electromagnetic-radiation due to resonant excitation of surface-waves," Phys. Rev. E **47**(6), 4623–4624 (1993).

[40]O. Sakai, J. Maeda, T. Shimomura, and K. Urabe, "Functional composites of plasmas and metamaterials: Flexible waveguides, and variable attenuators with controllable phase shift," Phys. Plasmas **20**(7), 073506 (2013).
[41]N. S. Erokhin and V. E. Zakharov, "On transillumination of wave barriers for electromagnetic radiation in an inhomogeneous plasma," Doklady Phys. **52**(9), 485–487 (2007).
[42]N. S. Erokhin, V. E. Zakharov, N. N. Zolnikova, and L. A. Mikhailovskaya, "Exactly solvable model of resonance tunneling of an electromagnetic wave in plasma containing short-scale inhomogeneities," Plasma Phys. Rep. **41**(2), 182–187 (2015).
[43]A. B. Shvartsburg, M. Marklund, G. Brodin, and L. Stenflo, "Superluminal tunneling of microwaves in smoothly varying transmission lines," Phys. Rev. E **78**(1), 016601 (2008).
[44]R. Lee, B. Wang, and M. A. Cappelli, "Plasma modification of spoof plasmon propagation along metamaterial-air interfaces," Appl. Phys. Lett. **111**(26), 261105 (2017).