Resonant Transmission through Dense Plasmas via Amplification of Evanescent Mode

N. Sternberg[1] **and A. I. Smolyakov**[2]

[1]Department of Mathematics and Computer Science, Clark University, Worcester, MA, USA
[2]Department of Physics and Engineering Physics, University of Saskatchewan, Saskatoon, Canada

Abstract— A study of electromagnetic wave propagation in a dense plasma layer when the plasma frequency is higher than the wave frequency is presented. Under such conditions, the wave amplitude is usually exponentially attenuated due to collisionless skin effect. It is shown that absolute, 100%, transparency can be achieved through resonant excitation of evanescent modes by a diffraction grating placed into vacuum preceding the plasma layer.

1. INTRODUCTION

The behavior of surface modes is the subject of much current research. Surface modes are (exponentially) localized eigen-modes which exist at the interfaces of two regions with opposite signs of the dielectric constants. A closely related field is the study of propagation of the electromagnetic radiation in metamaterials, which are materials with negative dielectric permittivity ϵ and negative permeability μ. The increased interest in surface modes has been driven by the tremendous potential to guide and manipulate electromagnetic radiation at the subwavelength scales below the diffraction limit in composite devices involving metallic nanostructures. The electromagnetic waves in these devices take the form of surface waves (Surface Plasmons) supported by free electrons in the metal and localized at the metal-dielectric interfaces [1–5].

Inside a dense region, electromagnetic wave energy is carried by evanescent modes. The general solution inside a medium of negative permittivity is a sum of two exponential functions, one that decays and the other one that grows with the distance. The corresponding component of the time averaged Poynting vector may become finite when the growing and decaying evanescent modes are superimposed with a finite phase shift. The condition of the absolute transparency is equivalent to the resonant condition for the excitation of the surface plasma mode. At resonance, the Poynting flux inside the slab becomes equal to that of the incident radiation, and the opaque plasma slab becomes absolutely transparent [6].

One can find in the literature studies of various media that allow signal transmission via amplification of evanescent waves through surface modes. General conditions for resonant signal transmission via a two-layer structure were studied in [7, 8], and a three-layer structure was considered in [9, 10]. In [8] a two-layer structure consisting of a layer of rarified plasma with $0 < \varepsilon_1 < 1$ of width a_1 and a layer of dense plasma with $\varepsilon_2 < 0$ of width a_2 was considered, and it was shown that total signal transmission is achieved if the effective dielectric constant of the total structure is zero, i.e., $\bar{\varepsilon} \equiv \varepsilon_1 a_1 + \varepsilon_2 a_2 = 0$. The three layer structure considered in [10] consisted of a dense plasma layer nested between two layers of rarified plasma. It was shown in [10] that in contrast to a two-layer structure, resonant signal transmission for such three-layer model can be achieved by choosing the width of each boundary layer comparable in size with the width of the dense plasma. Furthermore, the bandwidth of the transmitted wave is much larger than in the two-layer structure. In [11, 12], resonant transmission of an electromagnetic wave through a dense plasma layer was achieved by placing a diffraction grating into the vacuum layer on each side of the plasma.

In the present paper, we study resonant signal transmission through surface modes and possible applications to the problem of communication interruption with an aircraft surrounded by a dense plasma layer (the black-out problem) [13–15]. In that application, the model with two diffraction gratings is not useful, because one cannot place a diffraction grating on the exterior side of the plasma layer. The question is weather resonant transmission can be achieved by using only one diffraction grating (along the side of the aircraft) which precedes the dense plasma layer. This paper is an attempt to answer this question. Using a model for the diffraction grating similar to the one in [11, 12] for plasma parameters relevant to hypersonic flights [13], we show that by choosing appropriate parameters for the diffraction grating, resonant transmission of electromagnetic waves through a dense plasma layer is possible even if only one diffraction grating is being used.

2. GENERAL EQUATIONS AND BOUNDARY CONDITIONS FOR THE DIFFRACTION GRATING

We consider the propagation of electromagnetic radiation through a multi-layer structure as depicted in Fig. 1. A dense plasma layer is nested between two semi-infinite vacuum (air) layers. An electromagnetic wave is incident from a semi-infinite vacuum (air) region on the left. The transmitted wave propagates into a semi-infinite vacuum (air) region on the right. In general, there are incident and reflected waves on the left, but there is no reflected wave on the right. When a dense plasma layer is present within the structure, the reflection coefficient is large, and most of the radiation is reflected due to the skin effect screening. The question is whether by placing a diffraction grating into the vacuum region to the left of the dense plasma (at $z = 0$) it is possible to create conditions when the reflection is low and most of the radiation is transmitted through the structure.

We consider a p-polarized normal incident wave: $\mathbf{H} = (H_x, 0, 0)$, and $\mathbf{E} = (0, E_y, E_z)$. The wave equation for a non-homogenous (in y and in z) medium is then

$$\varepsilon \frac{\partial}{\partial y}\left(\frac{1}{\varepsilon}\frac{\partial H_x}{\partial y}\right) + \varepsilon \frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial H_x}{\partial z}\right) + \frac{\omega^2}{c^2}\varepsilon H_x = 0 \tag{1}$$

The dependence in the y direction is always periodic $H_x \sim \exp(ik_y y)$, with k_y real. We assume that the permittivity ε is defined by a step-function such that $\varepsilon = 1$ in the vacuum layer and $\varepsilon = \varepsilon_p < 0$ in the dense plasma layer. At the plasma-vacuum interfaces one finds from (1) the boundary conditions

$$[H_x]_-^+ = 0 \qquad \text{and} \qquad \left[\frac{1}{\varepsilon}\frac{dH_x}{dz}\right]_-^+ = 0 \tag{2}$$

where, "+" and "−" indicate the corresponding limits from the right and left at the plasma-vacuum interface.

To model the diffraction grating interface, we use an equation similar to the one in [11, 12]:

$$\frac{d^2}{dz^2}H_x - k_y^2 H_x + \frac{\omega^2}{c^2}\varepsilon H_x + \frac{\omega^2}{c^2}h_g\varepsilon_g\alpha\cos(qy)\,\delta(z)\,H_x = 0 \tag{3}$$

where q is a wave vector of the grating, α is a modulation parameter, h_g is the grating thickness, and ϵ_g is the grating dielectric constant. Note that due to the delta function, Equation (3) is used only in the neighborhood of the grating and produces the following boundary conditions:

$$[H_x]_-^+ = 0 \qquad \text{and} \qquad \left[\frac{dH_x}{dz}\right]_-^+ = -\frac{k_g\alpha}{2}\left(e^{iqy} + e^{-iqy}\right) H_x|_{z=0} \tag{4}$$

where $k_g = \varepsilon_g h_g \omega^2/c^2$, and "+" and "−" indicate the corresponding limits from the right and left at the diffraction grating.

The role of the diffraction grating is to generate the sideband harmonics so that the total solution (neglecting higher harmonics) is of the form

$$H_x = H_0 e^{ik_y y} + H_+ e^{i(k_y+q)y} + H_- e^{i(k_y-q)y} \tag{5}$$

where H_0 is the amplitude of the principal harmonics (the same as the incident wave), and H_+ and H_- are the amplitude of the sidebands. We will consider here only a normal incidence wave, thus, $H_+ = H_-$, and it is sufficient to consider only H_0 and H_+. Note that H_0 and H_+ are corresponding solutions of (1) and, in each layer, they can be represented as

$$H_0 = A\left(\exp(\gamma z) + \Gamma\exp(-\gamma z)\right) \qquad \text{and} \qquad H_+ = A^+\left(\exp(\gamma^+ z) + \Gamma^+\exp(-\gamma^+ z)\right) \tag{6}$$

for appropriate γ and γ^+ and corresponding A, A^+, Γ, Γ^+. In the vacuum regions, the incidence wave is propagating and $\gamma = ik_0$, $k_0 = \omega/c$, while the sideband are evanescent with $\gamma^+ = -\gamma_v^+$, where $\gamma_v^+ = \sqrt{q^2 - k_0^2}$, $q > k_0$. In the dense plasma layer, the incidence wave is evanescent with $\gamma = -\gamma_p$, with $\gamma_p = \sqrt{-\varepsilon_p}\omega/c$, and so are the sidebands with $\gamma^+ = -\gamma_p^+$, with $\gamma_p^+ = \sqrt{q^2 - \varepsilon_p\omega^2/c^2}$.

3. WAVE IMPEDANCE AND TRANSPARENCY

To study our model, we will use the impedance matching technique described in [10]. The local wave impedance is defined by

$$Z = -\frac{E_y}{H_x} = -\frac{i}{\omega\varepsilon_0\varepsilon}\frac{1}{H}\frac{\partial H_x}{\partial z} \tag{7}$$

In the semi-infinite vacuum region $z < 0$, the incidence wave is propagating and its impedance, according to (6) and (7) is

$$Z_3(z) = Z_0\frac{(\exp(ik_0z) - \Gamma_3\exp(-ik_0z))}{(\exp(ik_0z) + \Gamma_3\exp(-ik_0z))} \tag{8}$$

where $Z_0 = k_0/(\omega\varepsilon_0)$ is the vacuum characteristic impedance. The sideband is evanescent with $\Gamma_3^+ = 0$ and its impedance is

$$Z_3^+(z) = -\frac{i\gamma_v^+}{\omega\varepsilon_0} \tag{9}$$

In the vacuum region, $0 < z < a$, the corresponding impedances for the incidence wave and the sideband are

$$Z_2(z) = Z_0\frac{\exp(ik_0z) - \Gamma_2\exp(-ik_0z)}{\exp(ik_0z) + \Gamma_2\exp(-ik_0z)} \quad \text{and} \quad Z_2^+(z) = Z_0^+\frac{\exp(-\gamma_v^+z) - \Gamma_2^+\exp(\gamma_v^+z)}{\exp(-\gamma_v^+z) + \Gamma_2^+\exp(\gamma_v^+z)} \tag{10}$$

where $Z_0^+ = i\gamma_v^+/(\omega\varepsilon_0)$ is the characteristic impedance of the vacuum region for the sidebands. In the plasma region, $a < z < d = a + l$, the corresponding impedances are

$$Z_p(z) = Z_{ch}\frac{\exp(-\gamma_pz) - \Gamma_p\exp(\gamma_pz)}{\exp(-\gamma_pz) + \Gamma_p\exp(\gamma_pz)} \quad \text{and} \quad Z_p^+(z) = Z_{ch}^+\frac{\exp(-\gamma_p^+z) - \Gamma_p^+\exp(\gamma_p^+z)}{\exp(-\gamma_p^+z) + \Gamma_p^+\exp(\gamma_p^+z)} \tag{11}$$

where $Z_{ch} = i\gamma_p/(\omega\varepsilon_0\varepsilon_p)$ and $Z_{ch}^+ = i\gamma_p^+/(\omega\varepsilon_0\varepsilon_p)$. In the last semi-infinite vacuum region, $z > d = a + l$, the impedances of the transmitted propagating wave and of the corresponding sideband are

$$Z_1(z) = Z_0 \qquad \text{and} \qquad Z_1^+(z) = Z_0^+ \tag{12}$$

The quantity of primary interest is the reflection coefficient which is determined by the expression

$$\Gamma_3 = \frac{Z_0 - Z_3(0)}{Z_0 + Z_3(0)} \tag{13}$$

If $\Gamma = 0$ then there is a 100% wave transmission. Thus, the transparency condition is

$$Z_3(0) = Z_0 \tag{14}$$

4. MATCHING CONDITIONS AND RESONANCE

At the plasma-vacuum boundaries, the impedance is continuous. Hence,

$$Z_p(d) = Z_1(d) = Z_0 \qquad Z_p^+(d) = Z_1^+(d) = Z_0^+ \qquad Z_p(a) = Z_2(a) \qquad Z_p^+(a) = Z_2^+(a) \tag{15}$$

The matching condition at the diffraction grating follows from (4) and yields:

$$Z_3(0) = Z_2(0) - \frac{k_g^2\alpha^2}{2\omega^2\epsilon_0^2(Z_3^+(0) - Z_2^+(0))} \tag{16}$$

According to (14), resonance is achieved when

$$Z_3(0) = Z_0 = Re(Z_2(0)) \tag{17}$$

and

$$iIm(Z_2(0)) - \frac{\alpha^2k_g^2}{2\omega^2\epsilon_0^2(Z_3^+(0) - Z_2^+(0))} = 0 \tag{18}$$

Equation (17) holds if

$$L_p^2 - \frac{L_v^2 L_p^2}{\epsilon_p} - 2\frac{L_v L_p}{\sqrt{-\epsilon_p}} = 0 \tag{19}$$

where $L_p = \tanh(\gamma_p l)$, $L_v = \tan(k_0 a)$. From Equation (19), we find L_v, which yields the width of the vacuum layer a needed for resonant transmission to take place:

$$a = \frac{1}{k_0}\tan^{-1}(L_v) \tag{20}$$

Note that, using the equations from the previous section, we find

$$Z_3^+(0) - Z_2^+(0) = -Z_0^+ \frac{(1+L_v^+)(2+L_p^+(K+1/K))}{1+L_p^+/K + L_v^+ + L_p^+ L_v^+ K} \tag{21}$$

where $K = Z_{ch}^+/Z_0^+$, $L_v^+ = \tanh(\gamma_v^+ a)$ and $L_p^+ = \tanh(\gamma_p^+ l)$. On the other hand, form (18)

$$Z_3^+(0) - Z_2^+(0) = -i\frac{\alpha^2 k_g^2}{2\omega^2\epsilon_0^2 \mathrm{Im}(Z_2^0)} \tag{22}$$

and we obtain the dispersion relation

$$\gamma_v^+(1+L_v^+)\frac{2+L_p^+(K+1/K)}{1+L_p^+/K+L_v^+ + L_p^+ L_v^+ K} = \frac{\alpha^2 k_g^2}{2\omega\epsilon_0 \mathrm{Im}(Z_2^0)} \tag{23}$$

Our computations have shown that for an appropriate choice of parameters k_g and α, resonant transmission of electromagnetic waves is possible. In fact, if k_g and α are chosen so that $q \in (21.27, 21.284) \cup (21.39, \infty)$, the right-hand side of (23) is positive, and resonant transmission will always take place. This is illustrated in Fig. 2 which shows the dependence of the reflection coefficient on the diffraction wave vector q for $\alpha^2 k_g^2 = 4000$, $l = 0.02\,\mathrm{m}$ and $\epsilon_p = -35$. Resonant transmission takes place at about $q = 21.6$.

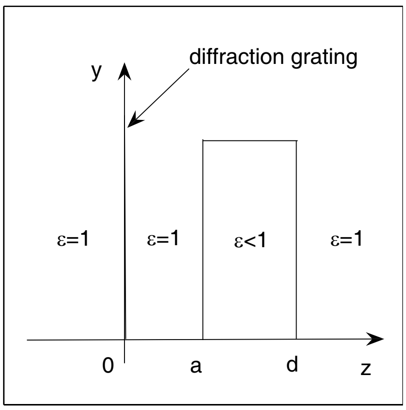

Figure 1: Plasma layer and a diffraction grating.

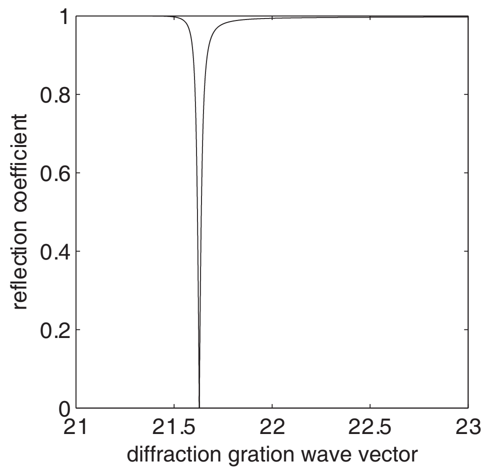

Figure 2: Reflection coecient versus diffraction wave vector.

5. CONCLUSION

Anomalously high transmission of electromagnetic radiation through a dense plasmas layer has been observed in experiments and numerical simulations. In those observations, electromagnetic wave transmission resulted from resonant amplification of evanescent waves by surface modes at the plasma boundary. In particular, in [11, 12], resonant transmission through dense plasma was achieved by placing the plasma layer between two diffraction gratings. In some applications, however, it is not possible to create such symmetric configuration. For example, to investigate the communication black-out problem during hypersonic flight, one could put the diffraction grating along the side of the aircraft, but not at the exterior side of the plasma layer. It is therefore important to understand whether resonant transmission can be achieved by placing the diffraction grating only on one side of the plasma layer. This is the configuration we have considered in the present paper. Using an equation for the diffraction grating similar to the one in [11, 12], we were able to demonstrate that for an appropriate choice of parameters, resonant transmission of electromagnetic waves through a dense plasma layer is possible even if only one diffraction grating is used. In our investigation, we neglected the effects of dissipation caused by electron-atom collisions and/or by electron thermal motion [16, 17]. Those effects could be especially detrimental for the narrow resonances situation. On the other hand, electron thermal motion effects could lead to the appearance of new resonant modes and additional transparency regimes [8]. All those questions go beyond the scope of this paper and are left for future studies.

ACKNOWLEDGMENT

This work was supported in part by AFOSR Award #FA9550-07-1-0415 and NSERC Canada. The authors wish to thank Yu. Bliokh and V. Godyak for helpful discussions.

REFERENCES

1. Alu, A. and N. Engheta, *IEEE Antennas and Wireless Propagation Letters*, Vol. 4, 417–420, 2005.
2. Hooper, I. R., T. W. Preist, and J. R. Sambles, "Making tunnel barriers (including metals) transparent," *Physical Review Letters*, Vol. 97, No. 5, 053902, 2006.
3. Pendry, J. B., "Negative refraction makes a perfect lens," *Physical Review Letters*, Vol. 85, No. 18, 3966–3969, 2000.
4. Tomita, S., T. Yokoyama, H. Yanagi, B. Wood, J. B. Pendry, M. Fujii, and S. Hayashi, "Resonant photon tunneling via surface plasmon polaritons through one-dimensional metal-dielectric metamaterials," *Optics Express*, Vol. 16, No. 13, 9942–9950, 2008.
5. Wegener, M., G. Dolling, and S. Linden, "Plasmonics — backward waves moving forward," *Nature Materials*, Vol. 6, No. 7, 475–476, 2007.
6. Fourkal, E., I. Velchev, C.-M. Ma, and A. Smolyakov, "Resonant transparency of materials with negative permittivity," *Phys. Letters A*, Vol. 361, 277–282, 2007.
7. Alù, A. and N. Engheta, "Pairing an ϵ-negative slab with a μ-negative slab: Resonance, tunneling and transparency," *IEEE Trans. Antennas and Propagation*, Vol. 51, No. 10, 2558, 2003.
8. Fourkal, E., I. Velchev, C.-M. Ma, and A. Smolyakov, "Evanescent wave interference and total transparency of warm high-density plasma slab," *Phys. Plasmas*, Vol. 13, 092113, 2006.
9. Dragila, R., B. Lutherdavies, and S. Vukovic, "High transparency of classically opaque metallic films," *Phys. Rev. Letters*, Vol. 55, 1117, 1985.
10. Sternberg, N. and A. Smolyakov, "Resonant transmission of electromagnetic waves in multilayer dense plasma structures," *IEEE Trans. Plasma Sci.*, 2009.
11. Bliokh, Yu. P., J. Felsteiner, and Y. Slutsker, "Total absorbtion of electromagnetic waves by an overdense plasma," *Phys. Rev. Lettes*, Vol. 95, 165003, 2005.
12. Bliokh, Yu. P., "Plasmon mechanism of light transmission through a metal film or a plasma layer," *Optics Communic.*, Vol. 259, 436, 2006.
13. Josyula, E. and W. Bailey, "Governing equations for weakly ionized plasma flowfields of aerospace vehicles," *J. of Spacecraft and Rockets*, Vol. 60, No. 6, 845, 2003.
14. White, M. and S. Sherer, "High-order simulation of communication through a weakly ionized plasma for reentry vehicles," *44th AIAA Aerospace Science Meeting and Exhibit*, Reno, Nevada, January 2006.
15. Nazarenko, S. V., A. C. Newell, and V. E. Zakharov, "Communication through plasma sheath via Raman (three-wave) scattering process," *Phys. Plasmas*, Vol. 1, 9, 1994.
16. Lieberman, M. and V. Godyak, "From Fermi accelaration to collisionless discharge heating," *IEEE Trans. Plasma Sci.*, Vol. 26, No. 3, 955–986, 1998.
17. Godyak, V., "Hot plasma effects in gas discharge plasma," *Phys. Plasmas*, Vol. 12, 055501, 2005.