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Contributions to Plasma Physics

Influence of flow shear on localized Rayleigh–Taylor and resistive drift wave instabilities

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Abstract

The impact of velocity shear on the localized solutions of Rayleigh–Taylor (RT) and resistive drift wave (DW) instabilities has been investigated. Slab geometry is used, and the plasma density gradient is assumed to have a finite spatial structure. It demonstrates that the velocity shear has quite different effects on these instabilities: while it stabilizes RT instability and causes tilting of the eddies of equipotential contour, it has a very mild impact on the resistive DW instability and simply shifts the eddies with no tilting.

KEYWORDS

localized solution, Rayleigh–Taylor instability, resistive drift wave instability, velocity shear

1 | INTRODUCTION

The stability properties of eigenmode solutions of different gradient drift-driven plasma waves has been investigated extensively since the 1960s, and it has been widely accepted that plasma instability will be quenched by the shear of plasma flow when the gradient of the flow exceeds the growth rate without the velocity shear.[1–4] A generic role of velocity shear in the suppression of plasma instability and turbulent transport can be estimated by finding the effective growth rate and using a mixing length rule.[5] However, almost all the analytical studies were conducted under an assumption of constant logarithmic gradient of equilibrium plasma density, $\Lambda_n = -dln(n_0)/dx$, which may be far from the reality of the edge of magnetically confined plasmas. The choice of plasma density profile may significantly affect the conclusions. For example, recently, it was shown that a modification of Λ_n from constant can significantly change the dynamics of the drift wave (DW).[6] Therefore, in this article, we will revisit the impact of velocity shear on the eigenmode solutions of different plasma instabilities by considering plasma with non-constant Λ_n.

We will consider both the interchange modes and resistive DW, which are related to plasma polarization caused, respectively, by cross-field magnetic drift of charged particles (e.g., interchange modes, ion temperature gradient modes, and resistive ballooning modes) and by electron dynamic along the magnetic field (e.g., resistive DW). The interchange mode appears when the pressure gradient is inverted with respect to an effective gravitational field (due to the curvature of confining magnetic field), and to a large extent, it can be characterized by the Rayleigh–Taylor (RT) instability of fluids, which will be considered in this article.

In what follows, we will specify $\Lambda_n(x)$ as $\Lambda_n(x) = \hat{n}/\cos h^2(x/w)$, where both $\hat{n}$ and w are constants characterizing the widths of the variation of density and its gradient, respectively. Such a plasma density profile is similar to that in the pedestal region of tokamak[7] and to that in the linear machine with a limiter.[8] In order to focus on the effect of such a non-constant Λ_n, we will limit this article to the linear regime, and the shear flow will be externally applied as $V_0(x) = V_0' x$, where V_0' is a constant to avoid the Kelvin-Helmholtz effect. The effects of a self-generated plasma flow via Reynolds stress on the plasma instabilities and of velocity shear on the transport driven by the interchange modes and resistive DW are beyond the scope of this article and be will be studied in future publications.

2 VELOCITY SHEAR FOR RT/INTERCHANGE MODES

In this section, we consider the RT instability of a stratified fluid in a gravity field characterized by acceleration g, which could be considered a proxy of the interchange plasma instability with effective acceleration $g_{eff} = 2T_e/m_i R$ for cold ions.[9] To be consistent with plasma notations, we assume that the gravitational acceleration is in the x-direction, and the flow is in y-direction. Then, in the Boussinesq approximation, small perturbations of fluid velocity stream function $\widetilde{\psi}$, defining the perturbation of fluid velocity, $\widetilde{\mathbf{V}} = \mathbf{e}_z \times \nabla\widetilde{\psi}$, is described by[10]

$$\frac{d^2\psi}{dx^2} - k_y^2\psi + \frac{d\ln(n_0)}{dx}\frac{gk_y^2}{\widetilde{\omega}^2}\psi = 0, \tag{1}$$

where the Fourier expansion of $\widetilde{\psi}$ in time and y-coordinate, $\widetilde{\psi} = \psi(x)exp(-i\omega t + ik_y y)$, have been used, and $\widetilde{\omega} = \omega - k_y V_0(x)$. Notice that, in the interchange mode, the perturbation of electrostatic potential obeys an equation similar to Equation (1), except that it has an extra term proportional to $V_0' dln(n_0)/dx$, which does not change the basic features of the impact of velocity shear on RT instability.

Equation (1) is usually solved as the eigenfunction–eigenvalue problem with the boundary condition of $\psi(x \to \pm\infty) = 0$, where the role of the eigenvalues is played by ω. Without velocity shear, an analogy of Equation (1) to the Schrödinger equation for an electron in potential well $U \propto \text{-}cosh^{-2}(x/w)$ and energy $E \propto -k_y^2$ allows us to find the RT instability growth rate versus integer mode number $m (m = 0,1,2\ldots)$.[11] Here, we are interested in the impact of velocity shear on the fastest growing mode, the growth rate, and eigenfunction, which corresponds to $m = 0$ read

$$\gamma_{RT}^2(\hat{\kappa}) = -\omega^2 = \overline{\gamma}_{RT}^2\frac{|\hat{\kappa}|}{1+|\hat{\kappa}|}, \quad \text{and} \quad \psi_{RT} = \cos h^{-|\hat{\kappa}|}(x/w), \tag{2}$$

where $\overline{\gamma}_{RT}^2 = g\hat{n}$ characterizes the growth rate, and $\hat{\kappa} = wk_y$. As we can see, without velocity shear, the unstable RT wave is purely growing.

Including the velocity shear, we could solve Equation (1) by using a successive approximation method assuming V_0' is small and expanding both corrections of frequency $\delta\omega = \omega - i\gamma_{RT}$ and the eigenfunction $\delta\psi = \psi(x) - \psi_{RT}(x)$ in powers of V_0'. With no restrictions, we assume that $\psi_{RT}(x)$ is real and limit our analysis through the second order of V_0'. In fact, these corrections can be found by combing Equation (1) and its integral expression

$$Q(\psi) = \int d\xi \left[-\left|\frac{d\psi}{d\xi}\right|^2 - \hat{\kappa}^2|\psi|^2 - \frac{\overline{\gamma}_{RT}^2}{\cos h^2(\xi)}\frac{\hat{\kappa}^2}{\widetilde{\omega}^2}|\psi|^2 \right] = 0, \tag{3}$$

where $\xi = x/w$. It should be noted that Equation (1) has an asymptotic solution, $\psi(|\xi| \to \infty) \propto \exp(-|\hat{\kappa}\xi|)$, such that integrals in Equation (3) converge at $|\xi| \to \infty$. As a result, from Equations (1) and (3), we can easily obtain the first-order corrections

$$\frac{\delta\omega_1}{\gamma_{RT}} = 0 \text{ and } \delta\psi_1(\xi) = -i2\hat{\kappa}^3\frac{\overline{\gamma}_{RT}^2}{\gamma_{RT}^2}\frac{V_0'}{\gamma_{RT}}\psi_{RT}(\xi)\int_0^{\xi}\frac{d\xi'}{\psi_{RT}^2(\xi)}\int_{\xi'}^{\infty}\frac{\xi''\psi_{RT}^2(\xi'')}{\cos h^2(\xi'')}d\xi'', \tag{4}$$

where $\delta\psi_1$ is the asymmetric function, and the second-order correction to eigenvalue is

$$\frac{\delta\omega_2}{\gamma_{RT}}|_{\hat{\kappa}\to\infty} \approx -\frac{i}{8}\left(\frac{V_0'}{\overline{\gamma}_{RT}}\right)^2\hat{\kappa}^2, \text{ and } \frac{\delta\omega_2}{\gamma_{RT}}|_{\hat{\kappa}\to 0} \approx -i2\left(\frac{V_0'}{\overline{\gamma}_{RT}}\right)^2\hat{\kappa}. \tag{5}$$

Therefore, we see that the impact of velocity shear on RT instability could be characterized by the effective Richardson number, which we define as $R_i = g|d\ln(n_0)/dx|(V_0')^{-2} = (\overline{\gamma}_{RT}/V_0')^2$.[10] It follows that the mode with small wavelength will

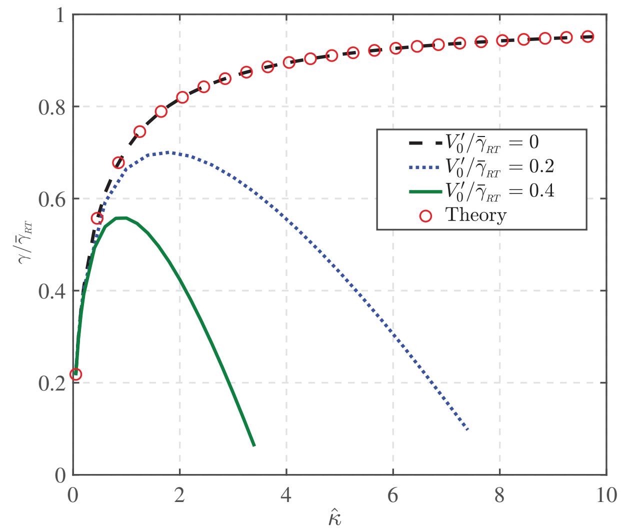

FIGURE 1 Growth rate of the most unstable Rayleigh–Taylor mode versus $\hat{\kappa}$. $V_0'/\bar{\gamma}_{RT} = 0$ (dashed black) and its analytical solution (red circle) from Equation (2), $V_0'/\bar{\gamma}_{RT} = 0.2$ (dotted blue) and $V_0'/\bar{\gamma}_{RT} = 0.4$ (solid green)

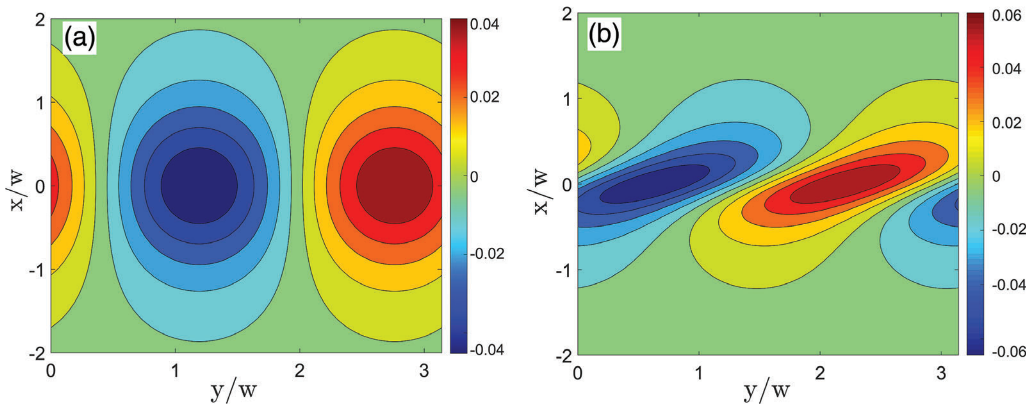

FIGURE 2 An impact of velocity shear on the eddies of electrostatic potential contour corresponding to the eigenfunctions in Figure 1 for $\hat{\kappa} = 2$. (a) $V_0'/\bar{\gamma}_{RT} = 0$ and (b) $V_0'/\bar{\gamma}_{RT} = 0.4$

first be stabilized by the velocity shear,[12] where the reduction of growth rate for large $\hat{\kappa}$ is mainly due to the correction of $\delta\psi_1$. On the other hand, for small $\hat{\kappa}$, there is virtually no stabilization of RT instability. The same trend of an impact of poloidal velocity shear on the growth rate of the resistive interchange modes[13] and ion temperature gradient (ITG) modes[14] has also been found.

From Equation (4), we find that the velocity shear leads to a small correction in the eigenfunction. However, taking into account that this correction is imaginary and asymmetric, it could cause a strong stretching of the eddies of equipotential contour. Such stretching of eddies can be accounted for as follows: from Equation (4), we know $\delta\psi_1(\xi)$ is negative (positive) for $\xi > 0$ ($\xi < 0$) such that the real part of $\tilde{\psi}$, $\tilde{\psi}_r = \psi_r(\xi)\cos(k_y y) - \psi_i(\xi)\sin(k_y y)$, will be tilted towards positive (x,y) given $-\psi_i(\xi)\sin(k_y y) > 0$.

To check these analyses, numerical solutions of Equation (1) have been found, as shown in Figure 1, for different $\hat{\kappa}$ and V_0'. Notice that both γ and V_0' are normalized by the parameter $\bar{\gamma}_{RT}$ such that γ tends to unity for the case without velocity shear as shown in Equation (2). It demonstrates that, for large values of $\hat{\kappa}$, the fastest growing modes are stabilized, whereas for small $\hat{\kappa}$, instability persists. We note that, in the simulations, the real part of ω for all cases is zero such that the RT modes are purely growing even in the presence of velocity shear, which agrees with our analysis ($\delta\omega_r = 0$ to the second order of V_0'). We also noted that the eigenfunctions for a marginally stable solution is similar to that in resistive interchange mode, which is strongly squeezed.[15] The eddies corresponding to the eigenfunctions for $\hat{\kappa} = 2$ are shown in Figure 2, demonstrating that velocity shear causes strong tilting of eddies.

However, we should keep in mind that the slab model of the RT instability does not account for many important effects in practical tokamak, including both poloidal and toroidal periodicities of tokamak geometry, centrifugal and Coriolis

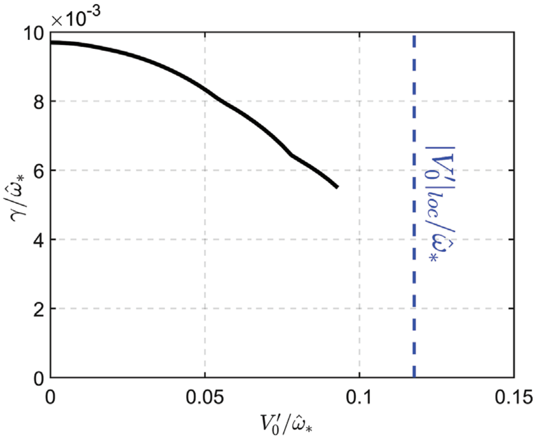

FIGURE 3 The growth rate of dissipative DW instability versus $V_0{}'$ for $\rho_s k_y = 0.3$, $w/\rho_s = 20$, and $\nu_\parallel/\widehat{\omega}_* = 10$

forces, and electromagnetic and other effects. As a result, theoretical assessment of the role of plasma flows, both poloidal and toroidal, on different instabilities becomes much more complex. Nonetheless, it appears that plasma flow shear is very efficient in reducing the growth rate of RT/interchange mode with small wavelength.

3 | VELOCITY SHEAR FOR RESISTIVE DW

In this section, we consider resistive DW occurring in a plasma with non-uniform density. If we ignore the electron temperature perturbation, the dispersion equation for the perturbed electrostatic potential reads[16]

$$\rho_s^2 \frac{d^2\phi}{dx^2} - \left[1 + \rho_s^2 k_y^2 - \frac{\omega_*}{\widetilde{\omega}} + i\frac{\widetilde{\omega} - \omega_*}{\nu_\parallel}\right]\phi = 0, \tag{6}$$

where $\omega_* = k_y \rho_s^2 \Omega_i \Lambda_n$, $\Omega_i = eB_0/m_i c$, $\rho_s = T_e M c^2/e^2 B_0^2$, and $\nu_\parallel = k_z^2 T_e/m\nu_{ei}$ with ν_{ei} being the electron–ion collision frequency. Equation (6) is obtained from cold ion fluid equation subject to $\mathbf{E}\times\mathbf{B}$ and polarization drifts, as well as quasi-neutrality condition, where the electrostatic potential perturbation, $\widetilde{\phi}$, takes the form of $e\widetilde{\phi}/T_e = \phi(x)exp(-i\omega t + ik_y y + ik_z z)$ for eigenmode solution. Here, we considered the case $V_0' \ll \Omega_i$ and $k_z \ll k_y$, the latter of which implies that the variation of the perturbations along the main equilibrium magnetic field has much larger scale than that in the poloidal direction. In fact, an equation similar to Equation (6) can be obtained for DW with Landau damping effect by assuming $\omega \ll k_z V_{Te}$.

Unlike RT instability, where the modes are purely growing, resistive DW instability has complex frequencies ω with the imaginary part (the growth rate denoted by γ) usually being much smaller than the real part. In the absence of velocity shear and for a wave-like solution, we have $\omega_r = \omega_*/(1 + \rho_s^2 k_\perp^2)$ and $\gamma \approx \omega_*^2 \rho_s^2 k_\perp^2/\nu_\parallel (1 + \rho_s^2 k_\perp^2)^3$, respectively, for constant ω_* and $\nu_\parallel \gg \omega_*$. Therefore, the mode with largest growth rate corresponds to $\rho_s^2 k_\perp^2 \approx 1/2$. As a result, the dependence of growth rate, computed from the Schrödinger equation, on the mode number m is more complicated compared with RT for the same $\Lambda_n(x)$.

Recalling the profile of $\Lambda_n(x)$, we have $\omega_* = \widehat{\omega}_* \cos h^{-2}(x/w)$, where $\widehat{\omega}_* = \rho_s k_y C_s \widehat{n}$ characterizes the DW frequency. Then, the considered problem is characterized by the dimensionless parameters of $\nu_\parallel/\widehat{\omega}_*$, $V_0'/\widehat{\omega}_*$, w/ρ_s and $\rho_s k_y$. In this article, we solve Equation (6) numerically, where the largest growth rate versus V_0' for $\rho_s k_y = 0.3$, $w/\rho_s = 20$, and $\nu_\parallel/\widehat{\omega}_* = 10$ has been illustrated in Figure 3. As can be seen, unlike the RT instability, there is quite a mild impact of V_0' on the growth rate of dissipative DW instability even though $V_0' \gg \gamma$. Considering that $V_0(x)$ is invariant under $x \to -x$, $V_0' \to -V_0'$, and all other terms in Equation (6) are even in x, the velocity shear effects on ω are independent of the sign of $V_0{}'$, and from simulations, we find that the real (imaginary) part of ω quadratically increases (decreases) with V_0'.

We note that, at a rather large velocity shear, localized solutions of Equation (6) are absent. This effect can be interpreted within eikonal approximation, where the wave packet could be considered an effective "particle", the dynamic of which is described by the "Hamiltonian" $\omega(k_x, x) = \omega_*(x)(1 + \rho_s^2 k_y^2 + \rho_s^2 k_x^2)^{-1} + V_0' k_y x$ and canonical variables x and k_x.[17] To obtain a localized solution of the wave packet, the motion of the particle should be bounded by two turning points corresponding to $k_x = 0$, which indicates that $\partial\omega(k_x = 0, x)/\partial x$ should have at least one root. As a result, for our case

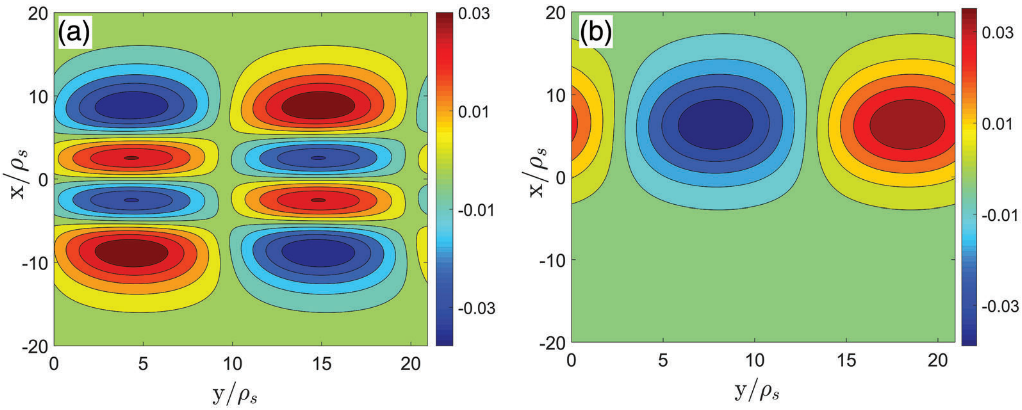

FIGURE 4 An impact of velocity shear on the eddies of electrostatic potential contour corresponding to the eigenfunctions in Figure 3 for $V_0' = 0$ (a) and $V_0'/\hat{\omega}_* = 0.08$ (b)

$\omega_* = \hat{\omega}_* \cos h^{-2}(x/w)$, it is easy to show that this is only possible for

$$|V_0'| < |V_0'|_{loc} = \frac{4}{3^{3/2}} \frac{\hat{\omega}_*}{wk_y(1+\rho_s^2 k_y^2)}. \tag{7}$$

Beyond this limit, no localized solution exists, which is in good agreement with the result of numerical simulations (e.g., see Figure 3). From this approach, we can also see that, if ω_* is constant, no localized solution is possible for Equation (6).

The manifestation of the different impacts of velocity shear on resistive DW and RT could also be seen from corresponding eigenfunctions, where the eddies related to resistive DW instability are shown in Figure 4. It can be seen that, unlike RT instability, the eddies corresponding to resistive DW are not tilted by the velocity shear, although the centre of eigenfunction has been shifted towards positive $V_0(x)$. We note that the mode number for the most unstable mode has been reduced due to the presence of velocity shear. However, such a number depends both on k_y and w, where the most unstable mode corresponds to small mode number for relatively large k_y and small w.

The fact that the eddies of DW are not tilted by the velocity shear can be explained as follows: For the case of $v_\| \gg \hat{\omega}_*$, the last term in Equation (6) has little impact on the eigenfunction but simply drives the instability. As a result, if we assume that the eigenfunction of Equation (6) without the driving term is real for $V_0' = 0$, then the imaginary part of the eigenfunction is in the order of or even smaller than $\hat{\omega}_*/v_\|$ and thus is negligible even for $V_0' \neq 0$. As a result, the velocity shear will not tilt the eddies of DW but simply change the structure of eigenfunctions (e.g., shift the eddies towards positive $V_0(x)$, and we note that the shift of eigenfunction in the radial direction due to the impact of velocity shear was also observed when considering constant Λ_n but with magnetic shear[18]).

4 | DISCUSSIONS AND CONCLUSIONS

This article focused on the impact of velocity shear on the localized solutions of resistive DW and RT instabilities, which are related to plasma polarization caused, respectively, by electron dynamic along the magnetic field and cross-field magnetic drift of charged particles, by considering Λ_n to be $\Lambda_n(x) \propto \cosh^{-2}(x/w)$. Such treatment of Λ_n introduces an effective "potential well" and thus allows us to find the localized solutions. It demonstrates that the velocity shear has quite different effects on these plasma instabilities. First, it causes the stabilization of a short wavelength mode of RT instability by slightly changing the eigenfunctions, although the instability of the mode with long wavelength will still persist. However, it has a very mild impact on the growth rate of resistive DW. Second, the eddies of electrostatic equipotential contour for RT are strongly tilted due to the impact of the velocity shear, but it is not the case for the DW, where the eddies are simply shifted along the radial direction without tilting.

Here, we provide a possible explanation for the different impacts of velocity shear on these two plasma instabilities. First, the RT instability is associated with the dynamics of density protrusions, and similarly, the interchange plasma instability is pertinent to the dynamics of plasma density perturbations with embedded electric charges originating from almost "irreversible" cross-field magnetic drift effects. Spatial distribution of these charges produces $\mathbf{E}\times\mathbf{B}$ drifts, which in turn drives the instability. Analogous processes are relevant to all plasma instabilities driven by magnetic drift effects (e.g., toroidal ITG and ballooning modes). Therefore, advection of plasma density perturbations with embedded electric charges by shear plasma flow inevitably alter such instabilities. The case of DW situation is very different. In this case, the electric field and related $\mathbf{E}\times\mathbf{B}$ drifts are due to a largely "reversible" response of fast parallel electron dynamics on plasma density perturbations. Even though advection of plasma density perturbations by velocity shear changed the "landscape" of density perturbations, the distribution of electric charges virtually has no "memory", and therefore, velocity shear has very mild impact on the growth rate of DW instabilities.

We note that the choice of $\Lambda_n \propto \cos h^{-2}(x/w)$ is somewhat idealized. However, it largely demonstrates the features of the effect of velocity shear on plasma wave instabilities when the gradient of plasma density has a finite radial scale similar to that in the pedestal region. Therefore, we believe it can provide considerable physical insight on the influence of velocity shear on different plasma instabilities.

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